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CERTAIN SUBCLASS OF M-FOLD SYMMETRIC BI-UNIVALENT FUNCTIONS' BOUNDS FOR INITIAL COEFFICIENTS

R. S. Dubey, N. Shekhawat, P. Vijaywargiya and K. Modi

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Abstract In this paper, we introduced a new subclass $S^{d,l}_{\sum_m}(\gamma,\lambda)$ of bi-univalent functions with m-fold symmetry in the open unit disk Δ. Further, we investigated a_n to obtain bounds for the initial coefficients of functions which belongs to this subclass.

1 Introduction

An analytic function ξ in domain D of the extended complex plane C is univalent, if $\xi(z_1) \neq \xi(z_2)$ whenever $z_1 \neq z_2$, z_1, $z_2 \in D$. Suppose A be the class containing a function $\xi(z)$ which is analytic in the open unit disk $\Delta = \{z : z \in C,\ and,\ |z| < 1\}$ and satisfies the following normalization conditions:

$$\xi\,(0) = \xi'\,(0) - 1 = 0,$$

and is given by:

$$\xi\,(z) = z + \Sigma_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

Suppose S is the subclass of A, containing functions that possess the property of univalence in the open unit disk Δ. According to the Koebe's theorem (see [1]), every univalent function has its inverse.

Suppose an analytic function $\xi \in$ A with the property that ξ and ξ^{-1} both are univalent in Δ, then ξ is called bi-univalent in Δ.

The class of bi-univalent functions is denoted by $\sum$, which is defined in equation (1.1). This class of bi-univalent functions was investigated by Lewin [2]. He proved $|a_2| < 1.51$ for bi-univalent functions. In this progress, Brannan and Clunie [3] gave conjecture $|a_2| \leq \sqrt{2}$. And it is seen that in recent years, many researchers showed their interest in investigating subclass of bi-univalent functions and obtained results on the initial coefficient bounds (see [4,5,6,7,8,9]). If a rotation of domain M about the origin through an angle $\frac{2\pi}{m}$ carries M on itself, then it is called the *m*-fold symmetric domain. Thus, an analytic function $\xi(z)$ in the open unit disk Δ is called *m*-fold symmetric for $m \in N$, if it satisfies the given below equation:

$$\xi\left(e^{2\pi i/m} z\right) = e^{2\pi i/m}\xi\,(z)\,.$$

Let us define the class of *m*-fold symmetric univalent functions by S_m. A function $\xi \in S_m$ is given as follows:

$$\xi\,(z) = z + \Sigma_{k=1}^{\infty} a_{mk+1} z^{mk+1}, \quad (z \in \Delta, m \in N)\,. \tag{1.2}$$

Every function $\xi \in S$ has the function, $d(z) = \sqrt[m]{\xi(z^m)}$, $(z \in \Delta,\ m \in N)$, which is univalent along with the property of mapping the unit disk Δ into a region with m-fold symmetry.

Initially, Srivastava et al. [10] described m-fold symmetric bi-univalent functions and have shown that for each $m \in N$, there is a function $\xi \in \sum$, that gives the m-fold symmetric bi-univalent function. Also, they gave the series expansion for ξ^{-1}, (ξ is given by equation (1.2)), which is as follows:

$$\begin{aligned}\eta\left(w\right) &= \xi^{-1}\left(w\right) = w - a_{m+1}w^{m+1} + \left[\left(m+1\right)a_{m+1}^{2} - a_{2m+1}\right]w^{2m+1} \\ &- \left[\tfrac{1}{2}\left(m+1\right)\left(3m+2\right)a^{3}{}_{m+1} - \left(3m+2\right)a_{m+1}a_{2m+1} + a_{3m+1}\right]w^{3m+1} + \ldots,\end{aligned} \tag{1.3}$$

where $\eta = \xi^{-1}$. The subclass of *m*-fold symmetric bi-univalent functions in the open unit disk Δ is given by $\sum_m$.

We study the recent works of mathematicians such as A. Zireh et al. [11], H. M. Srivastava et al. [12,13], and S. S. Eker [14], etc., to give the results of our paper.
In this research work, we introduce a new subclass $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$ containing bi-univalent functions with the property that ξ and ξ^{-1} are *m*-fold symmetric. We also try to provide results on initial coefficient bounds.The purpose of this paper is to provide a formula of the upper bounds for initial coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ of the functions in this new subclass $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$. Our results are motivated by the latest works of the researchers.

Definition 1.1: Suppose the functions $d,\ l:\ \Delta \to C$ are analytic and

$$d\left(z\right) = 1 + d_m z^m + d_{2m}z^{2m} + d_{3m}z^{3m} + \ldots, \tag{1.4}$$

$$l\left(w\right) = 1 + l_m w^m + l_{2m}w^{2m} + l_{3m}w^{3m} + \ldots, \tag{1.5}$$

such that $\min\left\{\mathrm{Re}\left(d\left(z\right)\right), \mathrm{Re}\left(l\left(z\right)\right)\right\} > 0\left(z \in \Delta\right).$

Let $\gamma \in C \backslash \{0\}$ and $\lambda \geq 1$. A function ξ given by equation (1.2) is said to be in subclass $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$, if it satisfies the following conditions:

$$1 + \frac{1}{\gamma}\left[\frac{z\xi'\left(z\right) + \lambda z^2\xi''\left(z\right)}{\left(1-\lambda\right)\xi\left(z\right) + \lambda z\xi'\left(z\right)} - 1\right] \in \mathrm{d}\left(\Delta\right),\ \left(z \in \Delta\right) \tag{1.6}$$

and

$$1 + \frac{1}{\gamma}\left[\frac{w\eta'\left(w\right) + \lambda w^2\eta''\left(w\right)}{\left(1-\lambda\right)\eta\left(w\right) + \lambda w\eta'\left(w\right)} - 1\right] \in l\left(\Delta\right),\ \left(w \in \Delta\right) \tag{1.7}$$

where η is given by equation (1.3).

2 Coefficient estimates for class $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$

Now, we find the bounds for the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ of the subclass $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$.

Theorem 2.1: Let the function $\xi(z)$ given by equation (1.2) be in the class $S_{\sum_m}^{d,l}\left(\gamma,\lambda\right)$, with $\gamma \in C \backslash \{0\}$ and $\lambda \geq 1$. Then,

$$|a_{m+1}| \leq \min\left\{\sqrt{\frac{|\gamma|^2\left(\left|d^{(m)}\left(0\right)\right|^2 + \left|l^{(m)}\left(0\right)\right|^2\right)}{2m^2(m!)^2(1+\lambda m)^2}}, \sqrt{\frac{|\gamma|\left(\left|d^{(2m)}\left(0\right)\right| + \left|l^{(2m)}\left(0\right)\right|\right)}{2m\left(2m!\right)\left|(m+1)(1+2\lambda m) - (1+\lambda m)^2\right|}}\right\}, \tag{2.1}$$

and

$$\left|a_{2m+1}\right| \leq \min\left\{ \frac{|\gamma|^2(m+1)\left(\left|d^{(m)}(0)\right|^2+\left|l^{(m)}(0)\right|^2\right)}{4m^2(m!)^2(1+\lambda m)^2} + \frac{|\gamma|\left(\left|d^{(2m)}(0)\right|+\left|l^{(2m)}(0)\right|\right)}{4m(2m!)(2\lambda m+1)]}, \frac{|\gamma|}{4m(2m!)}\left[\frac{\left|2(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|\left|d^{(2m)}(0)\right|+(1+m\lambda)^2\left|l^{(2m)}(0)\right|}{(1+2\lambda m)\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}\right]\right\}. \tag{2.2}$$

Furthermore, for any $\mu \in C$,

$$\left|a_{2m+1}-\mu a^2{}_{m+1}\right| \leq \min\left\{ \frac{|\gamma|^2|m+1-2\mu|\left(\left|d^{(m)}(0)\right|^2+\left|l^{(m)}(0)\right|^2\right)}{4m^2(m!)^2(1+\lambda m)^2} + \frac{|\gamma|\left(\left|d^{(2m)}(0)\right|+\left|l^{(2m)}(0)\right|\right)}{4m(2m!)(1+2\lambda m)}, \frac{|\gamma|}{4m}\left[\frac{\left|2(1+2\lambda m)(m+1-\mu)-(1+\lambda m)^2\right|\left|d^{(2m)}(0)\right|+\left[(1+m\lambda)^2+2|\mu|(1+2\lambda m)\right]\left|l^{(2m)}(0)\right|}{(2m!)(1+2\lambda m)\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}\right]\right\}. \tag{2.3}$$

Proof: First we write the equations (1.6) and (1.7) in equivalent forms,

,

$$1+\frac{1}{\gamma}\left[\frac{z\xi'(z)+\lambda z^2\xi''(z)}{(1-\lambda)\xi(z)+\lambda z\xi'(z)}-1\right]=d(z), \tag{2.4}$$

$$1+\frac{1}{\gamma}\left[\frac{w\eta'(w)+\lambda w^2\eta''(w)}{(1-\lambda)\eta(w)+\lambda w\eta'(w)}-1\right]=l(w), \tag{2.5}$$

respectively, here d and l follows the argument of Definition 1.1.

Now, using equations (1.4) and (1.5) in equations (2.4) and (2.5) respectively, and comparing the coefficients, we get:

$$\frac{m}{\gamma}(1+\lambda m)\,a_{m+1}=d_m, \tag{2.6}$$

$$\frac{1}{\gamma}\left[2m(1+2\lambda m)\,a_{2m+1}-m(1+\lambda m)^2a^2{}_{m+1}\right]=d_{2m}, \tag{2.7}$$

$$\frac{-m(1+\lambda m)}{\gamma}a_{m+1}=l_m, \tag{2.8}$$

and

$$\frac{1}{\gamma}\left[\left\{2m(m+1)(1+2\lambda m)-m(1+\lambda m)^2\right\}a^2{}_{m+1}-2m(1+2\lambda m)\,a_{2m+1}\right]=l_{2m}. \tag{2.9}$$

From equations (2.6) and (2.8), we obtain:

$$d_m=-l_m, \tag{2.10}$$

and

$$\frac{2m^2}{\gamma^2}(1+\lambda m)^2a^2{}_{m+1}=d_m^2+l_m^2. \tag{2.11}$$

Also, by adding equations (2.7) and (2.9), we obtain:

$$\frac{2m}{\gamma}\left[(m+1)(1+2\lambda m)-(1+\lambda m)^2\right]a^2{}_{m+1}=d_{2m}+l_{2m}, \tag{2.12}$$

By using equations (2.11) and (2.12), we get:

$$a^2{}_{m+1}=\frac{\gamma^2(d_m^2+l_m^2)}{2m^2(1+\lambda m)^2}, \tag{2.13}$$

and

$$a^2{}_{m+1} = \frac{\gamma\left(d_{2m}+l_{2m}\right)}{2m\left[\left(m+1\right)\left(1+2\lambda m\right)-\left(1+\lambda m\right)^2\right]}. \tag{2.14}$$

Taking absolute values in equations (2.13) and (2.14), we get:

$$|a_{m+1}|^2 \leq \frac{|\gamma|^2\left(\left|d^{(m)}\left(0\right)\right|^2+\left|l^{(m)}\left(0\right)\right|^2\right)}{2m^2(m!)^2(1+\lambda m)^2},$$

and

$$|a_{m+1}|^2 \leq \frac{|\gamma|\left(\left|d^{(2m)}(0)\right|+\left|l^{(2m)}(0)\right|\right)}{2m(2m!)\left|(m+1)(1+2m\lambda)-(1+m\lambda)^2\right|},$$

respectively, Hence, we obtain the result of inequality (2.1).

Now, to obtain the bound of a_{2m+1}, we subtract equation (2.9) from (2.7),

$$\frac{4m\left(1+2\lambda m\right)}{\gamma}a_{2m+1}-\frac{2m\left(m+1\right)\left(1+2\lambda m\right)}{\gamma}a^2{}_{m+1}=d_{2m}-l_{2m}. \tag{2.15}$$

Using equation (2.13) in equation(2.15), we get:

$$a_{2m+1}=\frac{\gamma^2\left(m+1\right)\left(d_m^2+l_m^2\right)}{4m^2(1+\lambda m)^2}+\frac{\gamma\left(d_{2m}-l_{2m}\right)}{4m\left(1+2\lambda m\right)}, \tag{2.16}$$

On taking absolute values, we get:

$$|a_{2m+1}| \leq \frac{|\gamma|^2\left(m+1\right)\left(\left|d^{(m)}(0)\right|^2+\left|l^{(m)}(0)\right|^2\right)}{4m^2(m!)^2(1+\lambda m)^2}+\frac{|\gamma|\left(\left|d^{(2m)}(0)\right|+\left|l^{(2m)}(0)\right|\right)}{4m\left(1+2\lambda m\right)\left(2m!\right)}. \tag{2.17}$$

Now by putting the value of a_{m+1}^2 from equation (2.14) in equation(2.15), we get:

$$a_{2m+1}=\frac{\gamma\left(m+1\right)\left(d_{2m}+l_{2m}\right)}{4m\left[\left(m+1\right)\left(1+2\lambda m\right)-\left(1+m\lambda\right)^2\right]}+\frac{\gamma\left(d_{2m}-l_{2m}\right)}{4m\left(1+2\lambda m\right)},$$

or

$$a_{2m+1}=\frac{\gamma}{4m}\left[\frac{\{2\left(m+1\right)\left(1+2\lambda m\right)-\left(1+\lambda m\right)^2\}d_{2m}+\left(1+\lambda m\right)^2 l_{2m}}{\left(1+2\lambda m\right)\left\{\left(m+1\right)\left(1+2\lambda m\right]-\left(1+m\lambda\right)^2\right\}}\right]. \tag{2.18}$$

Taking absolute value of the above equation, we get:

$$|a_{2m+1}| \leq \frac{|\gamma|}{4m(2m!)}\left[\frac{\left|2(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|\left|d^{(2m)}(0)\right|+(1+m\lambda)^2\left|l^{(2m)}(0)\right|}{(1+2\lambda m)\left|\left\{(m+1)(1+2\lambda m)-(1+m\lambda)^2\right\}\right|}\right]. \tag{2.19}$$

Equations (2.17) and (2.19) together give the desired inequality (2.2).
In the end, for any by using equations (2.13) and (2.16), we get:

$$a_{2m+1}-\mu a_{m+1}^2=\frac{\gamma^2(m+1)\left(d_m^2+l_m^2\right)}{4m^2(1+m\lambda)^2}+\frac{\gamma(d_{2m}-l_{2m})}{4m(1+2\lambda m)}-\frac{\mu\gamma^2(d_m^2+l_m^2)}{2m^2(1+\lambda m)^2},$$

or

$$a_{2m+1}-\mu a_{m+1}^2=\frac{[(m+1)-2\mu]\gamma^2\left(d_m^2+l_m^2\right)}{4m^2(1+\lambda m)^2}+\frac{\gamma(d_{2m}-l_{2m})}{4m(1+2\lambda m)}.$$

Taking absolute values of the above equation:

$$\begin{aligned}&\left|a_{2m+1}-\mu a^2{}_{m+1}\right|\\&\le \frac{|m+1-2\mu||\gamma|^2\left(\left|d^{(m)}(0)\right|^2+\left|l^{(m)}(0)\right|^2\right)}{4m^2(1+\lambda m)^2(m!)^2}\\&+\frac{|\gamma|\left(\left|d^{(2m)}(0)\right|+\left|l^{(2m)}(0)\right|\right)}{4m(1+2\lambda m)(2m!)}.\end{aligned} \tag{2.20}$$

Similarly, on repeating the above method, by using equation (2.14) in equation (2.18), we get:

$$\begin{aligned}&a_{2m+1}-\mu a^2{}_{m+1}\\&=\frac{\gamma}{4m}\left[\frac{\left\{2(1+2\lambda m)(m+1-\mu)-(1+\lambda m)^2\right\}d_{2m}+\left\{(1+\lambda m)^2-2\mu(1+2\lambda m)\right\}l_{2m}}{(1+2\lambda m)\left\{(m+1)(1+2\lambda m)-(1+\lambda m)^2\right\}}\right],\end{aligned}$$

thus

$$\begin{aligned}&\left|a_{2m+1}-\mu a^2{}_{m+1}\right|\\&\le \frac{|\gamma|}{4m}\left[\frac{\left|2(1+2\lambda m)(m+1-\mu)-(1+\lambda m)^2\right|\left|d^{(2m)}(0)\right|+\left\{(1+\lambda m)^2+2|\mu|(1+2\lambda m)\right\}\left|l^{(2m)}(0)\right|}{(2m!)(1+2\lambda m)\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}\right].\end{aligned} \tag{2.21}$$

Inequalities (2.20) and (2.21) give the desired estimate $\left|a_{2m+1}-\mu a^2{}_{m+1}\right|$, as asserted in inequality (2.3). Hence proved the theorem.

Remark 2.1: If we take, $d\,(z) = l\,(z) = \left(\frac{1+z^m}{1-z^m}\right)^p = 1 + pz^m + 2p^2z^{2m} + 2p^3z^{3m} + \ldots$, $0 < p \le 1$, in the subclass $S^{d,l}_{\sum_m}(\gamma, \lambda)$ with $\gamma \in C\setminus\{0\}$ and $\lambda \ge 1$ in theorem (2.1), we get the subsequent consequences.

Corollary 2.1: Let the function $\xi(z)$ satisfy the equation (1.2) exists in the subclass $S^{d,l}_{\sum_m}(\gamma, \lambda)$. Then we obtain:

$$|a_{m+1}| \le \min\left\{\frac{2p\,|\gamma|}{m\,(1+\lambda m)}, p\sqrt{\frac{2\,|\gamma|}{m\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}}\right\},$$

and

$$\begin{aligned}&|a_{2m+1}| \le\\&\min\left\{\frac{p^2(m+1)|\gamma|^2}{2m^2(1+\lambda m)^2}+\frac{p^2|\gamma|}{m(1+2\lambda m)}, \frac{p^2(m+1)|\gamma|}{m\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}\right\}.\end{aligned}$$

Remark 2.2: If we take,

$$\begin{aligned}d\,(z) = l\,(z) &= \left(\frac{1+z^m}{1-z^m}\right)^p\\&= 1 + pz^m + 2p^2z^{2m} + 2p^3z^{3m} + \ldots,\ 0 < p \le 1\end{aligned}$$

and $\gamma = \lambda = 1$ in Theorem 2.1, the class $S^{d,l}_{\sum_m}(\gamma, \lambda)$ reduces to class $S^{d,l}_{\sum_m}$ and we obtain the following results.

Corollary 2.2: Let the function $\xi(z)$ which satisfy the equation (1.2), lies in the subclass $S^{d,l}_{\sum_m}$. Hence:

$$|a_{m+1}| \le \min\left\{\frac{2p}{m\,(1+m)}, \frac{p}{m}\sqrt{\frac{2}{m+1}}\right\},$$

and

$$|a_{2m+1}| \le \min\left\{\frac{p^2\,(m+1)}{2m^2(1+m)^2}+\frac{p^2}{m\,(1+2m)}, \frac{p^2}{m^2}\right\}.$$

Remark 2.3: If we take, $d(z) = l(z) = \left(\frac{1+z^m}{1-z^m}\right)^p = 1 + pz^m + 2p^2z^{2m} + 2p^3z^{3m} + \ldots,\ 0 < p \leq 1$ and $\gamma = \lambda = m = 1$ in the Theorem 2.1, the class $S^{d,l}_{\sum_m}$ reduces to class $S^{d,l}_{\sum}$ and we obtain the following results.

Corollary 2.3: Let the function $\xi(z)$ given by equation (1.1) be in class $S^{d,l}_{\sum}$. Then we obtain:

$$|a_2| \leq p,\ and,\ |a_3| \leq p^2.$$

Remark 2.4: Now, taking $d(z) = l(z) = 1 + 2(1-q)z^m + 2(1-q)z^{2m} + 2(1-q)z^{3m} + \ldots, 0 \leq q < 1$, in the subclass, $S^{d,l}_{\sum_m}(\gamma, \lambda)$ with $\gamma \in C\backslash\{0\}$ and $\lambda \geq 1$ in theorem (2.1), we deduce the subsequent consequences.

Corollary 2.4: Suppose the function $\xi(z)$ defined in equation (1.2) exists in the subclass $S^{d,l}_{\sum_m}(\gamma, \lambda)$. Then we obtain:

$$|a_{m+1}| \leq \min\left\{\frac{2(1-q)|\gamma|}{m(1+\lambda m)}, \sqrt{\frac{4(1-q)|\gamma|}{m\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}}\right\},$$

and

$$|a_{2m+1}| \leq \min\left\{\frac{2(1+m)(1-q)^2|\gamma|^2}{m^2(1+\lambda m)^2} + \frac{(1-q)|\gamma|}{m(1+2\lambda m)}, \frac{(m+1)(1-q)|\gamma|}{m\left|(m+1)(1+2\lambda m)-(1+\lambda m)^2\right|}\right\}.$$

Remark 2.5: If we take

$$\begin{aligned} d(z) &= l(z) \\ &= 1 + 2(1-q)z^m + 2(1-q)z^{2m} + \ldots, 0 \leq q < 1, \end{aligned}$$

and set $\gamma = \lambda = 1$ in the theorem 2.1, then the class $S^{d,l}_{\sum_m}(\gamma, \lambda)$ reduces to the class $S^{d,l}_{\sum_m}$ and deduce the following result.

Corollary 2.5: Suppose the function $\xi(z)$ satisfying equation (1.2) exists in the subclass $S^{d,l}_{\sum_m}$.Then we obtain:

$$|a_{m+1}| \leq \min\left\{\frac{2(1-q)}{m(1+m)}, \sqrt{\frac{4(1-q)}{m^2(m+1)}}\right\},$$

and

$$|a_{2m+1}| \leq \min\left\{\frac{2(1-q)^2}{m^2(1+m)} + \frac{(1-q)}{m(1+2m)}, \frac{(1-q)}{m^2}\right\}.$$

Remark 2.6: By putting,

$$\begin{aligned} d(z) &= l(z) \\ &= 1 + 2(1-q)z^m + 2(1-q)z^{2m} + \ldots,\quad 0 \leq q < 1, \end{aligned}$$

$\gamma = \lambda = 1$ and setting $m = 1$ in the theorem 2.1, the class $S^{d,l}_{\sum_m}$ reduces to the class $S^{d,l}_{\sum}$.

Corollary 2.6: Suppose the function $\xi(z)$ satisfying equation (1.1) exists in the subclass $S^{d,l}_{\sum}$. Then we deduce:

$$|a_2| \leq (1-q),$$

and

$$|a_3| \leq (1-q).$$

3 Conclusion

We have tried to obtain coefficient bounds for the new subclass $S^{d,l}_{\sum_m}$ of bi-univalent functions, where $\xi(z)$ and $\xi^{-1}(z)$ both have the property of *m*-fold symmetry in the open unit disk Δ. Further, we get some results by using specific values in our main theorem. In the future, we will try to generalize the new subclass $S^{d,l}_{\sum_m}$ and will try to get upper bounds for the initial coefficients. Also, we can find the logarithmic coefficients for this subclass $S^{d,l}_{\sum_m}$.

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5 Data availability

There is no data used in the paper.

6 Conflict of Interest

There is no conflict of interest.

7 Authors Contribution

All authors contributed in writing the draft, calculations etc. And all reviewed and approved the final version of the manuscript.

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Author information

R. S. Dubey, Department of Mathematics, ASAS, AMITY University Rajasthan, Jaipur, 302002, India.
E-mail: ravimath13@gmail.com

N. Shekhawat, Department of Mathematics, ASAS, AMITY University Rajasthan, Jaipur, 302002, India.
E-mail: neetushekhawat1723@gmail.com

P. Vijaywargiya, Department of Mathematics, S. S. Jain Subodh Girls P. G. College, Sanganer, Jaipur, 302033, India.
E-mail: pramilavijay1979@gmail.com

K. Modi, Department of Mathematics, ASAS, AMITY University Rajasthan, Jaipur, 302002,.
E-mail: mangalkanak@gmail.com

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