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Some results on projective curvature tensor in an ϵ-Kenmotsu manifold

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Abstract. Some properties of projective curvature tensor in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection have been studied.

1 Introduction

In 1972, K. Kenmotsu studied a class of contact Riemannian manifolds satisfying some special conditions [10]. We call it Kenmotsu manifold. After that Kenmotsu manifolds have been studied by many authors in several ways to a different extent such as [4, 8, 16, 19]. In 1993, A. Bejancu and K. L. Duggal [1] introduced the concept of (ϵ)-Sasakian manifolds, which later on showed by X. Xufeng and C. Xiaoli [21] that the manifolds are real hypersurfaces of indefinite Kahlerian manifolds. (ϵ)-almost para-contact manifolds were introduced by M. M. Tripathi et al. [14]. While the concept of (ϵ)-Kenmotsu manifolds was introduced by U. C. De and A. Sarkar [17] who showed that the existence of new structure on an indefinite metrics influences the curvatures. Recently, A. Haseeb, M. A. Khan and M. D. Siddiqi [3] studied ϵ-Kenmotsu manifolds with a semi-symmetric metric connection.

In 1924, the idea of semi-symmetric linear connection on a differentiable manifold was introduced by A. Friedmann and J. A. Schouten [2]. In 1930, E. Bortolotti [7] gave a geometrical meaning of such a connection. In 1932, H. A. Hayden [9] defined and studied semi-symmetric metric connection. In 1970, K. Yano [11] started a systematic study of the semi-symmetric metric connection in a Riemannian manifold and this was further studied by various authors such as S. Ahmad and S. I. Hussain [15], M. M. Tripathi [13], C. Özgür et al. [5] and many others.

Let ∇ be a linear connection in an n-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇ are given respectively by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y],$$

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

The connection ∇ is said to be symmetric if its torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is said to be metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if it is the Levi-Civita connection.

A linear connection ∇ is said to be semi-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y,$$

where η is a 1-form.

Semi-symmetric connections play an important role in the study of Riemannian manifolds. There are various physical problems involving the semi-symmetric metric connection. For example, if a man is moving on the surface of the earth always facing one definite point, say Jerusalem or Mekka or the North pole, then this displacement is semi-symmetric and metric [2].

Motivated by the above studies, in this paper we study some properties of projective curvature tensor in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection. The paper is organized as follows : In Section 2, we give a brief introduction of an ϵ-Kenmotsu manifold and define semi-symmetric metric connection. In Section 3, we find the curvature tensor, the Ricci tensor and the scalar curvature in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection. Section 4 deals with the study of projective curvature tensor in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection. Projectively flat and ξ-projectively flat ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection are studied in Sections 5 and 6, respectively. In Section 7, we investigate partially Ricci-pseudosymmetric ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection and proved that such a manifold is an η-Einstein manifold. In Section 8, we have shown that a ϕ-Ricci symmetric ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is an η-Einstein manifold.

2 Preliminaries

An n-dimensional smooth manifold (M, g) is said to be an ϵ-almost contact metric manifold [17], if it admits a $(1,1)$ tensor field ϕ, a structure vector field ξ, a 1-form η and an indefinite metric g such that

$$\phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$g(\xi, \xi) = \epsilon, \tag{2.3}$$

$$\eta(X) = \epsilon g(X, \xi), \tag{2.4}$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y) \tag{2.5}$$

for all vector fields X, Y on M, where ϵ is 1 or -1 according as ξ is space like or time like vector field and rank ϕ is $(n-1)$. If

$$d\eta(X, Y) = g(X, \phi Y) \tag{2.6}$$

for every $X, Y \in TM$, then we say that $M(\phi, \xi, \eta, g, \epsilon)$ is an almost contact metric manifold. Also, we have

$$\phi\xi = 0, \ \eta(\phi X) = 0. \tag{2.7}$$

If an ϵ-contact metric manifold satisfies

$$(\nabla_X \phi)(Y) = -g(X, \phi Y) - \epsilon\eta(Y)\phi X, \tag{2.8}$$

where ∇ denotes the Levi-Civita connection with respect to g, then M is called an ϵ-Kenmotsu manifold [6].
An ϵ-almost contact metric manifold is an ϵ-Kenmotsu manifold, if and only if

$$\nabla_X \xi = \epsilon(X - \eta(X)\xi). \tag{2.9}$$

Moreover, the curvature tensor R, the Ricci tensor S and the Ricci operator Q in an ϵ-Kenmotsu manifold M with respect to the Levi-Civita connection satisfy [17]

$$(\nabla_X \eta)Y = [g(X, Y) - \epsilon\eta(X)\eta(Y)], \tag{2.10}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.11}$$

$$R(\xi, X)Y = \eta(Y)X - \epsilon g(X, Y)\xi, \tag{2.12}$$

$$R(\xi, X)\xi = -R(X,\xi)\xi = X - \eta(X)\xi, \tag{2.13}$$

$$\eta(R(X,Y)Z) = \epsilon[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \tag{2.14}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.15}$$

$$Q\xi = -\epsilon(n-1)\xi, \tag{2.16}$$

where $g(QX,Y) = S(X,Y)$. It yields to

$$S(\phi X, \phi Y) = S(X,Y) + \epsilon(n-1)\eta(X)\eta(Y). \tag{2.17}$$

We note that if $\epsilon = 1$ and the structure vector field ξ is space like, then an ϵ-Kenmotsu manifold is usual Kenmotsu manifold.

Definition 2.1. An ϵ-Kenmotsu manifold M is said to be an η-Einstein manifold if its Ricci tensor S of type $(0,2)$ is of the form ([12], [22])

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{2.18}$$

where a and b are smooth functions on M. If $b = 0$, then an η-Einstein manifold becomes an Einstein manifold.

Definition 2.2. The projective curvature tensor P in an ϵ-Kenmotsu manifold M of dimension n with respect to the connection ∇ is defined by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{(n-1)}[g(Y,Z)QX - g(X,Z)QY] \tag{2.19}$$

for any vector fields X, Y, Z on M, where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$. The manifold is said to be projectively flat if P vanishes identically on M.

A linear connection $\bar{\nabla}$ in M is called a semi-symmetric connection [5], if its torsion tensor

$$T(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y] \tag{2.20}$$

satisfies

$$T(X,Y) = \eta(Y)X - \eta(X)Y. \tag{2.21}$$

Further, a semi-symmetric connection is called a semi-symmetric metric connection [5], if

$$(\bar{\nabla}_X g)(Y,Z) = 0. \tag{2.22}$$

Let M be an n-dimensional ϵ-Kenmotsu manifold and ∇ be the Levi-Civita connection on M, the semi-symmetric metric connection $\bar{\nabla}$ on M is given by

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \tag{2.23}$$

3 Curvature tensor on an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection

If R and $\bar{R}$, respectively, are the curvature tensors of the Levi-Civita connection ∇ and the semi-symmetric metric connection $\bar{\nabla}$ on an ϵ-Kenmotsu manifold M. Then we have

$$\begin{aligned}\bar{R}(X,Y)Z = R(X,Y)Z + (2+\epsilon)[g(X,Z)Y - g(Y,Z)X] \\ +(1+\epsilon)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi \\ +(1+\epsilon)[\eta(Y)X - \eta(X)Y]\eta(Z),\end{aligned} \tag{3.1}$$

where

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$$

is the Riemannian curvature tensor of the connection ∇.
Contracting X in (3.1), we have

$$\bar{S}(Y,Z) = S(Y,Z) + [(\epsilon+2)(\epsilon-n)+2]g(Y,Z) + (1+\epsilon)(n-2\epsilon)\eta(Y)\eta(Z), \tag{3.2}$$

where $\bar{S}$ and S are the Ricci tensors of the connections $\bar{\nabla}$ and ∇, respectively on M.
This gives

$$\bar{Q}Y = QY + [(\epsilon+2)(\epsilon-n)+2]Y + (1+\epsilon)(n-2\epsilon)\eta(Y)\xi. \tag{3.3}$$

Contracting again Y and Z in (3.2), it follows that

$$\bar{r} = r + n[(\epsilon+2)(\epsilon-n)+2] + (1+\epsilon)(n-2\epsilon), \tag{3.4}$$

where $\bar{r}$ and r are the scalar curvatures of the connections $\bar{\nabla}$ and ∇, respectively on M.

Remark. Also in an n-dimensional ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection, the following relations hold [3] :

$$\bar{R}(X,Y)\xi = (1+\epsilon)[\eta(X)Y - \eta(Y)X], \tag{3.5}$$

$$\bar{R}(X,\xi)Y = -\bar{R}(\xi,X)Y = (1+\epsilon)[g(X,Y)\xi - \eta(Y)X], \tag{3.6}$$

$$\bar{R}(\xi,X)\xi = -\bar{R}(X,\xi)\xi = (1+\epsilon)[X - \epsilon\eta(X)\xi], \tag{3.7}$$

$$\bar{S}(Y,\xi) = -(n-1)(1+\epsilon)\eta(Y), \tag{3.8}$$

$$\bar{Q}\xi = -(n-1)(1+\epsilon)\xi, \tag{3.9}$$

$$\bar{\nabla}_X\xi = (1+\epsilon)X - 2\epsilon\eta(X)\xi, \tag{3.10}$$

$$\begin{aligned}\bar{S}(\phi X,\phi Y) = S(X,Y) + [(\epsilon+2)(\epsilon-n)+2]g(X,Y)\\ +\epsilon[n-3-(\epsilon+2)(\epsilon-n)]\eta(X)\eta(Y)\end{aligned} \tag{3.11}$$

for all $X, Y \in \chi(M)$.

4 Projective curvature tensor in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection

Analogous to the Definition 2.2, the projective curvature tensor $\bar{P}$ in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is given by

$$\bar{P}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-1)}[g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y] \tag{4.1}$$

for any vector fields X, Y, Z in M, where $\bar{Q}$ is the Ricci operator defined by $\bar{S}(X,Y) = g(\bar{Q}X,Y)$.
By interchanging X and Y in (4.1), we have

$$\bar{P}(Y,X)Z = \bar{R}(Y,X)Z - \frac{1}{(n-1)}[g(X,Z)\bar{Q}Y - g(Y,Z)\bar{Q}X]. \tag{4.2}$$

On adding (4.1) and (4.2) and using the fact that $R(X,Y)Z + R(Y,X)Z = 0$, we get

$$\bar{P}(X,Y)Z + \bar{P}(Y,X)Z = 0. \tag{4.3}$$

Next, from (3.1), (4.1) and the Bianchi's first identity $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0$ with respect to ∇, we obtain

$$\bar{P}(X,Y)Z + \bar{P}(Y,Z)X + \bar{P}(Z,X)Y = 0. \tag{4.4}$$

Thus equation (4.3) (resp.,(4.4)) shows that in an ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection the projective curvature tensor is skew-symmetric (resp., cyclic). From (2.19), (3.3), (3.11) and (4.1), we get

$$\begin{aligned}\bar{P}(X,Y)Z &= P(X,Y)Z + (2+\epsilon)[g(X,Z)Y - g(Y,Z)X] \\ &+(1+\epsilon)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi + (1+\epsilon)[\eta(Y)X - \eta(X)Y]\eta(Z) \\ &-\frac{1}{(n-1)}[((\epsilon+2)(\epsilon-n)+2)(g(Y,Z)X - g(X,Z)Y) \\ &+(1+\epsilon)(n-2\epsilon)(g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi)].\end{aligned} \tag{4.5}$$

Putting $Z = \xi$ in the last equation and using (2.2), (2.4), (2.11) and (2.19), we get

$$\bar{P}(X,Y)\xi = (\eta(X)Y - \eta(Y)X)(\frac{1+2\epsilon-n\epsilon}{n-1}) - \frac{1}{n-1}(\eta(Y)QX - \eta(X)QY). \tag{4.6}$$

5 Projectively flat ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection

Let us assume that the manifold M with respect to the semi-symmetric metric connection is projectively flat, that is, $\bar{P} = 0$. Then from (4.1), it follows that

$$\bar{R}(X,Y)Z = \frac{1}{(n-1)}[g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y]. \tag{5.1}$$

Using (3.1) and (3.2), we have

$$\begin{aligned}R(X,Y)Z &= -(2+\epsilon)[g(X,Z)Y - g(Y,Z)X] \\ &-(1+\epsilon)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi - (1+\epsilon)[\eta(Y)X - \eta(X)Y]\eta(Z) \\ &+\frac{1}{(n-1)}[g(Y,Z)(QX + ((\epsilon+2)(\epsilon-n)+2)X + (1+\epsilon)(n-2\epsilon)\eta(X)\xi) \\ &-g(X,Z)(QY + ((\epsilon+2)(\epsilon-n)+2)Y + (1+\epsilon)(n-2\epsilon)\eta(Y)\xi)].\end{aligned} \tag{5.2}$$

Taking inner product of (5.2) with ξ and using (2.3) and (2.4), we obtain

$$\begin{aligned}g(R(X,Y)Z,\xi) &= -\epsilon(2+\epsilon)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)] \\ &-(1+\epsilon)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] + \frac{1}{(n-1)}[g(Y,Z)S(X,\xi) \\ &+(\epsilon(\epsilon+2)(\epsilon-n) + 2\epsilon + (1+\epsilon)(n-2\epsilon))\eta(X)g(Y,Z) \\ &-g(X,Z)S(Y,\xi) - (\epsilon(\epsilon+2)(\epsilon-n) + 2\epsilon - (1+\epsilon)(n-2\epsilon))\eta(Y)g(X,Z)]\end{aligned} \tag{5.3}$$

which on using (2.15), reduces to

$$g(R(X,Y)Z,\xi) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X).$$

This gives

$$R(X,Y,Z,U) = -\epsilon[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)],$$

where $R(X,Y,Z,U) = g(R(X,Y)Z,U)$.
This shows that the manifold is isomorphic to the Hyperbolic space $H^n(-\epsilon)$. Hence we can state the following theorem:

Theorem 5.1. *An n-dimensional projectively flat ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is locally isomorphic to the Hyperbolic space $H^n(-\epsilon)$.*

6 ξ-projectively flat ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection

Definition 6.1. An ϵ-Kenmotsu manifold is said to be ξ-projectively flat with respect to the semi-symmetric metric connection if

$$\bar{P}(X,Y)\xi = 0, \qquad X, Y \in \chi(M). \tag{6.1}$$

From (4.1), we have

$$g[\bar{R}(X,Y)\xi - \frac{1}{(n-1)}(g(Y,\xi)\bar{Q}X - g(X,\xi)\bar{Q}Y), W] = 0. \tag{6.2}$$

Using (2.4) and (3.5) in the last equation, we have

$$(n-1)(1+\epsilon)[\eta(X)g(Y,W) - \eta(Y)g(X,W)] - \eta(Y)\bar{S}(X,W) + \eta(X)\bar{S}(Y,W) \tag{6.3}$$

which by taking $Y = \xi$ and using (2.2), (2.4) and (3.8) reduces to

$$\bar{S}(X,W) + (n-1)(1+\epsilon)g(X,W) = 0. \tag{6.4}$$

In view of (3.2), (6.4) takes the form

$$S(X,W) = -[\epsilon - n + 2]g(X,W) - (1+\epsilon)(n-2\epsilon)\eta(X)\eta(W). \tag{6.5}$$

Thus we can state the following theorem:

Theorem 6.2. *An n-dimensional ξ-projectively flat ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is an η-Einstein manifold.*

7 Partially Ricci-pseudosymmetric ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection

Definition 7.1. An ϵ-Kenmotsu manifold M is said to be partially Ricci-pseudosymmetric if and only if the relation [20]

$$R \cdot S = f(p)Q(g,S) \tag{7.1}$$

holds on the set $A = [x \in M : Q(g,S) \neq 0 \ \ at \ \ x]$, where $f \in C^\infty(A)$ for $p \in (A)$, $R \cdot S$ and $Q(g,S)$ are respectively defined by

$$(R(X,Y) \cdot S)(U,V) = -S(R(X,Y)U,V) - S(U,R(X,Y)V) \tag{7.2}$$

and

$$Q(g,S) = ((X \wedge_g Y) \cdot S)(U,V), \quad where \quad (X \wedge_g Y)Z = g(Y,Z)X - g(X,Z)Y \tag{7.3}$$

for all $X, Y, U \ \ and \ \ V \in \chi(M)$.

Let an n-dimensional $(n > 2)$ ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection be partially Ricci-pseudosymmetric. Then we have

$$(\bar{R}(X,Y) \cdot \bar{S})(U,V) = f(p)[((X \wedge_g Y) \cdot \bar{S})(U,V)]$$

for all $X, Y, U \ \ and \ \ V \in \chi(M)$. From the above relation it follows that

$$\bar{S}[\bar{R}(X,Y)U,V] + \bar{S}[U,\bar{R}(X,Y)V] = f(p)[\bar{S}((X \wedge_g Y)U,V) + \bar{S}(U,(X \wedge_g Y)V)]. \tag{7.4}$$

Taking $Y = V = \xi$ in (7.4), we have

$$\bar{S}[\bar{R}(X,\xi)U,\xi] + \bar{S}[U,\bar{R}(X,\xi)\xi] = f(p)[\bar{S}((X \wedge_g \xi)U,\xi) + \bar{S}(U,(X \wedge_g \xi)\xi)]. \tag{7.5}$$

Now by using (2.2)-(2.4) and (3.6)-(3.8) in (7.5), we find

$$(n-1)(1+\epsilon)^2 g(X,U) + (1+\epsilon)\bar{S}(X,U) = -f(p)[\epsilon\bar{S}(X,U) + (n-1)(1+\epsilon)g(X,U)]. \quad (7.6)$$

Thus we have

$$[\bar{S}(X,U) + (n-1)(1+\epsilon)g(X,U)][1+\epsilon+f(p)] = 0 \quad (7.7)$$

which in view of (3.2) takes the form

$$[S(X,U) + (\epsilon - n + 2)g(X,U) + (1+\epsilon)(n-2\epsilon)\eta(X)\eta(U)][1+\epsilon+f(p)] = 0. \quad (7.8)$$

This implies that either $S(X,U) = -(\epsilon - n + 2)g(X,U) - (1+\epsilon)(n-2\epsilon)\eta(X)\eta(U)$ or $f(p) = -1-\epsilon$.

Thus we can state the following theorem:

Theorem 7.2. *A partially Ricci-pseudosymmetric ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is an η-Einstein manifold, providing $f(p) \neq -1-\epsilon$.*

8 ϕ-Ricci symmetric ϵ-Kenmotsu manifolds with respect to the semi-symmetric metric connection

Definition 8.1. An ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is said to be ϕ-Ricci symmetric if the Ricci operator $\bar{Q}$ satisfies [18]

$$\phi^2((\bar{\nabla}_X \bar{Q})(Y)) = 0$$

for all vector fields X, Y on M.

Theorem 8.2. *An n-dimensional ϕ-Ricci symmetric ϵ-Kenmotsu manifold with respect to the semi-symmetric metric connection is an η-Einstein manifold.*

Proof. Let us assume that the manifold with respect to the semi-symmetric metric connection is ϕ-Ricci symmetric. Then we have

$$\phi^2((\bar{\nabla}_X \bar{Q})(Y)) = 0.$$

In view of (2.1), we have

$$-(\bar{\nabla}_X \bar{Q})Y + \eta((\bar{\nabla}_X \bar{Q})Y)\xi = 0. \quad (8.1)$$

Taking inner product of (8.1) with Z and using (2.4), we have

$$-g((\bar{\nabla}_X \bar{Q})Y, Z) + \epsilon\eta((\bar{\nabla}_X \bar{Q})Y)\eta(Z) = 0$$

from which, we have

$$-g(\bar{\nabla}_X \bar{Q}Y, Z) + \bar{S}(\bar{\nabla}_X Y, Z) + \epsilon\eta((\bar{\nabla}_X \bar{Q})Y)\eta(Z) = 0. \quad (8.2)$$

Now putting $Y = \xi$ in (8.2) and using (3.9) and (3.10), we get

$$2(n-1)(1+\epsilon)g(X,Z) + (1+\epsilon)\bar{S}(X,Z) + \epsilon\eta((\bar{\nabla}_X \bar{Q})\xi)\eta(Z) = 0. \quad (8.3)$$

Replacing X by ϕX and Z by ϕZ in (8.3), we have

$$\bar{S}(\phi X, \phi Z) = -2(n-1)g(\phi X, \phi Z). \quad (8.4)$$

Using (2.5) and (3.11) in the last equation, we get

$$S(X,Z) = -(1+2\epsilon-n\epsilon)g(X,Z) + (1+\epsilon)(2-n)\eta(X)\eta(Z). \quad (8.5)$$

This completes the proof. □

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