Palestine Journal of Mathematics

Vol. 6(Special Issue: II) (2017) , 141–148

3-difference cordiality of some graphs

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Communicated by Ayman Badawi

MSC 2010 Classifications: Primary 05C78.

Keywords and phrases: Path, Complete graph, Complete bipartite graph, Star.

Abstract Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of some graphs.

1 Introduction

Graphs considered here are finite and simple. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup \cdots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \bigcup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2 \dots, w_t$ and joining w_i to each vertex of Si $(1 \leq i \leq t)$. For a graph G, the splitting graph of G, $spl(G)$, is obtained from G by adding for each vertex v of G a new vertex v' so that v' is adjacent to every vertex that is adjacent to v. Recently Ponraj et al. [3], introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of star, m copies of star etc. Also they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$. In this paper we investigate 3-difference cordial labeling behavior of $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, mC_4, $spl(K_{1,n})$, $DS(B_{n,n})$, $C_n \odot K_2$, and some more graphs [3, 4, 5]. Terms are not defined here follows from Harary [2] and Gallian [1].

2 k-Difference cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

First we look into the corona graphs $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, and $C_n \odot K_2$.

Theorem 2.2. $K_{1,n} \odot K_2$ is 3-difference cordial.

Proof. Let $V(K_{1,n} \odot K_2) = \{u, u_i, v_i, w_i, x, y : 1 \leq i \leq n\}$ and $E(K_{1,n} \odot K_2) = \{ uu_i, u_iv_i, u_iw_i, v_iw_i, xy, ux, uy : 1 \leq i \leq n \}$. Assign the labels 1,2,3 to the vertices u,x,y respectively. Then assign the label 1 to all the vertices $u_i\,(1 \leq i \leq n)$. Then we assign the label 2 to the vertices $v_i\,(1 \leq i \leq n)$. Finally assign the label 3 to the vertices $w_i\,(1 \leq i \leq n)$. The edge and vertex condition of this labeling is as follows: $v_f(1) = v_f(2) = v_f(3) = n + 1$ and $e_f(1) = 2n + 2$, $e_f(0) = 2n + 1$. Hence f is a 3-difference cordial labeling. □

Theorem 2.3. $P_n \odot 3K_1$ is 3-difference cordial.

Proof. Let $u_1u_2 \dots u_n$ be the path P_n. Let x_i, y_i, z_i be the pendent vertices adjacent with u_i where $1 \leq i \leq n$. Let $n = 3t + r$where $0 \leq r < 3$. Assign the labels 1,2,3 to the first three vertices u_1, u_2, u_3 of the path P_n. Then assign the labels 1,2,3 to the next three vertices u_4, u_5, u_6 of the path. Continuing this way assign the next three vertices and so on. If $r = 0$ we have labeled all the vertices of the path. If $r = 1$ assign assign the label 1 to the next non labeled vertex of the path. If $r = 2$ then assign the labels 1,2 to the next non labeled vertices of the path P_n. Now our attention turn to the vertices x_i, y_i and z_i. First we assign the label 1 to all the vertices $x_i\,(1 \leq i \leq n)$ and assign the label 2 to all the vertices $y_i\,(1 \leq i \leq n)$. Finally assign the label 3 to all the vertices $z_i\,(1 \leq i \leq n)$.

Tables 1 and 2 establish that f is a 3-difference cordial labeling of $P_n \odot 3K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 3$	$2n$	$2n - 1$
$n \equiv 0, 2 \pmod 3$	$2n - 1$	$2n$

Table 1.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
$n \equiv 1 \pmod 3$	$\frac{4n+2}{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$
$n \equiv 2 \pmod 3$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$	$\frac{4n-2}{3}$

Table 2.

□

Theorem 2.4. $C_n \odot K_2$ is 3-difference cordial.

Proof. Let C_n be the cycle $u_1u_2 \dots u_nu_1$. Let $V(C_n \odot K_2) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(C_n \odot K_2) = E(C_n) \cup \{u_iv_i, u_iw_i, v_iw_i : 1 \leq i \leq n\}$. Note that $C_n \odot K_2$ has 3n vertices and 4n edges. First assign the label 2 to all the circle vertices $u_i\,(1 \leq i \leq n)$. Then assign the label 1 to the vertices $v_i\,(1 \leq i \leq n)$. Finally assign the label 3 to the vertices $w_i\,(1 \leq i \leq n)$. Clearly all the edges in the cycle and the edges $v_iw_i\,(1 \leq i \leq n)$ received the label 0. All the other edges received the label 1. It is easy to verify that $v_f(1) = v_f(2) = v_f(3) = n$ and $e_f(0) = e_f(1) = 2n$. Hence this labeling f is a 3-difference cordial labeling. □

Next is the m copies of the cycle C_4.

Theorem 2.5. mC_4 is 3-difference cordial.

Proof. Assign the labels 1,2,3,1 consecutively to the vertices of the first copy of the cycle C_4. Then assign 2,3,1,2 to the next copy of the cycle C_4. Next we assign the labels 3,2,1,3 to the vertices of the third copy of the cycle C_4. This labeling is displayed in figure 1.

Then assign the label to the vertices of the fourth copy as in first copy and the vertices of the fifth copy as in second copy. The vertices of sixth copy as in the third copy. In general assign the label to the vertices of i^{th} copy as in the $i - 3^{th}$ copy. Obviously this labeling pattern f is a 3-difference cordial labeling.

□

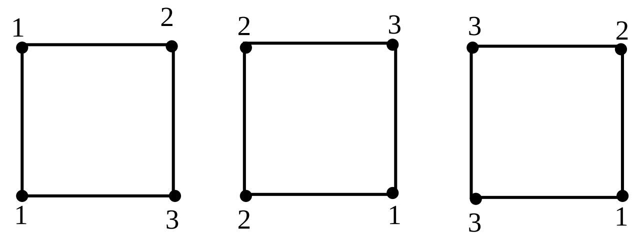

Figure 1.

Next is the splitting graph of the star.

Theorem 2.6. $spl(K_{1,n})$ is 3-difference cordial.

Proof. Let $V(spl(K_{1,n})) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(spl(K_{1,n}))$ $=\{ uu_i, uv_i, vv_i : 1 \leq i \leq n \}$. Note that $spl(K_{1,n})$ has $2n + 2$ vertices and 3n edges. First we assign the label 1 to the vertex u. Then assign the label 1 to the $\lceil \frac{n}{2} \rceil$ pendent vertices $u_1, u_2, \ldots u_{\lceil \frac{n}{2} \rceil}$. Then assign the label 2 to the remaining pendent vertices. Assign the label 2 to the vertex v. Then assign the label 3 to the vertices $v_1, v_2, v_4, v_5, v_7, v_8 \ldots$ etc. Next we assign the label 1 to the vertex $v_6, v_{12}, \ldots$ etc. Finally assign the label 2 to the vertex $v_3, v_9 \ldots$ etc.

Hence this labeling f is a 3-difference cordial labeling follows from the following table 3 and 4.

Nature of n	$e_f(0)$	$e_f(1)$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 3.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 3 \pmod 6$	$\frac{2n+3}{3}$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
$n \equiv 2, 5 \pmod 6$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$
$n \equiv 4 \pmod 6$	$\frac{2n+1}{3}$	$\frac{2n+4}{3}$	$\frac{2n+1}{3}$
$n \equiv 1 \pmod 6$	$\frac{2n+4}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$

Table 4.

□

The graph $P_n + K_1$ is called the fan graph and it is denoted by F_n.

Theorem 2.7. The fan graph F_n is 3-difference cordial.

Proof. Let $V(F_n) = \{u, u_i : 1 \leq i \leq n\}$ and $E(F_n) = \{uu_i, u_iu_{i+1} : 1 \leq i \leq n\}$. Note that F_n has n+1 vertices and 2n-1 edges.

Case 1. $n \equiv 0 \pmod 3$.

Assign the labels 1,3,2 to the first three vertices of the path u_1, u_2, u_3 respectively. Then assign the labels 1,3,2 to the next three vertices u_4, u_5, u_6 of the path. Proceeding like that, we assign the next three vertices and so on. It is obvious that the last vertex u_n received the label 2. Finally we assign the label 1 to the vertex u.

Case 2. $n \equiv 1 \pmod 3$.

Fix the label 1 to the vertex u_1. Then assign the labels 1,3,2 to the next three vertices u_2, u_3, u_4 respectively. Next we assign the labels 1,3,2 to the next three vertices of the path u_5, u_6, u_7 respectively. Continuing this pattern until we reach the last vertex u_n. It is clear that 2 is the label of the last vertex u_n. Finally assign the label 1 to the vertex u.

Case 3. $n \equiv 2 \pmod 3$.

Fix the labels 3,2 to the vertex u_1 and u_2 respectively. Assign the labels 1,3,2 to the next three vertices u_3, u_4, u_5 of the path. Next we assign the labels 1,3,2 to the next three vertices of the path u_6, u_7, u_8 respectively.Proceeding like this until we reach the last vertex u_n. Clearly the last vertex u_n received the label 2. Finally assign the label 1 to the vertex u.

The following table 5 and 6 shows that this labeling f is a 3-difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 3$	n	$n-1$
$n \equiv 1, 2 \pmod 3$	$n-1$	n

Table 5.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{n}{3}+1$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod 3$	$\frac{n+2}{3}$	$\frac{n+2}{3}$	$\frac{n-1}{3}$
$n \equiv 2 \pmod 3$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

Table 6.

□

The graph $P_n + 2K_1$ is called the double fan and it is denoted by DF_n.

Theorem 2.8. The double fan DF_n is 3-difference cordial.

Proof. Let P_n be the path $u_1u_2 \dots u_n$ and $V(2K_1) = \{u, v\}$.
Case 1. $n \equiv 0 \pmod 3$.
Subcase 1. $n \equiv 0 \pmod 6$.

Assign the labels 1,3,2,1,3,2 to the first six path vertices $u_1, u_2 \dots u_6$ respectively. Then assign the labels 2,3,1,2,3,1 to the next six vertices $v_7, v_8 \dots v_{12}$ of the path. Next we assign the labels 1,3,2,1,3,2 to the next six vertices and assign the labels 2,3,1,2,3,1 to the next six vertices . Continuing in this pattern until we reach the last vertex u_n received the label 2 or 1 according as $n \equiv 6 \pmod{12}$ or $n \equiv 0 \pmod{12}$. Finally we assign the labels 1,2 to the vertices u and v respectively.
Subcase 2. $n \equiv 3 \pmod 6$.

As in subcase 1, assign the label to the vertices u, v, u_i $(1 \le i \le n-3)$. Then we assign the labels 2,3,1 or 1,3,2 to the vertices u_{n-2}, u_{n-1}, u_n according as $n \equiv 0 \pmod 9$ or $n \equiv 6 \pmod 9$. In this cases $v_f(1) = v_f(2) = \frac{n+3}{3}$, $v_f(3) = \frac{n}{3}$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 3 \pmod 6$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
$n \equiv 0 \pmod 6$	$\frac{3n-2}{2}$	$\frac{3n}{2}$

Table 7.

Case 2. $n \equiv 1 \pmod 3$.

Fix the labels 1,3,2,3 to the vertices u_1, u_2, u_3, u_4 respectively. Then assign the labels 1, 3, 2, 1, 3, 2 to the next six vertices $v_5, v_6, \dots, v_{10}$ respectively. Next we assign the labels 2,3,1,2,3,1 to the next six vertices $u_{11}, u_{12}, \dots, u_{16}$ respectively. Then we assign the labels 1,3,2,1,3,2 to the next six vertices of the path $u_{17}, u_{18}, \dots, u_{22}$ and we assign the labels 2,3,1,2,3,1 to the next six vertices of the path. Proceeding like this, we assign the next six vertices and so on. Note that in this process, the last three vertices u_{n-2}, u_{n-1}, u_n received the labels 1,3,2 or 2,3,1 according as $n \equiv 7, 10 \pmod{12}$ or $n \equiv 1, 4 \pmod{12}$. Finally we assign the labels 1,2 respectively to the vertices u and v. The vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+2}{3}$ and edge condition of this labeling f is given in table 8.
Case 3. $n \equiv 2 \pmod 3$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 6$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
$n \equiv 4 \pmod 6$	$\frac{3n-2}{2}$	$\frac{3n}{2}$

Table 8.

Fix the labels 1,3,2,3,1 to the first five vertices u_1, u_2, u_3, u_4, u_5 respectively. Then assign the labels 1,3,2,1,3,2 to the next six vertices $v_6, v_7, \ldots, v_{11}$ respectively. Next we assign the labels 2,3,1,2,3,1 to the next six vertices $v_{12}, v_{13}, \ldots, v_{17}$ of the path respectively. Then assign the labels 1,3,2,1,3,2 to the next six path vertices and assign the labels 2,3,1,2,3,1 to the next six path vertices. Continuing this way, assign the labeling to the vertices of the path. It is easy to verify that in this pattern, the last three vertices u_{n-2}, u_{n-1}, u_n received the labels 1,3,2 or 2,3,1 according as $n \equiv 8, 11 \pmod{12}$ or $n \equiv 2, 5 \pmod{12}$. Finally we assign the labels 1,2 to the vertices u and v respectively. This labeling pattern is a 3-difference cordial labeling follows from the vertex condition $v_f(1) = \frac{n+4}{3}$, $v_f(2) = v_f(3) = \frac{n+1}{3}$ and table 9.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 5 \pmod 6$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
$n \equiv 2 \pmod 6$	$\frac{3n}{2}$	$\frac{3n-2}{2}$

Table 9.

□

Next investigation is about the degree splitting graph of the bistar $B_{n,n}$

Theorem 2.9. $DS(B_{n,n})$ is 3-difference cordial.

Proof. Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i : 1 \leq i \leq n\}$. Let $V(DS(B_{n,n})) = V(B_{n,n}) \cup \{w, z\}$ and $E(DS(B_{n,n})) = E(B_{n,n}) \cup \{wu_i, wv_i : 1 \leq i \leq n\} \cup \{uz, vz\}$. Note that $DS(B_{n,n})$ has 2n+4 vertices and 4n+3 edges.
Case 1. $n \equiv 0 \pmod 3$.

Assign the label 1 to the vertices $u_i, v_i\ (1 \leq i \leq \frac{n}{3})$. Then we assign the label 2 to the vertices $u_{i+\frac{n}{3}}, u_{i+\frac{n}{3}}\ (1 \leq i \leq \frac{n}{3})$. Next assign the label 3 to the vertices $u_{i+\frac{2n}{3}}, u_{i+\frac{2n}{3}}\ (1 \leq i \leq \frac{n}{3})$. Now we move to the vertices u and v. Assign the labels 2,2 to the vertices u and v respectively. Finally assign the label 1,1 to the vertices w,z respectively.
Case 2. $n \equiv 1 \pmod 3$.

Assign the label to the vertices $u, v, w, z, u_i, v_i\ (1 \leq i \leq n-1)$ as in case 1. Then assign the labels 1,3 to the vertices u_n, v_n respectively.
Case 3. $n \equiv 2 \pmod 3$.

As in case 2, Assign the label to the vertices $u, v, w, z, u_i, v_i\ (1 \leq i \leq n-1)$. Then assign the labels 2,1 to the vertices u_n, v_n respectively. Clearly this labeling f is 3-difference cordial labeling follows from $e_f(0) = 2n+1$, $e_f(1) = 2n+1$ and the table 10.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}+1$	$\frac{2n}{3}+1$	$\frac{2n}{3}+1$
$n \equiv 1 \pmod 3$	$\frac{2n}{3}+2$	$\frac{2n}{3}+2$	$\frac{2n}{3}+2$
$n \equiv 2 \pmod 3$	$\frac{2n}{3}+3$	$\frac{2n}{3}+3$	$\frac{2n}{3}+2$

Table 10.

□

Now we investigate the jellyfish graph.

Theorem 2.10. The jellyfish graph JF_n is 3-difference cordial.

Proof. Let $V(JF_n) = \{u, v, x, y, u_i, x_i : 1 \leq i \leq n\}$ and $E(JF_n)$ =$\{ux$, xv, uy, yv, xy, u_i, v_i : $1 \leq i \leq n\}$. Clearly JF_n has 2n+4 vertices and 2n+5 edges.

Case 1. $n \equiv 0 \pmod 3$.

Assign the labels 1,2,3,3 to the vertices u,v,x,y respectively. Then assign the labels 1,2,3 to the first three vertices u_1, u_2, u_3 respectively. Then assign the labels 1,2,3 to the next three vertices u_4, u_5, u_6 respectively. Proceeding like thiswe assign the next three vertices and so on. Assign the label to the vertices v_i $(1 \leq i \leq n)$ is same as the pattern of assign the label to the vertices u_i $(1 \leq i \leq n)$. Assign the labels 1,2,3 to the vertices v_1, v_2, v_3 respectively. Then assign the labels 1,2,3 to the next three vertices. Continuing this way we assign the next three vertices and so on. In this case the vertices u_n and v_n received the label 3.

Case 2. $n \equiv 1 \pmod 3$.

Assign the label to the vertices u, v, x, y, u_i, v_i $(1 \leq i \leq n-1)$ as in case 1. Then assign the labels 2,1 to the vertices u_n, v_n respectively.

Case 3. $n \equiv 1 \pmod 3$.

As in case 2, assign the label to the vertices u, v, x, y, u_i, v_i $(1 \leq i \leq n-1)$. Next we assign the labels 1,3 to the vertices u_n, v_n respectively. Then the following tables 11 and 12 shows that f is a 3-difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 3$	$n+3$	$n+2$
$n \equiv 1,2 \pmod 3$	$n+2$	$n+3$

Table 11.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}+1$	$\frac{2n}{3}+1$	$\frac{2n}{3}+2$
$n \equiv 1 \pmod 3$	$\frac{2n+4}{3}$	$\frac{2n+4}{3}$	$\frac{2n+4}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n+5}{3}$	$\frac{2n+2}{3}$	$\frac{2n+5}{3}$

Table 12.

□

Finally we investigate the graphs which are derived from cycles.

Theorem 2.11. Let C_n be the cycle $u_1u_2 \ldots u_nu_1$. Let G be the graph obtained from C_n with $V(G) = V(C_n) \cup \{v_i : 1 \leq i \leq \lceil \frac{n}{2} \rceil\}$ and $E(G) = \{u_iv_i, u_{i+1}v_i : 1 \leq i \leq n\}$. Then G is 3-difference cordial labeling.

Proof. Assign the label 1 to the $\lceil \frac{n}{2} \rceil$ cycle vertices $u_1, u_2, \ldots u_{\lceil \frac{n}{2} \rceil}$. Then assign the label 3 to the remaining cycle vertices. Next we assign the label 2 to all the vertices v_i $(1 \leq i \leq n)$. Then the following tables 13 and 14 shows that this labeling f is a 3-difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
n is even	n	n
n is odd	n	$n-1$

Table 13.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n is even	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$
n is odd	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$

Table 14.

□

Theorem 2.12. The graph G_n with the vertex set $V(G_n) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_nu_1, v_1u_1\} \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$ is 3-difference cordial.

Proof. Assign the label 1 to all the vertices $u_i\,(1 \leq i \leq n)$ and assign the label 2 to all the vertices $v_i\,(1 \leq i \leq n)$. Finally assign the label 3 to the vertices $w_i\,(1 \leq i \leq n)$. Clearly $v_f(1) = v_f(2) = v_f(3) = n$ and $e_f(0) = e_f(1) = 2n$. Hence this labeling f is a 3-difference cordial labeling. □

Theorem 2.13. Let C_3 be the cycle $u_1u_2u_3u_1$. Let G be a graph obtained from C_3 with $V(G) = V(C_3) \cup \{v_i, w_i, z_i : 1 \leq i \leq n\}$ and $E(G) = E(C_3) = \{u_1v_i, u_2w_i, u_3z_i : 1 \leq i \leq n\}$. Then G is 3-difference cordial if $n \equiv 0, 2, 3 \pmod 4$.

Proof. **Case 1.** $n \equiv 0 \pmod 4$.

Assign the labels 1,2,3 to the cycle vertices u_1, u_2, u_3 respectively. Then assign the labels 2,2,3,3 to the first four vertices v_1, v_2, v_3, v_4 respectively. Next we assign the label 2,2,3,3 to the next four vertices v_5, v_6, v_7, v_8 respectively. Proceeding like this, we assign the next four vertices and so on. Next we move to the vertices w_i. Assign the labels 1,1,1,2 to the first four vertices w_1, w_2, w_3, w_4 respectively. Next we assign the labels 1,1,1,2 to the next four vertices w_5, w_6, w_7, w_8 respectively. Continuing this process, we assign the next four vertices and so on. The last vertex w_n received the label 2. Now our attention move to the vertices z_i. Assign the labels 1,2,3,3 to the first four vertices z_1, z_2, z_3, z_4 respectively. Next we assign the label 1,2,3,3 to the next four vertices z_5, z_6, z_7, z_8 respectively. Continuing this pattern, we reach the last vertex z_n. Clearly 3 is the label of the last vertex z_n.

Case 2. $n \equiv 2 \pmod 4$.

Assign the label to the vertices $u_1, u_2, u_3, v_i, w_i, z_i\,(1 \leq i \leq n-2)$ as in case 1. Next we assign the label 3,3 to the vertices v_{n-1}, v_n respectively and assign the label 1,2 to the vertices w_{n-1}, w_n respectively. Finally assign the label 1,2 to the vertices z_{n-1}, z_n respectively.

Case 3. $n \equiv 3 \pmod 4$.

Assign the label to the vertices $u_1, u_2, u_3, v_i, w_i, z_i\,(1 \leq i \leq n-1)$ as in case 2. Next we assign the labels 3,2,1 to the vertices v_n, w_n, z_n respectively. The vertex condition for these three cases, $v_f(1) = v_f(2) = v_f(3) = n+1$. The edge condition is given in table 15. Hence f is a 3-difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{3n+2}{2}$	$\frac{3n+4}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n+4}{2}$	$\frac{3n+2}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n+3}{2}$	$\frac{3n+3}{2}$

Table 15.

□

References

[1] J.A.Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **19** (2012) #Ds6.

[2] F.Harary, Graph theory, *Addision wesley*, New Delhi (1969).

[3] R. Ponraj, M.Maria Adaikalam and R.Kala, k-Difference cordial labeling of graphs, international journal of Mathematical Combinatorics, 2(2016),121–131.

[4] R. Ponraj and M. Maria Adaikalam , 3-Difference cordial labeling of some union of graphs, Palestine Journal of Mathematics, 6(1)(2017)202 – 210.

[5] R.Ponraj and M. Maria Adaikalam, 3-Difference cordial labeling of some cycle related graphs, Journal of algorithm and computation, 47(2016),1–10.

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Received: December 20, 2015.

Accepted: May 10, 2016.