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Some Common Fixed Point Theorems For Weakly Subsequentially Continuous Mappings Via Implicit Relation

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Abstract. In this paper, we prove two common fixed point theorems for two weakly subsequentially continuous and compatible of type (E) for two pairs of self mappings, which satisfying implicit relation in metric spaces, an example is given to illustrate our results.

1 Introduction

Jungck[12] introduced the concept of commuting mappings to establish a common fixed point theorem for a pair of self mappings in complete metric space, Sessa[23] generalized it to the weakly commuting mappings notion, later Jungck[13] defined compatible mappings in metric space, which is weaker than the last notions. After that many authors gave various type of compatibility, compatibility of type (A), type (B), type (C) and type (P) for two self mappings S and T on metric space (X, d) respectively in [14], [18],[20] and [19] as follows: the pair(S, T) is compatible of type (A) if

$$\lim_{n\to\infty} d(STx_n, T^2x_n) = 0 \text{ and } \lim_{n\to\infty} d(TSx_n, S^2x_n) = 0,$$

S and T are compatible of type (B) if

$$\lim_{n\to\infty} d(STx_n, T^2x_n) \leq \frac{1}{2}\Big[\lim_{n\to\infty} d(STx_n, Sz) + \lim_{n\to\infty} d(Sz, S^2x_n)\Big] \text{ and}$$

$$\lim_{n\to\infty} d(TSx_n, S^2x_n) \leq \frac{1}{2}\Big[\lim_{n\to\infty} d(TSx_n, gz) + \lim_{n\to\infty} d(Tz, T^2x_n)\Big],$$

they are compatible of type (C) if

$$\lim_{n\to\infty} d(STx_n, T^2x_n) \leq \frac{1}{3}[\lim_{n\to\infty} d(STx_n, Sz) + \lim_{n\to\infty} d(Sz, T^2x_n) + \lim_{n\to\infty} d(Sz, T^2x_n)]$$

and

$$\lim_{n\to\infty} d(TSx_n, S^2x_n) \leq \frac{1}{3}[\lim_{n\to\infty} d(TSx_n, Tz) + \lim_{n\to\infty} d(Tz, T^2x_n) + \lim_{n\to\infty} d(Tz, S^2x_n)],$$

and said to be compatible of type(P) if

$$\lim_{n\to\infty} d(S^2x_n, T^2x_n) = 0,$$

whenever in the all above definitions, $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$, for some $z \in X$.
Aamri and Moutawakil [1] defined two self maps S and T on a metric space (X, d) are said to be satisfy property (E,A), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z,$$

for some z in X.

2 Preliminaries

Pant[16] introduced the notion of reciprocal continuity as follows:

Definition 2.1. Self maps S and T of a metric space (X, d) are said to be reciprocally continuous, if $\lim_{n\to\infty} STx_n = St$ and $\lim_{n\to\infty} TSx_n = Tt$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

In 2009, Bouhadjera and Godet Thobie [8] introduced the concept of subcompatibility and subsequential continuity as follows:
Two self-mappings S and T on a metric space (X, d) are said to be subcompatible, if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad \text{and} \quad \lim_{n\to\infty} d(STx_n, TSx_n) = 0,$$

for some $t \in X$

Definition 2.2. The pair (S, T) is called to be subsequentially continuous, if there exists a sequence $\{x_n\}$ in X, such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$ and $\lim_{n\to\infty} STx_n = St$, $\lim_{n\to\infty} TSx_n) = Tt$.

Now, as a generalization to the Definition 2.2, define:

Definition 2.3. [6] Let S and T be two self maps of a metric space (X, d), the pair (S, T) is said to be weakly subsequentially continuous (shortly wsc), if there exists a sequence $\{x_n\}$ in X, such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in X$ and $\lim_{n\to\infty} STx_n = St$ or, $\lim_{n\to\infty} TSx_n) = Tt$.

Notice that subsequentially continuous or, reciprocally continuous maps are weakly subsequentially continuous, but the converse may be not.

Definition 2.4. The pair (S, T) is said to be S-subsequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$ and $\lim_{n\to\infty} STx_n = St$.

Definition 2.5. The pair (S, T) is said to be S-subsequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in X$ and $\lim_{n\to\infty} TSx_n = Tt$.

Example 2.6. Let $X = [0, 2]$ and d is the euclidian metric, we define S, T as follows:

$$Sx = \begin{cases} 1 + x, & 0 \le x \le 1 \\ \frac{x+1}{2}, & 1 < x \le 2 \end{cases}, \quad Tx = \begin{cases} 1 - x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$$

Clearly that S and T are discontinuous at 1.
We consider a sequence $\{x_n\}$, which defined for each $n \ge 1$ by: $x_n = \dfrac{1}{n}$,
clearly that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 1$, also we have:

$$\lim_{n\to\infty} STx_n = \lim_{n\to\infty} S(1 - \frac{1}{n}) = 2 = S(1),$$

then (S, T) is S-subsequentially continuous, so it is wsc .
On other hand, let $\{y_n\}$ be a sequence which defined for each $n \ge 1$ by: $y_n = 1 + \dfrac{1}{n}$, we have

$$\lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ty_n = 1,$$

but

$$\lim_{n\to\infty} TSy_n = \lim_{n\to\infty} T(1 + \frac{1}{2n}) = 1 \ne T(1),$$

then (S, T) is never reciprocally continuous.

Singh and Mahendra Singh [24] introduced the notion of compatibility of type (E), and gave some properties about this type as follows:

Definition 2.7. Self maps S and T on a metric space (X, d), are said to be compatible of type (E), if $\lim_{n\to\infty} T^2x_n = \lim_{n\to\infty} TSx_n = St$ and $\lim_{n\to\infty} S^2x_n = \lim_{n\to+\infty} STx_n = Tt$, whenever $\{x_n\}$ is a sequence in X, such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

Remark 2.8. If $St = Tt$, then compatible of type (E) implies compatible (compatible of type (A), compatible of type (B), compatible of type (C), compatible of type (P)), however the converse may be not true. Generally compatibility of type (E) implies compatibility of type (B).

Definition 2.9. Two self maps S and T of a metric space (X, d) are S-compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} STx_n = Tt$, for some $t \in X$.
The pair (S, T) is said to be T-compatible of type (E), if $\lim_{n\to\infty} T^2x_n = \lim_{n\to\infty} TSx_n = St$, for some $t \in X$.

Notice that if S and T are compatible of type (E), then they are S-compatible and T-compatible of type (E), but the converse is not true.

Example 2.10. Let $X = [0, \infty)$ endowed with the euclidian metric, we define S, T as follows:

$$Sx = \begin{cases} 2, & 0 \le x \le 2 \\ x+1, & x > 2 \end{cases} \qquad Tx = \begin{cases} \frac{x+2}{2}, & 0 \le x \le 2 \\ 0, & x > 2 \end{cases}$$

Consider the sequence $\{x_n\}$ which defined by: $x_n = 2 - \dfrac{1}{n}$, for all $n \ge 1$.

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 2,$$

$$\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} STx_n) = 2 = T(2)$$

$$\lim_{n\to\infty} T^2x_n = \lim_{n\to\infty} TSx_n) = 2 = S(2)$$

then (S, T) is compatible of type (E).

Let $\mathcal{F}$ is the set of of all continuous functions $F : \mathbb{R}_+^6 \to \mathbb{R}_+$ such:

(F_1) : F is non decreasing in t_5 and t_6.

(F_2) : :For every $u > 0$ we have $F(u, u, 0, 0, u, u) < 0$

Example 2.11.

$$F(t_1, t_2, t_3, t_4, t_5, t_6) = at_1^2 - bt_2^2 + \frac{ct_5t_6}{dt_3^2 + et_4^2 + 1},$$

where $c, d, e \ge 0, a > 0$ and $b > a + c$. Obviously that $F(u, u, 0, 0, u, u) = (a + c - b)u^2 < 0$, for all $u > 0$.

Example 2.12.

$$F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^2 - \left[at_2^2 + \frac{bt_3^2 + ct_4^2}{t_5t_6 + 1}\right]^{\frac{1}{2}},$$

where $0 \le b, c < 1, a > 1$. Obviously that $F(u, u, 0, 0, u, u) = (1 - \sqrt{a})u < 0$, for all $u > 0$.

Example 2.13.

$$F(t_1, t_2, t_3, t_4, t_5) = t_1 - [at_2^p, at_3^p, at_4^p]^{\frac{1}{p}} + d\sqrt{t_5t_6},$$

where $a > (1 + d)^p$ and $0 \le c, b < 1$, $p \in \mathbb{N}^*$.

Example 2.14.

$$F(t_1, t_2, t_3, t_4, t_5) = t_1 - [at_2^p, at_3^p, at_4^p]^{\frac{1}{p}} + \frac{1}{2}\left(\frac{t_1}{1 + \sqrt{t_5t_6}}\right),$$

where $0 \le c, b < \frac{1}{2^p}$, $p \in \mathbb{N}^*$.

The aim of this paper is to prove the existence and the uniqueness of a common fixed point, for two pairs of self-mappings in metric space, which satisfying implicit relation, by using the weak subsequential continuity with compatibility of type (E), due to Singh et al.[24], also to support our results we will give an example.

3 Main results

Theorem 3.1. *Let (X,d) be a metric space, and let A, B, S and T be self mappings on X such for all $x, y \in X$ we have:*

$$F(d(Sx,Ty), d(Ax,By), d(Ax,Sx), d(By,Ty), d(Ax,Ty), d(By,Sx)) \geq 0, \tag{3.1}$$

where $F \in \mathcal{F}$, if the two pairs (A,S) and (B,T) are weakly subsequentially continuous (wsc) and compatible of type (E), then A, B, S and T have a unique common fixed point in X.

Proof. Since (A,S) is wsc, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, $\lim_{n\to\infty} SAx_n = Sz$ again $\{,S\}$ is compatible of type (E) implies that

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A^2x_n = Sz$$

and

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S^2x_n = Az,$$

then $Az = Sz$, and z is a coincidence point for A and S.
Similarly for B and T, since (B,T) is wsc (suppose that it is B-subsequentially continuous) there exists a sequence $\{y_n\}$ in X such:

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t,$$

for some $t \in X$ and

$$\lim_{n\to\infty} BTy_n = Bt,$$

again (B,T) is compatible of type (E) implies that

$$\lim_{n\to\infty} BTy_n = \lim_{n\to\infty} B^2y_n = Tt$$

$$\lim_{n\to\infty} TBy_n = \lim_{n\to\infty} T^2y_n = Bt,$$

hence $Bt = Tt$.
We claim $Az = Bt$, if not by using (3.1) we get:

$$F(d(Sz,Tt), d(Az,Bt), d(Az,Sz), d(Bt,Tt), d(Az,Tt), d(Bt,Sz)) =$$

$$F(d(Az,Bt), d(Az,Bt), 0, 0, d(Az,Bt), d(Az,Bt)) \geq 0,$$

which is a contradiction with (F_2).
Now we will prove $z = Az$, if not by using(3.1) we get:

$$F(d(Sx_n,Tt), d(Ax_n,Bt), d(Ax_n,Sx_n), d(Bt,Tt), d(Ax_n,Tt), d(Bt,Sx_n)) \geq 0,$$

letting $n \to \infty$ we get:

$$F(d(z,Tt), d(z,Bt), 0, 0, d(z,Tt), d(Bt,z)) =$$

$$F(d(z,Az), d(z,Az), 0, 0, d(z,Az), d(z,Az)) \geq 0,$$

which is a contradiction, then $z = Az = Sz$.
Nextly we shall prove $z = t$, if not by using (3.1) we get:

$$F(d(Sx_n,Ty_n), d(Ax_n,By_n), d(Ax_n,Sx_n), d(By_n,Ty_n), d(Ax_n,Ty_n), d(By_n,Sx_n)) \geq 0,$$

letting $n \to \infty$ we get:

$$F(d(z,t), d(z,t), 0, 0, d(z,t), d(t,z)) \geq 0,$$

which is a contradiction. Consequently, z is a fixed point for A, B, S and T. For the uniqueness suppose that there is another fixed point w and using (3.1) we get:

$$F(d(Sz,Tw), d(Az,Bw), d(Az,Sw), d(Bw,Tw), d(Az,Tw), d(Bw,Sz)) =$$

$$F(d(z,w), d(z,w), 0, 0, d(z,w), d(z,w)) \geq 0,$$

which contradicts (F_2), then z is unique. □

Corollary 3.2. *Let (X,d) be a metric space and let $S,A:X\to X$ be two self mappings such for all $x,y\in X$ we have:*

$$F(d(Sx,Sy),d(Ax,Ay),d(Ax,Sx),d(Ay,Sy),d(Ax,Sy),d(Ay,Sx))\geq 0,$$

where $F\in\mathcal{F}$, assume that the pair (A,S) is wsc compatible of type (E), then A and S have a unique common fixed point in X.

If we combine Theorem 3.1 with Example 2.14 ($d=0$), we obtain:

Corollary 3.3. *For the self mappings A,B,S and T on a metric space (X,d) such for all $x,y\in X$ we have:*

$$d^p(Sx,Ty))\geq ad^p(Ax,By)+bd^p(Ax,Sx)+cd^p(By,Ty),$$

where $a>1,\ \ c\geq 0,\ b<1$ and $p\in\mathbb{N}^*$, assume that the following conditions hold:

(i) (A,S) is A-subsequentially continuous and A-compatible of type (E),

(ii) (B,T) is B-subsequentially continuous and B-compatible of type (E).

Then A,B,S and T have a unique common fixed point.

Theorem 3.4. *Let (X,d) be a space metric and let A,B,S and T be self mappings on X, such for all $x,y\in X$ we have:*

$$F(d(Sx,Ty)),d(Ax,By),d(Ax,Sx),d(By,Ty),d(Ax,Ty),d(By,Sx))\geq 0,$$

where $F\in\mathcal{F}$ if the four mappings satisfying:

(i) (A,S) is S-subsequentially continuous and S-compatible of type (E),

(ii) (B,T) is B-subsequentially continuous and B-compatible of type (E),

then A,B,S and T have a unique common fixed point.

Remark 3.5. Theorem 3.4 remains true, if we replace:

- (A,S) is A-subsequentially continuous and A-compatible of type (E),
- (B,T) is T-subsequentially continuous and T-compatible of type (E).

Theorem 3.6. *Let (X,d) be a space metric and let A,B and $\{S_i\}_{i\in\mathbb{N}^*}$ be self mappings on X, such for all $x,y\in X$ we have:*

$$F(d(S_ix,S_{i+1}y)),d(Ax,By),d(Ax,S_ix),d(By,S_{i+1}y),d(Ax,S_{i+1}y),d(By,S_ix))\geq 0,\quad (3.2)$$

where $F\in\mathcal{F}$, moreover if the mappings satisfying:

(i) (A,S_i) is wsc and compatible of type (E), for each $i\in\mathbb{N}^*$.

(ii) (B,S_{i+1}) is wsc and compatible of type (E),for each $i\in\mathbb{N}^*$.

Then A,B and $\{S_i\}_{i\in\mathbb{N}^*}$ have a unique common fixed point.

Proof. For $i=1$, we get that the four mappings A,B,S_1 and S_2 satisfying the hypotheses of Theorem 3.1, then they have a unique common fixed point z in X.
For $i=2$, the self mappings A,B,S_2 and S_3 have a unique common fixed point w. If $z\neq w$, by using 3.2 we get:

$$F(d(S_2z,S_3w),d(Az,Bw),d(Az,S_2z),d(B,S_3w),d(Az,S_3w),d(Bw,S_2z))=$$

$$F(d(z,w),d(z,w),0,0,d(z,w),d(z,w))\geq 0,$$

which is a contradiction with (F_2), hence $z=w$.
By continuing in this way, we get A,B and $\{S_i\}_{i\in\mathbb{N}^*}$ have a unique common fixed point z in X, for each $i\in\mathbb{N}^*$. □

Theorem 3.6 improves and generalizes theorem 3.4 of Djoudi[10].

Corollary 3.7. *Let (X, d) be a metric space and let A, B and S three self mappings satisfying for all $x, y \in X$:*

$$F(d(Sx, Sy), d(Ax, Ay), d(Ax, Sx), d(Ay, Sy), d(Ax, Sy), d(Ay, Sx)) \geq 0,$$

where $F \in \mathcal{F}$, if the pairs (A, S) and (B, S) are wsc compatible of type (E), then A and S have a unique common fixed point in X.

Example 3.8. Let $X = [0, 4]$ and d is the euclidian metric, we define A, B, S and T by

$$Ax = Bx = \begin{cases} 2, & 0 \leq x \leq 2 \\ \frac{7}{4}, & 2 < x \leq 4 \end{cases} \qquad Sx = \begin{cases} x, & 0 \leq x \leq 2 \\ 1, & 2 < x \leq 4 \end{cases}$$

$$Tx = \begin{cases} 4 - x, & 0 \leq x \leq 2 \\ 3, & 2 < x \leq 4 \end{cases} .$$

We consider a sequence $\{x_n\}$ which defined for each $n \geq 1$ by:
$x_n = 2 - \frac{1}{n}$, clearly that $\lim_{n\to\infty} Ax_n = 2$ and $\lim_{n\to\infty} Sx_n = 2$, also we have:

$$\begin{aligned} \lim_{n\to\infty} ASx_n &= 2 = A(2) \\ &= S(2), \\ \lim_{n\to\infty} A^2 x_n &= S(2) = 2, \end{aligned}$$

then (A, S) is A-subsequentially continuous and A-compatible of type (E).
On the other hand, consider a sequence defined by: $y_n = 2$, for all $n \geq 0$. It is clear that

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 2,$$

and

$$\begin{aligned} \lim_{n\to\infty} BTy_n &= B(2) \\ &= T(2) = 2, \\ \lim_{n\to\infty} B^2 y_n &= T(2) = 2 \end{aligned}$$

this yields that (B, T) is B-subsequentially continuous and B-compatible of type (E).
We will utilize Corollary 3.3, with $a = \frac{4}{3}$, $b = c = \frac{1}{3}$ and $p = 1$, so for the inequality (**??**), we have the following cases:

(i) For $x, y \in [0, 2]$, we have

$$\begin{aligned} d(Sx, Ty) = 4 - (x + y) &\geq \frac{1}{3}[4 - (x + y)] \\ &= \frac{1}{3}[4d(Ax, Ay) + d(Ax, Sx) + d(By, Ty)]. \end{aligned}$$

(ii) For $x \in [0, 2]$ and $y > 2$, we have

$$\begin{aligned} d(Sx, Ty) = 3 - x &\geq \frac{1}{12}[17 - 4x] \\ &= \frac{1}{3}[4d(Ax, By) + d(Ax, Sx) + d(By, Ty)]. \end{aligned}$$

(iii) For $x \in (2, 4]$ and $y \in [0, 2]$, we have

$$\begin{aligned} d(Sx, Ty) = 3 - y &\geq \frac{1}{12}[15 - 4y] \\ &= \frac{1}{3}[4d(Ax, By) + d(Ax, Sx) + d(By, Ty)]. \end{aligned}$$

(iv) For $x, y \in (2, 4]$, we have

$$d(Sx, Ty) = 2 \geq \frac{2}{3} = \frac{1}{3}[4d(Ax, By) + d(Ax, Sx) + d(By, Ty)].$$

Consequently all hypotheses of Corollary 3.3 are satisfied, therefore 2 is the unique common fixed for A, S and T.

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