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ADDITIVITY OF MULTIPLICATIVE ISOMORPHISMS IN GAMMA RINGS

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Abstract. In this paper, some results given by Martindale III and Rickart are generalized to the $\Gamma-$rings. Using generalized Peirce decomposition of a $\Gamma-$ring given by Mukherjee, it is obtained that any multiplicative isomorphism of $\Gamma-$ring M onto an arbitrary $\Gamma-$ring N is additive.

1 Introduction and Preliminaries

Let R and S be arbitrary associative rings (not necessarily with identity elements). A one-to-one mapping σ of R onto S such that $\sigma(xy) = \sigma(x)\sigma(y)$ for all $x, y \in R$ is called a multiplicative isomorphism of R onto S. The question of when a multiplicative isomorphism is additive has been considered by Rickart [8] and also by Johnson [3]. Martindale III is generalized the main theorem of Rickart's paper in [6] and removed a condition from the theorem. Martindale III, using Peirce decomposition of a ring, showed that any multiplicative isomorphism of R onto an arbitrary ring S is additive.

The concept of a $\Gamma-$ring was introduced by Nobusawa in [5] as a generalization of the ring theory and generalized by Barnes [1] as follows: Let $(M, +)$ and $(\Gamma, +)$ be additive Abelian groups. If there exists a mapping $M \times \Gamma \times M \to M$ (the image of (a, α, b) is denoted by $a\alpha b$ where $a, b \in M$ and $\alpha \in \Gamma$) satisfying the conditions

(i) $(x + y)\alpha z = x\alpha z + y\alpha z,$

(ii) $x\alpha(y + z) = x\alpha y + x\alpha z,$

(iii) $x(\alpha + \beta)z = x\alpha z + x\beta z,$

(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$

for all x, y, z in M and α, β in Γ, then M is called a Γ-ring.

Every ring is a $\Gamma-$ring and many notions on the ring theory are generalized to the $\Gamma-$ring.

Mukherjee [7] is generalized and extended some results on $\Gamma-$rings obtained by some researchers.

In this paper, some results given by Martindale III and Rickart are generalized to the $\Gamma-$rings. Using generalized Peirce decomposition of a $\Gamma-$ring given by Mukherjee, it is obtained that any multiplicative isomorphism of $\Gamma-$ring M onto an arbitrary $\Gamma-$ring N is additive.

A $\Gamma-$ring M is said to be a prime gamma ring if and only if $a\Gamma M\Gamma b = 0$ for $a, b \in M$ implies $a = 0$ or $b = 0$ and M is called completely prime if and only if $a\Gamma b = 0$ implies $a = 0$ or $b = 0$.

Theorem 1.1. [9] Let M be a prime gamma ring. U be a nonzero ideal of M. Then, for $a, b \in M$,

(i) if $U\Gamma a = 0$ or $a\Gamma U = 0$ then $a = 0$,

(ii) if $a\Gamma U\Gamma b = 0$ then $a = 0$ or $b = 0$.

An element e in a Γ-ring is said to be an idempotent, if there exists $\gamma \in \Gamma$ such that $e\gamma e = e$. In this case we also say that e is γ-idempotent.

The following result can be termed as generalized Peirce Decomposition of a gamma ring M.

Theorem 1.2. [7] If e is an idempotent of M then

$$M = e\gamma M\gamma e \oplus e\gamma M\gamma(1 - e) \oplus (1 - e)\gamma M\gamma e \oplus (1 - e)\gamma M\gamma(1 - e).$$

In this Theorem, taking e_1, e_2 instead of e and $1 - e$, respectively, we can write the Peirce

Decomposition of a gamma ring M as

$$M = e_1\gamma M\gamma e_1 \oplus e_1\gamma M\gamma e_2 \oplus e_2\gamma M\gamma e_1 \oplus e_2\gamma M\gamma e_2.$$

Then letting $M_{ij}= e_i\gamma M\gamma e_j$, we may write M as

$$M = M_{11}\oplus M_{12}\oplus M_{21}\oplus M_{22}.$$

It is also known that $e_i\gamma e_j= e_i$, if $i = j$, and $e_i\gamma e_j= 0$, if $i \neq j$.

2 The Main Part

Definition 2.1. Let M and N gamma rings. A one-to-one mapping φ of M onto N such that $\varphi(x\gamma y) = \varphi(x)\gamma\varphi(y)$ for all $x, y \in M$ will be called a multiplicative isomorphism of M onto N.

In this part, e is an idempotent element of M such that $e \neq 0$ and $e \neq 1$ (M need not have an identity) and φ is a multiplicative isomorphism of M onto N. Also $e_1= e$ and $e_2= 1 - e$.

Theorem 2.1. Let M and N be two Γ-rings. Then $\varphi(0) = 0$.

Proof. Since $0 \in N$ and φ is onto, $\varphi(x) = 0$ for some $x \in M$. Then we have

$$\varphi(0) = \varphi(0\gamma x) = \varphi(0)\gamma\varphi(x) = 0.$$

Theorem 2.2. Let M be a prime Γ-ring, N be a Γ-ring. Then

(i) $\varphi(x_{11}+x_{12}) = \varphi(x_{11}) + \varphi(x_{12})$,
(ii) $\varphi(x_{11}+x_{21}) = \varphi(x_{11}) + \varphi(x_{21})$,
(iii) $\varphi(x_{22}+x_{12}) = \varphi(x_{22}) + \varphi(x_{12})$,
(iv) $\varphi(x_{22}+x_{21}) = \varphi(x_{22}) + \varphi(x_{21})$
where $x_{ij}\in M_{ij}$.

Proof. (i) For $x_{11}, x_{12}\in M$, since $\varphi(x_{11}) + \varphi(x_{12}) \in N$ and φ is onto, we have an element $y \in M$ such that $\varphi(y) = \varphi(x_{11}) + \varphi(x_{12})$. Taking $x_{11}= e_1\gamma m\gamma e_1$, $x_{12}= e_1\gamma m\gamma e_2$ and $a_{11}= e_1\gamma n\gamma e_1$, for $a_{11}\in M_{11}$, where e_1 is an idempotent element and $e_2= 1 - e_1$, we have $(x_{11}+ x_{12})\gamma a_{11} = x_{11}\gamma a_{11} + x_{12}\gamma a_{11} = x_{11}\gamma a_{11}$ since $x_{12}\gamma a_{11} = 0$. Then we get, ,

$$\begin{aligned}
\varphi(y\gamma a_{11}) &= \varphi(y)\gamma\varphi(a_{11}) \\
&= (\varphi(x_{11}) + \varphi(x_{12}))\gamma\varphi(a_{11}) \\
&= \varphi(x_{11})\gamma\varphi(a_{11}) + \varphi(x_{12})\gamma\varphi(a_{11}) \\
&= \varphi(x_{11}\gamma a_{11}) + \varphi(x_{12}\gamma a_{11}) \\
&= \varphi((x_{11}+x_{12})\gamma a_{11}) + \varphi(0) \\
&= \varphi((x_{11}+x_{12})\gamma a_{11}).
\end{aligned}$$

Hence we obtain $y\gamma a_{11} = (x_{11}+ x_{12})\gamma a_{11}$ since φ is one to one. Similarly we can see that $y\gamma a_{12}= (x_{11}+ x_{12})\gamma a_{12}$ for $a_{12}\in M_{12}$, $y\gamma a_{21}= (x_{11}+x_{12})\gamma a_{21}$ for $a_{21}\in M_{21}$, $y\gamma a_{22}= (x_{11}+ x_{12})\gamma a_{22}$ for $a_{22}\in M_{22}$. Hence since $a_{11}+a_{12}+a_{21}+a_{22}= a \in M$, it is obtained that $(y - (x_{11}+x_{12}))\gamma M = 0$. Since M is a prime Γ-ring, by Theorem 1.1, we have $y - (x_{11}+x_{12}) = 0$ or $y = x_{11}+ x_{12}$. That is

$$\varphi(x_{11} + x_{12}) = \varphi(x_{11}) + \varphi(x_{12}).$$

(ii) It is obtained $M\gamma(y - (x_{11}+x_{12})) = 0$ with similar operations. Since M is a prime Γ-ring, by Theorem 1.1, we get $\varphi(x_{11}+ x_{21}) = \varphi(x_{11}) + \varphi(x_{21})$, consequently.

(iii) and (iv) is can be seen similarly.

Theorem 2.3. Let M be a prime Γ-ring, N be a Γ-ring. Then

$$\varphi(u_{12}+v_{12}) = \varphi(u_{12}) + \varphi(v_{12})$$

for all $u_{12}, v_{12}\in M_{12}$.

Proof: Since $\varphi(u_{12}) + \varphi(v_{12}) \in N$ and φ is onto, we have an element $y \in M$ such that $\varphi(y) = \varphi(u_{12}) + \varphi(v_{12})$. For $a_{11} \in M_{11}$, taking $a_{11} = e_1\gamma n\gamma e_1$, $u_{12} = e_1\gamma m\gamma e_2$ and $v_{12} = e_1\gamma r\gamma e_2$, we have $u_{12}\gamma a_{11} = 0$ and $v_{12}\gamma a_{11} = 0$. Hence

$$\begin{aligned}\varphi(y\gamma a_{11}) &= \varphi(y)\gamma\varphi(a_{11}) \\ &= (\varphi(u_{12}) + \varphi(v_{12}))\gamma\varphi(a_{11}) \\ &= \varphi(u_{12})\gamma\varphi(a_{11}) + \varphi(v_{12})\gamma\varphi(a_{11}) \\ &= \varphi(u_{12}\gamma a_{11}) + \varphi(v_{12}\gamma a_{11}) \\ &= \varphi(0) + \varphi(0) \\ &= 0.\end{aligned}$$

Since φ is one to one, we get $y\gamma a_{11} = 0$. Similarly, we see that $y\gamma a_{12} = 0$ for $a_{12} \in M_{12}$. Also for $a_{21} = e_2\gamma k\gamma e_1 \in M_{21}$, using the fact that $e_1\gamma a_{21} = 0$, $e_1\gamma v_{12}\gamma a_{21} = v_{12}\gamma a_{21}$ and $u_{12}\gamma v_{12}\gamma a_{21} = 0$, we obtain

$$\begin{aligned}\varphi(y\gamma a_{21}) &= \varphi(y)\gamma\varphi(a_{21}) \\ &= [\varphi(u_{12}) + \varphi(v_{12})]\,\gamma\varphi(a_{21}) \\ &= [\varphi(e_1) + \varphi(u_{12})]\,\gamma\,[\varphi(a_{21}) + \varphi(v_{12}\gamma a_{21})] \\ &= \varphi(e_1 + u_{12})\gamma\varphi(a_{21} + v_{12}\gamma a_{21}), \text{ by Theorem 2.2 (i) and (ii)} \\ &= \varphi((e_1 + u_{12})\gamma(a_{21} + v_{12}\gamma a_{21})) \\ &= \varphi\left[(u_{12} + v_{12})\gamma a_{21}\right].\end{aligned}$$

Hence since φ is one to one, we get $y\gamma a_{21} = (u_{12} + v_{12})\gamma a_{21}$. Similarly we see that $y\gamma a_{22} = (u_{12} + v_{12})\gamma a_{22}$. Therefore, it follows that $(y - (u_{12} + v_{12}))\gamma M = 0$, and so by Theorem 1.1., $y = u_{12} + v_{12}$, that is $\varphi(u_{12} + v_{12}) = \varphi(u_{12}) + \varphi(v_{12})$.

Theorem 2.4. Let M be a prime Γ-ring, N be a Γ-ring. Then $\varphi(u_{11} + v_{11}) = \varphi(u_{11}) + \varphi(v_{11})$ for all $u_{11}, v_{11} \in M_{11}$.

Proof. Since $\varphi(u_{11}) + \varphi(v_{11}) \in N$ and φ is onto, we have an element $y \in M$ such that $\varphi(y) = \varphi(u_{11}) + \varphi(v_{11})$. For $a_{12} \in M_{12}$, we get, since $u_{11}\gamma a_{12}$, $v_{11}\gamma a_{12} \in M_{12}$,

$$\begin{aligned}\varphi(y\gamma a_{12}) &= \varphi(y)\gamma\varphi(a_{12}) \\ &= [\varphi(u_{11}) + \varphi(v_{11})]\,\gamma\varphi(a_{12}) \\ &= \varphi(u_{11})\gamma\varphi(a_{12}) + \varphi(v_{11})\gamma\varphi(a_{12}) \\ &= \varphi(u_{11}\gamma a_{12}) + \varphi(v_{11}\gamma a_{12}) \\ &= \varphi(u_{11}\gamma a_{12} + v_{11}\gamma a_{12}), \text{ by Theorem 2.3.}\end{aligned}$$

Since φ is one to one, this shows that $y\gamma a_{12} = u_{11}\gamma a_{12} + v_{11}\gamma a_{12}$. That is, $(y - (u_{11} + v_{11}))\gamma a_{12} = 0$ or $(y - (x_{11} + u_{11}))\gamma M_{12} = 0$. Now let $y = y_{11} + y_{12} + y_{21} + y_{22}$. Then since $e_1\gamma u_{11} = u_{11}$, $e_1\gamma v_{11} = v_{11}$, $e_1\gamma y_{11} = y_{11}$, $e_1\gamma y_{12} = y_{12}$, $e_1\gamma y_{21} = 0$ and $e_1\gamma y_{22} = 0$, we obtain

$$\begin{aligned}\varphi(y) &= \varphi(u_{11}) + \varphi(v_{11}) \\ &= \varphi(e_1\gamma u_{11}) + \varphi(e_1\gamma v_{11}) \\ &= \varphi(e_1)\gamma\varphi(u_{11}) + \varphi(e_1)\gamma\varphi(v_{11}) \\ &= \varphi(e_1)\gamma\,[\varphi(u_{11}) + \varphi(v_{11})] \\ &= \varphi(e_1)\gamma\varphi(y) \\ &= \varphi(e_1)\gamma\varphi(y_{11} + y_{12} + y_{21} + y_{22}) \\ &= \varphi\,[e_1\gamma(y_{11} + y_{12} + y_{21} + y_{22})] \\ &= \varphi(y_{11} + y_{12}).\end{aligned}$$

Since φ is one to one, we have $y = y_{11} + y_{12}$. Furthermore, we get

$$\begin{aligned}\varphi(y) &= \varphi(u_{11}) + \varphi(v_{11}) \\ &= \varphi(u_{11}\gamma e_1) + \varphi(v_{11}\gamma e_1), \text{ since } u_{11}\gamma e_1 = u_{11}, \text{ and } v_{11}\gamma e_1 = v_{11} \\ &= \varphi(u_{11})\gamma\varphi(e_1) + \varphi(v_{11})\gamma\varphi(e_1) \\ &= (\varphi(u_{11}) + \varphi(v_{11}))\gamma\varphi(e_1) \\ &= \varphi(y)\gamma\varphi(e_1) \\ &= \varphi(y_{11} + y_{12})\gamma\varphi(e_1) \\ &= \varphi((y_{11} + y_{12})\gamma e_1)\end{aligned}$$

$= \varphi(y_{11}\gamma e_1 + y_{12}\gamma e_1)$
$= \varphi(y_{11})$, since $y_{12}\gamma e_1 = 0$.

Since φ is one to one, we have $y = y_{11} \in M_{11}$. Therefore $y - (x_{11}+u_{11}) \in M_{11}$. Then, by theorem 1.1, $(y - (x_{11}+u_{11}))\gamma M_{12} = 0$ implies $y - (u_{11}+v_{11}) = 0$, that is, $y = u_{11}+v_{11}$. So we obtain that $\varphi(u_{11}+v_{11}) = \varphi(u_{11}) + \varphi(v_{11})$ for all $u_{11}, v_{11} \in M_{11}$.

Theorem 2.5. Let M be a prime Γ-ring, N be a Γ-ring and $\varphi : M \to N$ be multiplicative isomorphism. Then φ is additive on $M_{11}+M_{12}$.

Proof. Let $x, y \in M_{11}+M_{12}$. For any $a, b \in M_{11}$ and $c, d \in M_{12}$, we have $x = a+c$, $y = b+d$. Then

$\varphi(x+y) = \varphi((a+c)+(b+d))$
$= \varphi((a+b)+(c+d))$, $a+b \in M_{11}$ and $c+d \in M_{12}$
$= \varphi(a+b) + \varphi(c+d)$, by Theorem 2.2. (i), since $a+b \in M_{11}$, $c+d \in M_{12}$
$= \varphi(a) + \varphi(b) + \varphi(c) + \varphi(d)$, by Theorem 2.4. and Theorem 2.3.
$= \varphi(a+c) + \varphi(b+d)$, by Theorem 2.2.(i)
$= \varphi(x) + \varphi(y)$.

Theorem 2.6. Let M be a prime Γ-ring, N be a Γ-ring. Then any multiplicative gamma isomorphism φ of M onto N is additive.

Proof: Since $\varphi(x) + \varphi(y) \in N$ for $x, y \in M$ and φ is onto, we have an element $z \in M$ such that $\varphi(z) = \varphi(x) + \varphi(y)$.

Let $t \in e\gamma M$. Since

$e\gamma M = e_1\gamma M$
$= e_1\gamma(e_1\gamma M\gamma e_1 + e_1\gamma M\gamma e_2 + e_2\gamma M\gamma e_1 + e_2\gamma M\gamma e_2)$
$= e_1\gamma M\gamma e_1 + e_1\gamma M\gamma e_2$
$= M_{11} + M_{12}$,

we obtain

$\varphi(t\gamma z) = \varphi(t)\gamma\varphi(z)$
$= \varphi(t)\gamma(\varphi(x) + \varphi(y))$
$= \varphi(t)\gamma\varphi(x) + \varphi(t)\gamma\varphi(y)$
$= \varphi(t\gamma x) + \varphi(t\gamma y)$
$= \varphi(t\gamma x + t\gamma y)$, by Theorem 2.5.

So, since φ is one-to-one, we have $t\gamma z = t\gamma x + t\gamma y$. Then $t\gamma(z - (x+y)) = 0$ or $e\gamma M\gamma(z - (x+y)) = 0$. By Theorem 1.1. (ii), we have $z = x+y$. Then we obtained that $\varphi(x+y) = \varphi(x) + \varphi(y)$ for all $x, y \in M$.

Definition 2.2. A gamma ring M is called a Boolean gamma ring if $m\gamma m = m$ for all $m \in M$,$\gamma \in \Gamma$.

Theorem 2.7. Let M be a Boolean gamma ring. Then $m = -m$ for all $m \in M$.

Proof. Since M Boolean gamma ring, $(m+m)\gamma(m+m) = m+m$. Then we have

$m+m = (m+m)\gamma(m+m)$
$= m\gamma m + m\gamma m + m\gamma m + m\gamma m$
$= m+m+m+m$.

Using the cancellation rule in the gamma ring M, we get $m+m = 0$ or $m = -m$.

Theorem 2.8. If M is a Boolean gamma ring, then M is commutative.

Proof. Since M Boolean gamma ring, $(m+n)\gamma(m+n) = m+n$. Then we have

$m+n = (m+n)\gamma(m+n)$
$= m\gamma m + m\gamma n + n\gamma m + n\gamma n$
$= m + m\gamma n + n\gamma m + n$.

Using the cancellation rule in the gamma ring M, we get $m\gamma n + n\gamma m = 0$. Hence, by Theorem 2.7, we obtain $m\gamma n = n\gamma m$.

Theorem 2.9: Let M be a Boolean $\Gamma-$ring and N arbitrary gamma ring. Then any multiplicative isomorphism φ of M onto N is additive.

Proof: Let φ multiplicative mapping from M onto N. Then N is also a Boolean gamma ring.

Let x and y arbitrary elements in M. Since $\varphi(x)+\varphi(y)\in N$ and φ is onto, there exist $m\in M$ so that $\varphi(m)=\varphi(x)+\varphi(y)$. The following equations can be obtained using mapping φ is multiplicative,

$$\begin{aligned}\varphi(x\gamma m+y\gamma m) &= \varphi((x+y)\gamma m)\\ &= \varphi(x+y)\gamma\varphi(m)\\ &= \varphi(x+y)\gamma(\varphi(x)+\varphi(y))\\ &= \varphi(x+y)\gamma\varphi(x)+\varphi(x+y)\gamma\varphi(y))\\ &= \varphi(x\gamma x+y\gamma x)+\varphi(x\gamma y+y\gamma y)\\ &= \varphi(x+y\gamma x)+\varphi(x\gamma y+y),\end{aligned} \tag{1}$$

$$\begin{aligned}\varphi(x\gamma m) &= \varphi(x)\gamma\varphi(m)\\ &= \varphi(x)\gamma(\varphi(x)+\varphi(y))\\ &= \varphi(x\gamma x)+\varphi(x\gamma y)\\ &= \varphi(x)+\varphi(x\gamma y),\end{aligned} \tag{2}$$

and similarly

$$\varphi(y\gamma m)=\varphi(y)+\varphi(x\gamma y). \tag{3}$$

Our aim is to show $\varphi(x+y)=\varphi(x)+\varphi(y)$ for all $x,y\in M$. In the above equalities, if $x\gamma y=0$ (so $y\gamma x=0$ by commutativity), we have for (1), (2) and (3)

$$\varphi(x\gamma m+y\gamma m)=\varphi(x)+\varphi(y)=\varphi(m), \tag{4}$$

$$\varphi(x\gamma m)=\varphi(x), \tag{5}$$

$$\varphi(y\gamma m)=\varphi(y) \tag{6}$$

respectively. Since the mapping φ is one-to-one, equations (4), (5) and (6) imply $x\gamma m+y\gamma m=m$, $x\gamma m=x$ and $y\gamma m=y$. It follows that $m=x+y$ and thus we obtain

$$\varphi(x+y)=\varphi(x)+\varphi(y). \tag{7}$$

If $x\gamma y=y$, then we get the following for (1), (2) and (3), respectively,

$$\begin{aligned}\varphi(x\gamma m+y\gamma m) &= \varphi(x+y)+\varphi(y+y)\\ &= \varphi(x+y)+\varphi(0) \text{ by Teorem 2.7.}\\ &= \varphi(x+y),\end{aligned} \tag{8}$$

$$\varphi(x\gamma m)=\varphi(x)+\varphi(y)=\varphi(m), \tag{9}$$

$$\varphi(y\gamma m)=\varphi(y+y)=0. \tag{10}$$

Since the mapping is one-to-one, equations (8), (9) and (10) imply $x\gamma m+y\gamma m=x+y$, $x\gamma m=m$ and $y\gamma m=0$.Thus, since $m=x+y$, it follows that $\varphi(x+y)=\varphi(x)+\varphi(y)$.

Now, $x+y$ can be written as $x+y=(x+x\gamma y)+(y+x\gamma y)$ and also we have $(x+x\gamma y)\gamma(y+x\gamma y)=0$ by Theorem 2.7. So, using the result of the first case in the above, we obtain

$$\varphi(x+y)=\varphi((x+x\gamma y)+(y+x\gamma y))=\varphi(x+x\gamma y)+\varphi(y+x\gamma y). \tag{11}$$

Furthermore, since $x\gamma(x\gamma y)=x\gamma y$ and $y\gamma(x\gamma y)=x\gamma y$ (by comutativity), using the result of the second case in the above, we have

$$\varphi(x+x\gamma y)=\varphi(x)+\varphi(x\gamma y),\varphi(y+x\gamma y)=\varphi(y)+\varphi(x\gamma y). \tag{12}$$

Substituting the obtained equations in (12) to (11), we obtain

$$\varphi(x+y)=\varphi(x)+\varphi(y)$$

for all $x,y\in M$.

References

[1] Barnes, W., *On the Γ-Rings of Nobusawa*, Pasific J.Math, 18 (1966), 411-22.

[2] Chakraborty, S., Paul, A.C., *On jordan isomorphisms of 2-torsion free prime gamma rings*, Novi Sad J. Math., Vol.40, No.2, 2010, 1-5.

[3] Johnson, R. E., *Rings with unique addition*, Proc. Amer. Math. Soc. 9 (1958), 57-61.

[4] Luh, J., *On the theory of simple gamma rings*, Michigan Math. J., Volume 16, Issue 1 (1969), 65-75.

[5] Nobusawa, N., *On a generalization of the ring theory*, Osaka J. Math. 1 (1964), 81-89.

[6] Wallance, S., Martindale, *When are multiplicative mappings additive*, Proc. Amer. Math. Soc. 21(1969) 695-698.

[7] Mukherjee, R.N., *Some results on Γ-Rings*, Indian J. Pure Apply. Math., 34(6) : 991-994.

[8] Rickart, C.E., *One-to-one mappings of ring and lattices*, Bull. Amer.Math.Soc. 54 (1948), 758-764.

[9] Soytürk, M., *The commutativity in prime gamma rings with derivation*, Tr. J. Mathematics, 18 (1994), 149-155.

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