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MODIFIED γ GRAPH-$G(\gamma_m)$ OF SOME GRID GRAPHS

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Abstract Gerd H.Frickle et.al [1] introduced γ-graph of a graph. Consider the family of all γ-sets in a graph G and we define $G(\gamma) = (V(\gamma), E(\gamma))$ to be the graph whose vertices correspond 1 to 1 with the γ-sets of G and two γ-sets say S_1 and S_2 are adjacent in $G(\gamma)$ if there exist a vertex $v \in S_1$ and a vertex $w \in S_2$ such that v is adjacent to w and $S_1 = S_2 - \{w\} \cup \{v\}$ or equivalently $S_2 = S_1 - \{v\} \cup \{w\}$. The concept of γ-graph inspired us to define Modified γ-graph of a graph. Consider the family of all γ-sets of a graph G and define the modified γ-graph $G(\gamma_m) = (V(\gamma_m), E(\gamma_m))$ of G to be the graph whose vertices $V(\gamma_m)$ correspond 1-1 with the γ-sets of G and two γ-sets S_1 and S_2 form an edge in $G(\gamma_m)$ if there exists a vertex $v \in S_1$ and $w \in S_2$ such that $S_1 = S_2 - \{w\} \cup \{v\}$ and $S_2 = S_1 - \{v\} \cup \{w\}$. In this paper we determine $G(\gamma_m)$ of some grid graphs.

1 Introduction

By a graph we mean a finite, undirected, connected graph without loops and multiple edges. For graph theoretical terms we refer Harary [2] and for terms related to domination we refer Haynes et al.[3, 4]. A set $S \subseteq V$ is said to be a dominating set of G if every vertex in $V - S$ is adjacent to some vertex in S . The domination number of G is the minimum cardinality taken over all dominating sets of G and is denoted by $\gamma(G)$. A graph G is *regular* of degree r if every vertex of G has degree r. Such graphs are called r-*regular* graphs.

A *path* is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, ...,$ e_{n-1}, v_n, which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \leq i \leq n-1$. A path on n vertices is denoted by P_n. A path $v_1, e_1, v_2, e_2, ..., e_{n-1},$ v_n, e_n, v_1 is called a cycle and a cycle on n vertices is denoted by C_n. A graph $G = (V, E)$ is called a bipartite graph if $V = V_1 \cup V_2$ and every edge of G joins a vertex of V_1 to a vertex of V_2. If $|V_1| = m$, $|V_2| = n$ and if every vertex of V_1 is adjacent to every vertex of V_2, then G is called a complete bipartite graph and is denoted by $K_{m,n}$. $K_{1,n}$ is called a star. The bistar $B_{n,n}$ is the graph obtained by joining the centers of two copies of $K_{1,n}$ by an edge. If G is a graph on n vertices in which every vertex is adjacent to every other vertex, then G is called a complete graph and is denoted by K_n.

For any graph G, its complement $\overline{G}$ is defined to be the graph whose vertex set is same as that of G and two vertices in $\overline{G}$ are adjacent if and only if they are not adjacent in G. Let G_1 and G_2 be two graphs with disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their *Cartesian product* $G_1 \times G_2$ is defined to be the graph whose vertex set is $V_1 \times V_2$ and edge set is $\{(u_1, v_1), (u_2, v_2)|$ either $u_1 = u_2$ and $v_1v_2 \in E_2$ or $v_1 = v_2$ and $u_1u_2 \in E_1\}$. A *Grid graph* is the Cartesian product of two paths.

Gerd H.Frickle et.al [1] introduced γ-graph of a graph. Consider the family of all γ-sets in a graph G and we define $G(\gamma) = (V(\gamma), E(\gamma))$ to be the graph whose vertices correspond 1 to 1 with the γ-sets of G and two γ-sets say S_1 and S_2 are adjacent in $G(\gamma)$ if there exist a vertex $v \in S_1$ and a vertex $w \in S_2$ such that v is adjacent to w and $S_1 = S_2 - \{w\} \cup \{v\}$ or equivalently $S_2 = S_1 - \{v\} \cup \{w\}$. The concept of γ-graph inspired us to define Modified γ-graph of a graph.

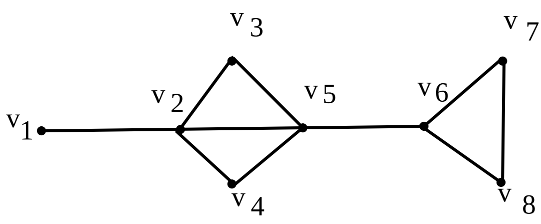


Fig. 2.1

2 Main results

Definition 2.1. Consider the family of all $\gamma-$sets of a graph G and define the modified $\gamma-$graph $G(\gamma_m) = (V(\gamma_m), E(\gamma_m))$ of G to be the graph whose vertices $V(\gamma_m)$ correspond 1-1 with the $\gamma-$sets of G and two $\gamma-$sets S_1 and S_2 form an edge in $G(\gamma_m)$ if there exists a vertex $v \in S_1$ and $w \in S_2$ such that $S_1 = S_2 - \{w\} \cup \{v\}$ and $S_2 = S_1 - \{v\} \cup \{w\}$. Thus two $\gamma-$sets are said to be adjacent if they differ by one vertex.

Example 2.2. Consider the graph G given in Fig. 2.1. Here $S_1 = \{v_2, v_6\}, S_2 = \{v_2, v_7\}, S_3 = \{v_2, v_8\}$ are the γ-sets of G. The Modified γ- graph $G(\gamma_m)$ is given in Fig. 2.2.

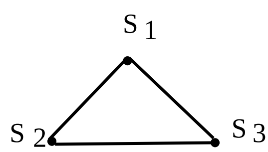


Fig. 2.2

Proposition 2.3. $P_{3k}(\gamma_m) \cong K_1$.

Proposition 2.4. $P_{3k+2}(\gamma_m) \cong P_{k+2}$.

Proposition 2.5. $P_4(\gamma_m) \cong C_4$.

Proof. Let v_1, v_2, v_3, v_4 be the vertices of the path P_4. Then it has 4 γ-sets namely $S_1 = \{v_1, v_3\}, S_2 = \{v_1, v_4\}, S_3 = \{v_2, v_3\}, S_4 = \{v_2, v_4\}$. For $i = 1, 2, 3, 4$, deg $S_i = 2$. Hence $P_4(\gamma_m)$ has 4 vertices and each vertex is of deg 2 so that $P_4(\gamma_m) \cong C_4$. □

Proposition 2.6. *$P_{3k+1}(\gamma_m)$ is isomorphic to the graph of order $\frac{k^2+5k+2}{2}$ for $k \geq 2$.*

Proof. **Case (1):** $k = 2$

The path obtained is P_7 and it has 8 γ-sets namely $S_1 = \{v_2, v_5, v_7\}, S_2 = \{v_2, v_5, v_6\}$, $S_3 = \{v_2, v_4, v_6\}, S_4 = \{v_2, v_3, v_6\}, S_5 = \{v_2, v_4, v_7\}, S_6 = \{v_1, v_4, v_7\}, S_7 = \{v_1, v_4, v_6\}$ and $S_8 = \{v_1, v_3, v_6\}$. The total number of γ-sets of P_7 is 8. So the order of $P_7(\gamma_m)$ is 8.

Case (2): $k \geq 3$

Step (i): Let $v_1, v_2, v_3, \ldots, v_{3k+1}$ be the vertices of the path P_{3k+1}. Consider the 4 $\gamma-$ sets $S_1 = \{v_1, v_4, v_7, \ldots, v_{3k-2}, v_{3k+1}\}, S_2 = \{v_1, v_4, v_7, \ldots, v_{3k-2}, v_{3k}\}$
$S_3 = \{v_2, v_5, v_8, \ldots, v_{3k-1}, v_{3k+1}\}$, $S_4 = \{v_2, v_5, v_8, v_{3k-1}, v_{3k}\}$ of P_{3k+1}. S_1 is the only γ-set with first the vertex v_1 and last vertex v_{3k+1}.

Step (ii): Now fixing the first and last vertices of S_2 and changing from the 2nd vertex we get $S_5 = \{v_1, v_3, v_6, v_9, \ldots, v_{3k+1}\}$. Similarly changing from the $3^{rd}, 4^{th}, 5^{th}, \ldots, k^{th}$ vertex we get $(k-2)$ γ-sets. Thus in step (ii) we get $(k-1)$ γ-sets.

Step (iii): Now fixing the first and last vertices of S_3 and changing from the 2^{nd} vertex we get $\{v_2, v_4, v_7, v_{10}, v_{3k-2}, v_{3k+1}\}$. Similarly by changing from the third ,fourth, fifth, $\ldots, k^{th}$ vertex we get $(k-2)$ γ-sets. Thus step(iii) contains $(k-1)$ k-sets.

Step (iv): $(k-1)$ γ-sets have 2 adjacent vertices. They are $\{v_2, v_3, v_6, v_9, \ldots, v_{3k}\}$, $\{v_2, v_5, v_6, v_9, \ldots, v_{12}, \ldots, v_{3k}\}, \ldots, \{v_2, v_5, v_8, \ldots, v_{3k-4}, v_{3k-3}, v_{3k}\}$. Thus this step contains $(k-1)$ γ-sets. [since $\{v_2, v_5, v_8, \ldots, v_{3k-4}, v_{3k-1}, v_{3k}\} = S_4$].

Step (v): The last γ-set of step (iv) is $\{v_2, v_5, v_8, v_{11}, \ldots, v_{3k-4}, v_{3k-3}, v_{3k-3}\}$.....$(1)$. Fixing the first vertex and last two vertices of (1) changing from the 2^{nd} vertex we get $\{v_2, v_4, v_7, v_{10}, \ldots, v_{3k-5}, v_{3k-3}, v_{3k}\}$. Then changing from the $3^{rd}, 4^{th}, 5^{th}, \ldots, (k-1)^{th}$ vertex we get $(k-3)$ γ-sets. Thus step (v) has $(k-2)$ γ-sets. [Here the last γ-set is $\{v_2, v_5, v_8, \ldots, v_{3k-7}, v_{3k-5}, v_{3k-3}, v_{3k}\}$].....(2).

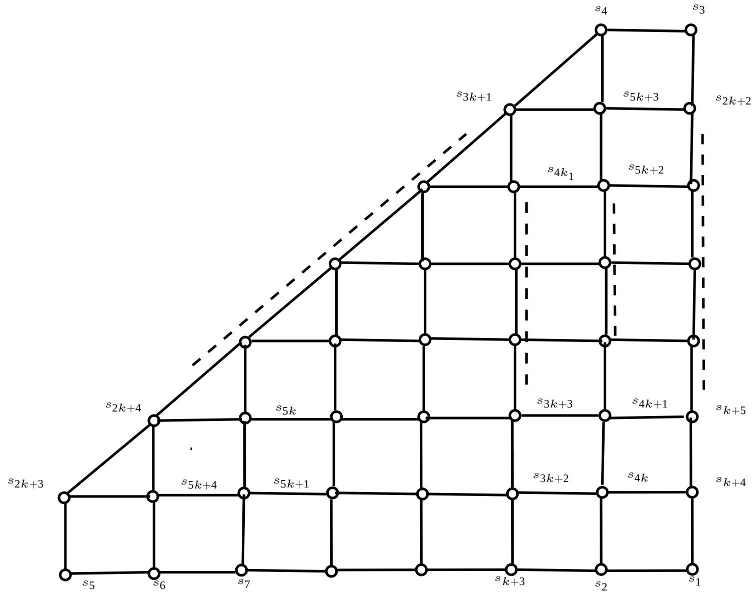
Step(vi): Now consider the γ-set $\{v_2, v_5, v_8, \ldots, v_{3k-7}, v_{3k-4}, v_{3k-2}, v_{3k}\}$....(3). Fixing the first vertex and last two vertices of (3) and changing from the 2nd vertex we get $\{v_2, v_4, v_7, v_{10}, \ldots, v_{3k-2}, v_{3k}\}$. Similarly changing from th $3^{rd}, 4^{th}, 5^{th}, \ldots, (k-1)^{th}$ vertex we get $(k-2)$ γ-sets. Thus step (vi) has $(k-1)$ γ-sets including (3).

Step (vii): Now consider all the γ-sets containing 3 alternate vertices. They are $\{v_2, v_4, v_6, v_9, v_{12}, \ldots, v_{3k-6}, v_{3k-3}, v_{3k}\}$, $\{v_2, v_5, v_7, v_9, v_{12}, v_{15}, \ldots, v_{3k-6}, v_{3k-3}, v_{3k}\}, \ldots, \{v_2, v_5, v_8, \ldots, v_{3k-10}, v_{3k-8}, v_{3k-6}, v_{3k-3}, v_{3k}\}$. Thus step (vii) has $(k-3)$ γ-sets. [The last 2 γ-sets are (2) of step (v) and (3) of step (vi)].

Step (viii): Using the above $(k-3)$ γ-sets we can write $(k-3)C_2\gamma$-sets with 2 pairs of alternate vertices with first vertex v_2 and last 2 vertices v_{3k-3}, n_{3k}. There are no γ-sets other than the γ-sets got by the above 8 steps. Thus total number of γ-sets

$$\begin{aligned}
&= 4 + k - 1 + k - 1 + k - 1 + k - 2 + k - 1 + k - 3 + (k-3)C_2 \\
&= 6k - 5 + \frac{k^2 - 7k + 12}{2} \\
&= \frac{12k - 10 + k^2 - 7k + 12}{2} \\
&= \frac{k^2 + 5k + 2}{2.}
\end{aligned} \tag{2.1}$$

□



$Fig.2.3$

Remark 2.7. (1) Vertices of steps $(v), (vi), (vii)$ and $(viii)$ are of deg 4. Vertices of step (ii) except S_5, vertices of step (iii) and vertices of step (iv) are of deg 3 and S_1, S_3, S_5, are the only 3 vertices of deg 2.
(2) Each dominating set is some number of swaps from $S_1, S_2, S_5, S_6, S_7,$
$\ldots, S_{k+3}$ and hence $P_{3k+1}(\gamma_m)$ is a connected graph and is isomorphic to the graph given in Fig. 2.3.

Theorem 2.8. $(P_2\Box P_2)(\gamma_m)$ *is a* 4-*regular graph of 6 vertices.*

Proof. Let $\{u_1, u_2, v_1, v_2\}$be the vertices of the grid $P_2\Box P_2$. Let $S_1 = u_1, u_2, S_2 = u_1, v_1, S_3 = \{u_1, v_2\}, S_4 = \{u_2, v_1\}, S_5 = \{u_2, v_2\}, S_6 = \{v_1, v_2\}$ are the 6 γ- sets of $P_2\Box P_2$. Here S_1 is adjacent to S_2, S_3, S_4, S_6; S_2 is adjacent to S_1, S_3, S_4, S_6; S_3 is adjacent to S_1, S_2, S_5, S_6; S_4 is adjacent to $S_1 S_2, S_5, S_6$; S_5 is adjacent to S_1, S_3, S_4, S_6 and S_6 is adjacent to S_1, S_3, S_4, S_6. □

Theorem 2.9. $(P_2\Box P_4)(\gamma_m)$ *is a* 3- *regular graph with 12 vertices.*

Proof. Consider the grid $P_2\Box P_4$ given in Fig. 2.4.

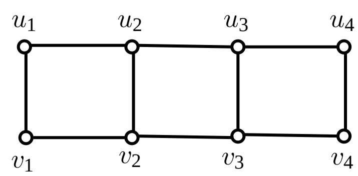


Fig.2.4

Let u_1, u_2, u_3, u_4 and v_1, v_2, v_3, v_4 be the vertices of the first and second row of the grid $P_2\Box P_4$. $S_1 = \{u_1, v_3, v_4\}$,$S_2 = \{u_1, v_3, u_4\}$,$S_3 = \{u_1, v_3, u_3\}$,$S_4 = \{v_1, v_3, u_3\}$, $S_5 = \{u_1, v_2, u_4\}$, $S_6 = \{u_1, u_2, v_4\}$,$S_7 = \{u_1, u_3, v_4\}$,$S_8 = \{v_1, u_3, u_4\}$, $S_9 = \{v_1, u_2, u_4\}$,$S_{10} = \{v_1, v_2, u_4\}$,$S_{11} = \{u_2, v_2, u_4\}$,$S_{12} = \{u_2, v_2, v_4\}$ are the γ-sets of $P_2\Box P_4$. Here S_1 is adjacent to S_2, S_3, S_6, S_2 is adjacent to S_1, S_3, S_5, S_3 is adjacent to S_1, S_2, S_4, S_4 is adjacent to S_3, S_7, S_8; S_5 is adjacent to S_2, S_{10}, S_{11}; S_6 is adjacent to S_1, S_9, S_{12}; S_7 is adjacent to S_4, S_8, S_9; S_8 is adjacent to S_4, S_7, S_{10}; S_9 is adjacent to S_6, S_7, S_{12}; S_{10} is adjacent to S_5, S_8, S_{11}; S_{11} is adjacent to S_5, S_{10}, S_{12} and S_{12} is adjacent to S_6, S_9, S_{11}. The graph $(P_2\Box P_4)$ (γ_m) is given in Fig. 2.5.

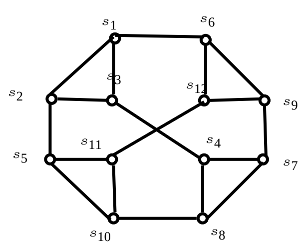


Fig. 2.5

Thus $(P_2\Box P_4)(\gamma_m)$ is a cubic graph with 12 vertices. □

Theorem 2.10. $(P_2\Box P_6)(\gamma_m)$ *is isomorphic to the graph G given in Fig.* 2.6.

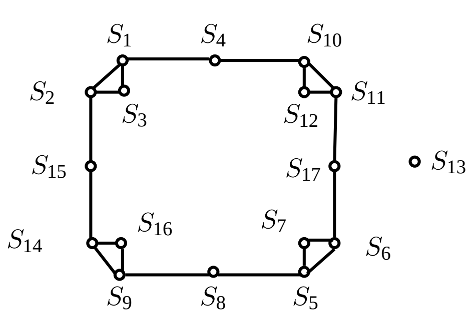


Fig.2.6

Proof. Consider the grid $P_2\square P_6$ given in Fig. 2.7. Let $u_1, u_2, u_3, u_4, u_5, u_6$ and $v_1, v_2, v_3, v_4, v_5, v_6$ be the vertices of the first and second rows of the grid $P_2\square P_4$. S_1=$\{u_1, v_3, u_5, v_6\}$, S_2=$\{u_1, v_3, u_5, v_6\}$, S_3=$\{u_1, v_3, u_5, v_5\}$, S_4=$\{u_1, v_3, u_4, v_6\}$, S_5=$\{v_1, u_3, v_5, u_6\}$, S_6=$\{v_1, u_3, v_5, v_6\}$, S_7=$\{v_1, u_3, v_5, u_6\}$, S_8=$\{v_1, u_3, v_4, u_6\}$, S_9=$\{v_1, u_3, v_4, u_6\}$, S_{10}=$\{u_1, v_2, u_4, v_6\}$, S_{11}=$\{v_1, v_2, u_4, v_6\}$, S_{12}=$\{u_2, v_2, u_4, v_6\}$, S_{13}=$\{u_2, v_2, u_5, v_5\}$, S_{14}=$\{u_1, u_2, v_4, u_6\}$, S_{15}=$\{u_1, v_3, v_4, u_6\}$, S_{16}=$\{u_2, v_2, v_4, u_6\}$, S_{17}=$\{v_1, u_3, u_4, v_6\}$ are the γ-sets of $P_2\square P_6$. Here S_1 is adjacent to S_2, S_3, S_4; S_2 is adjacent to S_1, S_3, S_{15}; S_3 is adjacent to S_1, S_2; S_4 is adjacent to S_1, S_{10}; S_5 is adjacent to S_6, S_7, S_8; S_6 is adjacent to S_5, S_7, S_{17}; S_7 is adjacent to S_5, S_6; S_8 is adjacent to S_5, S_9; S_9 is adjacent to S_8, S_{14}, S_{16}; S_{10} is adjacent to S_4, S_{11}, S_{12}; S_{11} is adjacent to S_{10}, S_{12}, S_{17}; S_{12} is adjacent to S_{10}, S_{11}; S_{14} is adjacent to S_9, S_{15}, S_{16}; S_{15} is adjacent to S_2, S_{14}; S_{16} is adjacent to S_9, S_{14}; S_{17} is adjacent to S_6, S_{11} and S_{13} is an isolated vertex. Thus we get the graph given in Fig. 2.6.

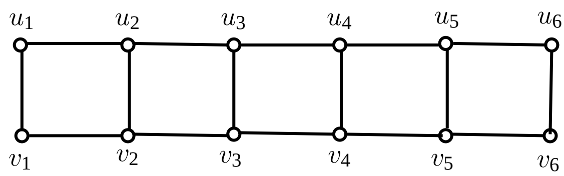


Fig.2.7

□

Theorem 2.11. $(P_2\square P_n)(\gamma_m)$ *where* $n = 2k, k \geq 4$ *is isomorphic to the graph* G *with order* $4\left\lfloor\frac{n+1}{2}\right\rfloor$ *of which 8 vertices have deg 3 and the remaining vertices have deg 2. The graph* G *is given in Fig.* 2.8.

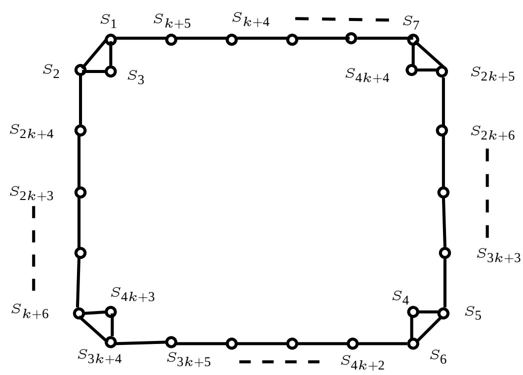


Fig.2.8

Proof.

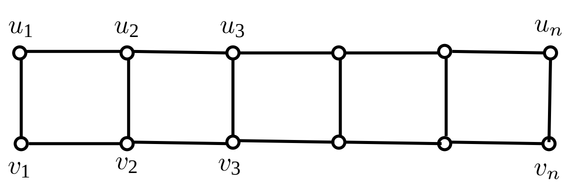


Fig. 2.9

Consider the grid $P_2\square P_n$ when $n = 2k$ that is given in Fig. 2.9. Let $u_1, u_2, u_3, \ldots, u_n$ and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the 1^{st} and 2^{nd} rows of the grid $P_2\square P_n$ when $n = 2k$. We know that $P_2\square P_n$ has domination number $\left\lfloor\frac{n+1}{2}\right\rfloor$. Consider the 6 γ-sets $S_1 = \{u_1, v_3, u_5, v_7, \ldots, u_{n-3}, u_{n-1}, u_n\}, S_2 = \{u_1, v_3, u_5, v_7, \ldots, v_{n-3}, u_{n-1}, v_n\}, S_3 = \{u_1, v_3, u_5, v_7, \ldots, v_{n-3}, u_{n-1}, v_{n-1}\}, S_4 = \{v_1, u_3, v_5, v_7, \ldots, u_{n-3}, v_{n-1}, u_n\}, S_5 = \{v_1, u_3, v_5, u_7, \ldots, u_{n-3}, v_{n-1}, u_n\}, S_6 = \{v_1, u_3, v_5, u_7, \ldots, u_{n-3}, v_{n-1}, v_n\}$ of $P_2\square P_n$.

Case(1): k is odd

Step(i): Fixing the first and last vertices of S_1 and changing from the 2^{nd} vertex we get $S_7 = \{u_1, u_2, v_4, u_6, u_8, u_{10} \ldots, u_{n-4}, v_{n-2}, u_n\}$. Fixing the first 2 vertices and changing from the 3^{rd} vertex we get $S_8 = \{u_1, v_3, v_4, u_6, v_8, u_{10} \ldots, v_{n-2}, u_n\}$. Proceeding like this, fixing the $(k-1)$ vertices and changing from the k^{th} vertex we get $S_{k+5} = \{u_1, v_3, u_5, v_7, u_9, \ldots, v_{n-3}, v_{n-2}, u_n\}$. Thus we get $(k-1)$ γ-sets in Step(i).

Step(ii): Now fixing the first vertex of S_2 and changing the 2^{nd} vertex we get $S_{k+6} = \{u_1, v_2, u_4, v_6, u_8, v_{10} \ldots, u_{n-2}, v_n\}$. Fixing first 2 vertices of S_2 and changing from the 3^{rd} vertex we get $\{u_1, v_3, u_4, v_6, u_8, v_{10} \ldots, v_{n-4}, u_{n-2}, v_n\}$. Continuing upto the change of k^{th} vertex of S_2 we get $(k-1)$ γ-sets. Here the last γ-set is $\{u_1, v_3, u_5, v_7, \ldots, u_{n-5}, v_{n-3}, u_{n-2}, v_k\} = S_{2k+4}$.

Step(iii): S_3 is the only γ-set with first vertex u_1 and first 2 vertices u_{n-1}, v_{n-1} and S_4 is the only γ-set with first vertex v_1 and last 2 vertices v_{k-1}, u_{k-1}.

Step(iv): Fixing the first vertex of S_5 and changing from the 2^{nd} vertex we get $S_{2k+5} = \{v_1, u_2, v_4, u_6, v_8, u_{10}, \ldots, u_{n-4}, v_{n-2}, u_n\}$. Fixing the first 2 vertices and changing from the 3^{rd} vertex we get $\{v_1, u_3, v_4, u_6, v_8, u_{10} \ldots, v_{n-2}, u_n\}$. Continuing upto the change in k^{th} vertex we get $S_{3k+3} = \{v_1, u_3, v_5, u_7, \ldots, u_{n-3}, v_{n-2}, u_n\}$. Thus we get step (iv) has $(k-1)$ γ-sets.

Step(v): Fixing the first vertex of S_6 and changing from the 2^{nd} vertex we get $S_{3k+4} = \{v_1, v_2, u_4, v_6, v_8, v_{10}, \ldots, v_{n-4}, u_{n-2}, u_n\}$. Fixing the first 2 vertices of S_6 and changing from the 3^{rd} vertex we get $\{v_1, u_3, u_4, v_6, u_8, v_{10}, \ldots, v_{n-4}, u_{n-2}, u_n\}$. Proceeding in a similar manner we arrive at the set $S_{4k+2} = \{v_1, u_3, v_5, u_7, \ldots, u_{n-3}, u_{n-2}, v_n\}$. Thus we get $(k-1)$ γ-sets.

Step(vi): $S_{4k+3} = \{u_2, v_2, u_4, v_6, u_8, v_{10} \ldots, u_{n-2}, v_n\}$ and $S_{4k+4} = \{u_2, v_2, v_4, u_6, v_8, u_{10}, \ldots, v_{n-2}, u_n\}$ are the 2 γ-sets with first 2 vertices u_2, v_2 and last 2 vertices v_n, u_n respectively. Thus total number of γ-sets

$$\begin{aligned} &= 6 + k - 1 + k - 1 + k - 1 + 2 \\ &= 4k - 4 + 8 \\ &= 4k + 4 \\ &= 4(k+1) \\ &= 4\left\lfloor \frac{2k+1}{2} \right\rfloor \end{aligned} \tag{2.2}$$

Case(2): k is even

Step(i): Fixing the first vertex of S_1 and changing from the 2^{nd} vertex we get $S_7 = \{u_1, v_2, u_4, v_6, \ldots, v_{n-2}, u_n\}$. Fixing the first 2 vertices of S_1 and changing from the 3^{rd} vertex we get $S_8 = \{u_1, v_3, u_4, v_6, u_8, \ldots, v_{n-2}, u_n\}$. Proceeding like this we get (by changing from the k^{th} vertex) $S_{k+5} = \{u_1, v_3, u_5, v_7 \ldots, u_{n-3}, v_{n-2}, u_n\}$. Thus step(i) has $(k-1)$ γ-sets.

Step(ii): Fixing the first vertex of S_2 and changing from the 2^{nd} vertex we get $S_{k+6} = \{u_1, u_2, v_4, u_6, v_8, \ldots, v_{n-4}, v_{n-2}, v_n\}$. Fixing the first 2 vertices of S_2 and changing from the 3^{rd} vertex we get $S_{k+7} = \{u_1, v_3, v_4, u_6, v_8, \ldots, u_{n-2}, v_n\}$. Proceeding like this we arrive at the set $\{u_1, v_3, u_5, u_7, \ldots, u_{n-3}, u_{n-2}, v_n\}$. Thus this step contains $(k-1)$ γ-sets.

Step(iii): S_3 and S_4 are the only 2 γ-sets with last 2 vertices u_{n-1}, v_{n-1} and first vertex u_1 and v_1 repectively.

Step(iv): Fixing the first vertex of S_5 and changing from the 2^{nd} vertex we get $S_{2k+5} = \{v_1, v_2, v_4, v_6, u_8, v_{10} \ldots, v_{n-2}, v_n\}$. Fixing the first 2 vertices of S_5 and changing from the 3^{rd} vertex we get $S_{2k+6} = \{v_1, u_3, v_4, v_6, u_8, \ldots, v_{n-2}, u_n\}$. Proceeding like this we get $S_{3k+3} = \{v_1, u_3, v_5, u_7, \ldots, v_{n-3}, v_{n-2}, u_n\}$. Thus Step(iv) has $(k-1)$ γ-sets.

Step(v): Fixing the first vertex of S_6 and changing from the 2^{nd} vertex we get $S_{3k+4} = \{v_1, u_2, v_4,$

$u_6, \ldots, u_{n-2}, v_n\}$. Fixing the first 2 vertices of S_6 and changing from the 3^{rd} vertex we get $S_{3k+5} = \{v_1, u_3, v_4, u_6, \ldots, u_{n-2}, v_n\}$. Proceeding like this by changing k^{th} vertex we get $S_{4k+2} = \{v_1, u_3, v_5, u_7, \ldots, v_{n-3}, v_{n-2}, v_n\}$. Thus Step(v) has $(k-1)$ $\gamma-$sets.

Step(vi): $S_{4k+3} = \{u_2, v_2, u_4, v_6, v_8, \ldots, v_{n-2}, u_n\}$ and $S_{4k+4} = \{u_2, v_2, u_4, u_6, v_8, \ldots, v_{n-2}, v_n\}$ are only 2 γ-sets with first 2 vertices u_2, v_2 and the last vertices u_n, v_n respectively. Thus the total number of γ-sets $= 6 + k - 1 + k - 1 + k - 1 + 2 = 4(k+1) = 4\left\lfloor \frac{2k+1}{2} \right\rfloor$. Thus in both cases we get the total number of γ-sets of $P_2 \square P_n = 4\left\lfloor \frac{n+1}{2} \right\rfloor$.

Here S_1, S_2, S_3; S_4, S_5, S_6; S_7, S_{2k+5}, S_{4k+4}; $S_{k+6}, S_{3k+4}, S_{4k+3}$ form a triangle. $S_1, S_{k+5}, S_{k+4}, S_{k+3}, \ldots, S_7$ form a path ; $S_{2k+5}, S_{2k+6}, S_{2k+7}, \ldots, S_{3k+2}, S_{3k+3}, S_5$ form a path; $S_2, S_{2k+4}, S_{2k+3}, \ldots, S_{k+6}$ form a path ; $S_{3k+4}, S_{3k+5}, \ldots S_{4k+2}, S_6$ form a path in $(P_2 \square P_n)(\gamma_m)$.

Thus $(P_2 \square P_n)(\gamma_m)$, where $n = 2k, k \geq 4$ is connected and is given in Fig. 2.8. □

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