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GENERALIZED RECURRENT AND GENERALIZED RICCI RECURRENT GENERALIZED SASAKIAN SPACE FORMS

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Abstract The purpose of the present paper is to study the notion of generalized recurrent and generalized Ricci recurrent generalized Sasakian space forms and to study various properties related with the existence of such notion.

1 Introduction

In differential geometry, the curvature of a Riemannian manifold (M, g) plays a fundamental role. As well known the sectional curvature of a manifold determine the curvature tensor R-completely. A Riemannian manifold with constant sectional curvature c is called a real-space form and its curvature tensor is given by the equation

$$R(X, Y)Z = c\{g(Y, Z)X - g(X, Z)Y\}, \tag{1.1}$$

for any vector fields X, Y, Z on M. Models for these spaces are the Euclidean space $(c = 0)$, the sphere $(c > 0)$ and the Hyperbolic space $(c < 0)$.

A Sasakian manifold $M(\phi, \xi, \eta, g)$ is said to be a Sasakian space form if all the ϕ-sectional curvatures $K(X \wedge \phi X)$ are equal to a constant c, where $K(X \wedge \phi X)$ denotes the sectional curvature of the section spanned by a unit vector field X, orthogonal to ξ and ϕX. In such a case, Riemannian curvature tensor of M is given by

$$\begin{aligned} R(X, Y)Z \;=\;& \frac{c+3}{4}\{g(Y, Z)X - g(X, Z)Y\} \\ &+\frac{c-1}{4}\{g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z\} \\ &+\frac{c-1}{4}\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ &-g(Y, Z)\eta(X)\xi\}. \end{aligned} \tag{1.2}$$

In 2004, P. Alegre, D. E. Blair and A. Carriazo [1] introduced the concept of generalized Sasakian space forms.

A generalized Sasakian space form is an almost contact metric manifold $M(\phi, \xi, \eta, g)$ whose curvature tensor is given by

$$\begin{aligned} R(X, Y)Z \;=\;& f_1\{g(Y, Z)X - g(X, Z)Y\} \\ &+f_2\{g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z\} \\ &+f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ &-g(Y, Z)\eta(X)\xi\}, \end{aligned} \tag{1.3}$$

where f_1, f_2, f_3 are differentiable functions on M and X, Y, Z are vector fields on M. Sasakian space forms appear as natural examples of generalized Sasakian space forms, with constant functions $f_1 = \frac{c+3}{4}$, $f_2 = \frac{c-1}{4}$ and $f_3 = \frac{c-1}{4}$, where c denotes constant ϕ-sectional curvature. The generalized Sasakian space forms have been extensively studied by [2, 3, 4, 8, 10, 17, 19] and many others.

A Riemannian manifold (M, g) is called generalized recurrent [9] if its curvature tensor R satisfies the condition

$$(\nabla_W R)(X,Y)Z = \alpha(W)R(X,Y)Z + \beta(W)[g(Y,Z)X - g(X,Z)Y],$$

where α and β are two 1-forms, β is non zero and these are defined by

$$\alpha(X) = g(X, \rho_1) \text{ and } \beta(X) = g(X, \rho_2), \forall X \in TM,$$

ρ_1 and ρ_2 being the vector fields associated to the 1-form α and β.

Again a non-flat Riemannian manifold M is called a generalized Ricci recurrent manifold [15] if its Ricci tensor S satisfies the condition

$$(\nabla_X S)(Y,Z) = \alpha(X)S(Y,Z) + \beta(X)g(Y,Z), \tag{1.4}$$

where ∇ is Levi-Civita connection of the Riemannian metric g and α, β are 1-forms on M and these are defined by

$$\alpha(X) = g(X, \rho_1) \text{ and } \beta(X) = g(X, \rho_2), \forall X \in TM,$$

being ρ_1 and ρ_2 the vector fields associated to the 1-forms α and β.

In particular, if the 1-form β vanishes identically, then M reduces to the well known Ricci-recurrent manifold [16].

The notion of generalized recurrent manifolds was introduced by U. C. De and N. Guha [9]. In [12], Q. Khan studied generalized recurrent and generalized Ricci recurrent Sasakian manifolds. In 2002, Kim, Prasad and Tripathi [13] studied generalized Ricci recurrent Trans-Sasakian manifolds. In 2014, V. J. Khairnar [11] studied generalized recurrent and Ricci recurrent Lorentzian Trans-Sasakian manifolds. On the other hand, many authors recently have studied the basic results with some of its properties.

The purpose of this paper is to study generalized recurrent and generalized Ricci recurrent generalized Sasakian space form. The paper is arranged as follows:

After the introduction in Section 2, we give the preliminaries of generalized Sasakian space forms needed for this paper. In section 3, we give the notion of generalized recurrent generalized Sasakian space forms and obtain the relations between, the associated 1-forms α and β. We also obtain the separate theorem for the relationship between the 1-forms α and β. Section 4 is devoted to a generalized concircular recurrent generalized Sasakian space forms. In Section 5, for generalized Ricci recurrent generalized Sasakian space forms, a relation between the 1-forms α and β is established. In the last section, an expression for Ricci tensor of a generalized Ricci recurrent generalized Sasakian space form with cyclic Ricci tensor is obtained.

2 Preliminaries

An odd dimensional manifold $M^{2n+1} (n \geq 1)$ is said to admit an almost contact structure, sometimes called a (ϕ, ξ, η)-structure, if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying [6, 7]:

$$\eta(\xi) = 1, \tag{2.1}$$

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.2}$$

$$g(X, \xi) = \eta(X), \tag{2.3}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.4}$$

$$(\nabla_X \eta)Y = g(\nabla_X \xi, Y), \tag{2.5}$$

$$g(X,\phi Y)=-g(\phi X,Y), \tag{2.6}$$

for any vector fields X, Y on M^{2n+1}. In particular, in an almost contact metric manifold we also have

$$\phi\xi=0 \quad \text{and} \quad \eta\circ\phi=0. \tag{2.7}$$

Such a manifold is said to be a contact metric manifold if $d\eta=\Phi$, where

$$\Phi(X,Y)=g(X,\phi Y), \tag{2.8}$$

is called the fundamental 2-form of M. If, in addition, ξ is a killing vector field, then M is said to be a K-contact manifold. It is well-known that a contact metric manifold is a K-contact manifold if and only if

$$\nabla_X\xi=-\phi X, \tag{2.9}$$

for any vector field X on M. On the other hand, the almost contact metric structure of M is said to be normal if

$$[\phi,\phi](X,Y)=-2d\eta(X,Y)\xi,$$

for any X, Y on M, where $[\phi,\phi]$ denotes the Nijenhuis torsion of ϕ, given by

$$[\phi,\phi](X,Y)=\phi^2[X,Y]+[\phi X,\phi Y]-\phi[\phi X,Y]-\phi[X,\phi Y].$$

A normal contact metric manifold is called a Sasakian manifold. It can be proved that an almost contact metric manifold is Sasakian if and only if

$$(\nabla_X\phi)Y=g(X,Y)\xi-\eta(Y)X, \tag{2.10}$$

for any X, Y.

On the other hand, given an almost contact metric manifold $M(\phi,\xi,\eta,g)$, we say that M is a generalized Sasakian space form if there exist three functions f_1, f_2, f_3 on M such that the curvature tensor R is given by

$$\begin{aligned} R(X,Y)Z \;=\;& f_1\{g(Y,Z)X-g(X,Z)Y\} \\ &+f_2\{g(X,\phi Z)\phi Y-g(Y,\phi Z)\phi X+2g(X,\phi Y)\phi Z\} \\ &+f_3\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X+g(X,Z)\eta(Y)\xi \\ &-g(Y,Z)\eta(X)\xi\}, \end{aligned} \tag{2.11}$$

for any vector fields X, Y, Z on M [5]. Such a manifold is denoted by $M^{2n+1}(f_1,f_2,f_3)$. This kind of manifold appears as a generalization of the well known Sasakian space form, which can be obtained as a particular case of generalized Sasakian space forms by taking $f_1=\frac{c+3}{4}$, $f_2=f_3=\frac{c-1}{4}$.

In a $(2n+1)$-dimensional generalized Sasakian space form $M^{2n+1}(f_1,f_2,f_3)$, we have the following relations [4]:

$$R(X,Y)\xi=(f_1-f_3)[\eta(Y)X-\eta(X)Y], \tag{2.12}$$

$$R(\xi,X)Y=(f_1-f_3)[g(X,Y)\xi-\eta(Y)X)], \tag{2.13}$$

$$R(\xi,X)\xi=(f_1-f_3)[\eta(X)\xi-X)], \tag{2.14}$$

$$\eta(R(X,Y)Z)=(f_1-f_3)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)], \tag{2.15}$$

$$S(X,Y)=(2nf_1+3f_2-f_3)g(X,Y)-(3f_2+(2n-1)f_3)\eta(X)\eta(Y), \tag{2.16}$$

$$QX=(2nf_1+3f_2-f_3)X-(3f_2+(2n-1)f_3)\eta(X)\xi, \tag{2.17}$$

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X), \tag{2.18}$$

$$S(\phi X, \phi Y) = S(X,Y) - 2n(f_1 - f_3)\eta(X)\eta(Y), \tag{2.19}$$

$$S(\xi,\xi) = 2n(f_1 - f_3), \tag{2.20}$$

$$Q\xi = 2n(f_1 - f_3)\xi, \tag{2.21}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{2.22}$$

where R, S and r denote the curvature tensor, Ricci tensor of type $(0,2)$ and scalar curvature of the space form, respectively, and Q is the Ricci operator defined by $g(QX,Y) = S(X,Y)$. We know that [4], the ϕ-sectional curvature of a generalized Sasakian space form $M^{2n+1}(f_1, f_2, f_3)$ is $f_1 + 3f_2$.

3 Generalized Recurrent Generalized Sasakian Space Forms

Definition 3.1. A generalized Sasakian space form $M^{2n+1}(\phi,\xi,\eta,g)$ is called generalized recurrent if its curvature tensor R satisfies the condition

$$(\nabla_W R)(X,Y)Z = \alpha(W)R(X,Y)Z + \beta(W)[g(Y,Z)X - g(X,Z)Y], \tag{3.1}$$

where α and β are two 1-forms, β is non zero and these are defined by

$$\alpha(W) = g(W,\rho_1) \text{ and } \quad \beta(W) = g(W,\rho_2), \forall W \in TM,$$

ρ_1 and ρ_2 being the vector fields associated to the 1-form α and β.

Theorem 3.2. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a generalized recurrent generalized Sasakian space form. Then*

$$\beta(X) = X(f_1 - f_3) - (f_1 - f_3)\alpha(X), \tag{3.2}$$

for any vector field X.

In particular, we get

$$\beta(\xi) = \xi(f_1 - f_3) - (f_1 - f_3)\alpha(\xi). \tag{3.3}$$

Proof. Let us consider a generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$ $(n > 1)$, which is generalized recurrent. Then the curvature tensor of $M^{2n+1}(\phi,\xi,\eta,g)$ satisfies the condition (3.1) for all vector fields X, Y, Z, W. Replacing $X = Y = \xi$ in (3.1), we have

$$(\nabla_X R)(\xi,Z)\xi = \alpha(X)R(\xi,Z)\xi + \beta(X)[\eta(Z)\xi - Z]. \tag{3.4}$$

Also, we know that

$$\begin{aligned}(\nabla_X R)(\xi,Z)\xi &= \nabla_X R(\xi,Z)\xi - R(\nabla_X\xi,Z)\xi \\ &\quad -R(\xi,\nabla_X Z)\xi - R(\xi,Z)\nabla_X\xi.\end{aligned} \tag{3.5}$$

Now from (2.9) and (3.5), we get

$$\begin{aligned}(\nabla_X R)(\xi,Z)\xi &= \nabla_X R(\xi,Z)\xi + R(\phi X,Z)\xi \\ &\quad -R(\xi,\nabla_X Z)\xi - R(\xi,Z)\phi X.\end{aligned} \tag{3.6}$$

Using (2.5) and (2.14) in (3.6), we obtain

$$(\nabla_X R)(\xi,Z)\xi = X(f_1 - f_3). \tag{3.7}$$

In view of (3.4) and (3.7), we have

$$\alpha(X)R(\xi,Z)\xi+\beta(X)[\eta(Z)\xi-Z]=X(f_1-f_3). \tag{3.8}$$

By virtue of (2.14) and (3.8), we get

$$[(f_1-f_3)\alpha(X)-X(f_1-f_3)+\beta(X)][\eta(Z)\xi-Z]=0,$$

which implies that

$$\beta(X)=X(f_1-f_3)-(f_1-f_3)\alpha(X),$$

for any vector field X.

Replacing $X=\xi$, we obtain

$$\beta(\xi)=\xi(f_1-f_3)-(f_1-f_3)\alpha(\xi).$$

This proves the theorem. □

Theorem 3.3. *In a generalized recurrent generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$, the scalar curvature r of $M^{2n+1}(\phi,\xi,\eta,g)$ satisfies*

$$[4n(f_1-f_3)-r]\eta(\rho_1)=2n(2n-1)\eta(\rho_2). \tag{3.9}$$

Proof. Suppose that $M^{2n+1}(\phi,\xi,\eta,g)$ is a generalized recurrent generalized Sasakian space form. Then using second Bianchi's identity in (3.1), we have

$$\begin{aligned} &\alpha(X)R(Y,Z)W+\beta(X)[g(Z,W)Y-g(Y,W)Z] \\ &+\alpha(Y)R(Z,X)W+\beta(Y)[g(X,W)Z-g(Z,W)X] \\ &\alpha(Z)R(X,Y)W+\beta(Z)[g(Y,W)X-g(X,W)Y] \\ =\;& 0. \end{aligned} \tag{3.10}$$

So by a suitable contraction from (3.10), we obtain

$$\begin{aligned} &\alpha(X)S(Z,W)+(2n-1)\beta(X)g(Z,W) \\ &-(2n-1)\beta(Z)g(X,W)-\alpha(Z)S(X,W) \\ &+R(Z,X,W,\rho_1)=0. \end{aligned} \tag{3.11}$$

Contracting (3.11) with respect to Z,W we get

$$r\alpha(X)+2n(2n-1)\beta(X)-2S(X,\rho_1)=0. \tag{3.12}$$

Replacing $X=\xi$ in (3.12) and using (2.18), we obtain (3.9). Our theorem is proved. □

4 On generalized concircular Recurrent generalized Sasakian space forms

In this section, we consider a generalized concircular recurrent generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$.

Definition 4.1. A generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$ is called generalized concircular recurrent if its concircular curvature tensor C (Yano, K; Kon, M, 1984)

$$C(X,Y)Z=R(X,Y)Z-\frac{r}{2n(2n+1)}[g(Y,Z)X-g(X,Z)Y], \tag{4.1}$$

satisfies the condition [18]:

$$(\nabla_W C)(X,Y)Z=\alpha(W)C(X,Y)Z+\beta(W)[g(Y,Z)X-g(X,Z)Y], \tag{4.2}$$

where α and β are two 1-forms, β is non zero and these are defined by

$$\alpha(W)=g(W,\rho_1)\ \text{ and }\quad \beta(W)=g(W,\rho_2), \forall W\in TM,$$

ρ_1 and ρ_2 being the vector fields associated to the 1-forms α and β.

Theorem 4.2. *In a generalized concircular recurrent generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$, the 1-forms α and β are related as*

$$\beta(X) = X[(f_1 - f_3) - \frac{r}{2n(2n+1)}] - [(f_1 - f_3) - \frac{r}{2n(2n+1)}]\alpha(X), \tag{4.3}$$

for any vector field X, where $X[r]$ denotes the covariant derivative of the scalar curvature r with respect to the vector field X.

Proof. Let us consider a generalized Sasakian space forms $M^{2n+1}(\phi,\xi,\eta,g)$ $(n > 1)$, which is generalized concircular recurrent. Then the concircular curvature tensor C of $M^{2n+1}(\phi,\xi,\eta,g)$ satisfies the condition (4.2) for all vector fields X, Y, Z, W. Putting $Y = W = \xi$ in (4.2), we have

$$(\nabla_X C)(\xi, Z)\xi = \alpha(X)C(\xi, Z)\xi + \beta(X)[\eta(Z)\xi - Z]. \tag{4.4}$$

Now using (2.13) in (4.1), we obtain

$$(\nabla_X C)(\xi, Z)\xi = \{((f_1 - f_3) - \frac{r}{2n(2n+1)})\alpha(X) + \beta(X)\}[\eta(Z)\xi - Z]. \tag{4.5}$$

On the other hand, from the definition of covariant derivative, it is well-known that

$$(\nabla_X C)(\xi, Z)\xi = \nabla_X C(\xi, Z)\xi - C(\nabla_X \xi, Z)\xi - C(\xi, \nabla_X Z)\xi - C(\xi, Z)\nabla_X \xi. \tag{4.6}$$

Then in view of (2.9) the equation (4.7) can be written as:

$$(\nabla_X C)(\xi, Z)\xi = \nabla_X C(\xi, Z)\xi + C(\phi X, Z)\xi - C(\xi, \nabla_X Z)\xi + C(\xi, Z)\phi X. \tag{4.7}$$

Hence by the use of (2.5) and (4.3) in (4.7) it can be easily seen that:

$$(\nabla_X C)(\xi, Z)\xi = X[(f_1 - f_3) - \frac{r}{2n(2n+1)}][\eta(Z)\xi - Z]. \tag{4.8}$$

Therefore from the equality of left-hand sides of the equations (4.5) and (4.8), we get

$$\begin{aligned} & \{((f_1 - f_3) - \frac{r}{2n(2n+1)})\alpha(X) + \beta(X)\}[\eta(Z)\xi - Z] \qquad (4.9)\\ = \quad & X[(f_1 - f_3) - \frac{r}{2n(2n+1)}][\eta(Z)\xi - Z]. \end{aligned}$$

On simplification, we have

$$\begin{aligned} & \{X[(f_1 - f_3) - \frac{r}{2n(2n+1)}] \qquad (4.10)\\ & -\{((f_1 - f_3) - \frac{r}{2n(2n+1)})\alpha(X) + \beta(X)\}\}[\eta(Z)\xi - Z] \\ = \quad & 0, \end{aligned}$$

which implies that

$$\begin{aligned} \beta(X) = {} & X[(f_1 - f_3) - \frac{r}{2n(2n+1)}] \qquad (4.11)\\ & -[(f_1 - f_3) - \frac{r}{2n(2n+1)}]\alpha(X). \end{aligned}$$

This proves the theorem. □

5 Generalized Ricci recurrent generalized Sasakian space form

Definition 5.1. A generalized Sasakian space form $M^{2n+1}(\phi,\xi,\eta,g)$ is called generalized Ricci recurrent if its Ricci tensor S satisfies the condition

$$(\nabla_X S)(Y, Z) = \alpha(X)S(Y, Z) + \beta(X)g(Y, Z), \tag{5.1}$$

where α and β are two 1-forms and these are defined by

$$\alpha(W) = g(W, \rho_1) \quad \text{and} \qquad \beta(W) = g(W, \rho_2), \forall W \in TM,$$

ρ_1 and ρ_2 being the vector fields associated to the 1-form α and β.

Theorem 5.2. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a generalized Ricci recurrent generalized Sasakian space form. Then, the 1-forms α and β are related by*

$$\beta(X) = 2n\{X(f_1 - f_3) - (f_1 - f_3)\alpha(X)\}. \tag{5.2}$$

In particular, we get

$$\beta(\xi) = 2n\{\xi(f_1 - f_3) - (f_1 - f_3)\alpha(\xi)\}. \tag{5.3}$$

Proof. We know that

$$(\nabla_X S)(Y,Z) = XS(Y,Z) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z). \tag{5.4}$$

Using (5.1) in (5.4), we have

$$\alpha(X)S(Y,Z) + \beta(X)g(Y,Z) = XS(Y,Z) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z).$$

Now putting $Y = Z = \xi$ in the above equation, we obtain

$$\alpha(X)S(\xi,\xi) + \beta(X)g(\xi,\xi) = XS(\xi,\xi) - S(\nabla_X \xi, \xi) - S(\xi, \nabla_X \xi). \tag{5.5}$$

Using (2.3),(2.9) and (2.20) in (5.5), we have

$$\beta(X) = 2n\{X(f_1 - f_3) - (f_1 - f_3)\alpha(X)\}.$$

Substituting $X = \xi$, we get equation (5.3).

This proves the theorem. □

Let α^* and β^* be the associated vector fields of α and β, that is, $g(X,\alpha^*) = \alpha(X)$ and $g(X,\beta^*) = \beta(X)$.

Corollary 5.3. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a generalized Ricci recurrent generalized Sasakian space form , we have*

$$\beta = -2n(f_1 - f_3)\alpha.$$

Thus, the associated vector fields α^* and β^* are in opposite directions.

Proof. From the equation (5.2), the proof follows immediately. □

6 Generalized Ricci recurrent generalized Sasakian space forms with cyclic Ricci tensor

A Riemannian manifold is said to admit cyclic Ricci tensor if

$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0. \tag{6.1}$$

Now, we prove the following:

Theorem 6.1. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ generalized Ricci recurrent generalized Sasakian space forms with cyclic Ricci tensor, the Ricci tensor satisfies:*

$$\begin{aligned} \alpha(\xi)S(X,Y) &= 2n\{\xi(f_1 - f_3) - (f_1 - f_3)\alpha(\xi)\}g(X,Y) \\ &\quad -2n\{\eta(Y)X(f_1 - f_3) - \eta(X)Y(f_1 - f_3)\}. \end{aligned} \tag{6.2}$$

Proof. Suppose that $M^{2n+1}(\phi,\xi,\eta,g)$ is a generalized Ricci symmetric manifold admitting cyclic Ricci tensor. Then by virtue of (5.1) and (6.1), we get

$$\begin{aligned} 0 &= \alpha(X)S(Y,Z) + \alpha(Y)S(Z,X) + \alpha(Z)S(X,Y) \\ &\quad +\beta(X)g(Y,Z) + \beta(Y)g(Z,X) + \beta(Z)g(X,Y). \end{aligned}$$

Moreover, if $M^{2n+1}(\phi,\xi,\eta,g)$ is a generalized Sasakian space form, putting $Z = \xi$, in the above equation, we find

$$\begin{aligned} \alpha(\xi)S(X,Y) &= -\beta(\xi)g(X,Y) - \alpha(X)S(Y,\xi) - \alpha(Y)S(X,\xi) \\ &\quad -\beta(X)\eta(Y) - \beta(Y)\eta(X). \end{aligned}$$

Using (2.18) in (5.3), we obtain (6.2). This proves the theorem. □

References

[1] P. Alegre, D. E. Blair and A. Carriazo, *Generalized Sasakian-space-forms*, Israel J. Math., **141**, 157–183 (2004).

[2] P. Alegre and A. Carriazo, *Structures on Generalized Sasakian-space-form*, Diff. Geo. and its Application, **26**(6), 656–666 (2008).

[3] P. Alegre and A. Carriazo, *Generalized Sasakian-space-forms and Conformal Changes of the Metric*, Results in Mathematics, **59**(3–4), 485–493 (2011).

[4] P. Alegre, D. E. Blair and A. Carriazo, *On generalized Sasakian-space-form*, Proceedings of the Ninth International Workshop on Diff. Geom., **9**, 31–39 (2005).

[5] A. Basari and C. Murathan, *On generalized ϕ –recurrent kenmotsu manifolds*, Sdu Fen Edebiyat Fakultesi Fen Dergisi (E-DERGI), **310**, 91–97 (2008).

[6] D. E. Blair, *Contact manifolds in Riemannian geometry* , Lecture notes in Math.509, Springer verlag, (1976).

[7] D. E. Blair, *Riemannian Geometry of contact and symplectic manifolds*, Birkhauser, Boston, (2002).

[8] A. Carriazo, V. Martin-Molina and M. M. Tripathi, *Generalized $(k;\mu)$–space forms*, arXiv:0812.2605v1 [math.DG] (14 Dec 2008).

[9] U. C. De, N. Guha and D. Kamilya, *On generalized Ricci–recurrent manifolds*, Tensor (N.S.), **56**(3), 312–317 (1995).

[10] U. C. De and A. Sarkar, *Some Results on Generalized Sasakian-space-forms*, Thai Journal of Mathematics, **8**(1), 1–10 (2010).

[11] V. J. Khairnar, *Generalized recurrent and Ricci–recurrent Lorentzian Trans-Sasakian manifolds*, IOSR Journal of Mathematics, **10**(4), 38–43 (2014).

[12] Q. Khan, *On generalized recurrent Sasakian Manifolds*, Kyungpook Math. J., **44**(2), 167–177 (2004).

[13] J. S. Kim, R. Prasad and M. M. Tripathi, *On generalized Ricci–recurrent trans–Sasakian manifolds*, J. Korean Math. Soc., **39**(6), 953–961 (2002).

[14] M. Kon and K. Yano, *Structures on manifolds*, Series in Pure Math., **3** World Sci, (1984).

[15] Hakan Ozturk, et.al. *On α –Kenmotsu manifolds Satisfying Certain conditions,* pl. sciences, **12**, 115–126 (2010).

[16] E. M. Patterson, *Some theorems on Ricci-recurrent spaces*, J. London Math. Soc., **27**, 287–295 (1952).

[17] S. Kisor and A. Singh, *Generalized ϕ –Recurrent Generalized Sasakian-Space-Forms*, Ganita, **67**(2), 175–188 (2017).

[18] T. Takahashi, *Sasakian ϕ–symmetric spaces,* Tohoku Math j., **29**, 91–113 (1977).

[19] A. K. Srivastava, D. L. Suthar and S. Yadav, *Some Results on $M(f_1, f_2, f_3)$ $(2n+1)$*–Manifolds, International Journal of Pure and Applied Mathematics, **70**(3), 415–423 (2011).

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