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METRIC DIMENSION VULNERABILITY IN TREES UNDER EDGE CONTRACTION

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Abstract For a simple connected graph $G = (V(G), E(G))$ where $V(G)$ is the set of vertices and $E(G)$ is the edge set, the distance between two vertices u and v denoted by $d_G(u, v)$ or simply $d(u, v)$ is the minimum number of edges between u and v. A vertex x resolves two distinct vertices u and v if $d(x, u) \neq d(x, v)$. An ordered set R is said to be a resolving set for the graph G if for any two vertices u and v of G there exists atleast one element R such that it resolves u and v. If the cardinality (the number of elements in R) of R is minimum then R is said to be a metric basis for G and the cardinality of the metric basis is called the metric dimension of G. Here we study the vulnerability in metric dimension of any tree under an arbitrary edge contraction.

1 Introduction

We consider a simple, finite and connected graph $G = (V(G), E(G))$, where $V(G)$ and $E(G)$ represents the vertex set and the edge set, respectively. The distance between two distinct vertices say u and v is denoted by $d_G(u, v)$ or without ambiguity $d(u, v)$. A set $W = \{w_1, w_2, \ldots, w_k\}$ on which the ordering $(w_1, w_2, \ldots, w_k)$ has been established is referred to as the ordered set of vertices. A vertex y resolves two distinct vertices u and v if $d(u, y) \neq d(v, y)$. The *code* of a vertex u with respect to an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ is denoted by $code(u|W)$ and is defined by

$$code(u|W) = (d(u, w_1), d(u, w_2), \ldots, d(u, w_k)).$$

If for every pair of arbitrary vertices u and v in $V(G)$ there is a vertex $x \in W$ such that $d(x, u) \neq d(x, v)$, then the set W is said to be a resolving set in G. The minimum cardinality (the number of elements in the set) of the resolving set of G is referred to as the metric dimension of G and it is denoted by $\beta(G)$. We observe that there may be many resolving set in $V(G)$ of different sizes hence it becomes utmost important and interesting to study the minimal one. The resolving set with minimum cardinality is termed as the metric basis of G[2]. Finding a minimum resolving set for a graph G is an NP- complete problem [17].

Metric dimension of a general metric space was first introduced in the year 1953 in [6]. Slater [39, 40] gave the concept of resolving set for a connected graph G, where he referred it to be a locating set and the minimum resolving set as the reference set for G. The location number of G was defined as the cardinality of the minimum resolving set. These concepts were well utilized in sonar stations, robot navigation [30] and also in the different fields of chemistry [27, 28]. These graph parameters were studied in many research articles like [1, 3, 11, 34, 41, 42, 45]. Harary and Melter independently worked on these ideas and gave the term metric dimension instead of locating number. The concept of metric dimension was first analysed taking into consideration the navigation system in different graphical networks which has a very wide range of applications in daily life. The robot covers different vertices and the landmark are those vertices which helps the robot to establish its location in the network [19]. In a network, the problem of determining the smallest set of landmarks becomes a problem of establishing a smallest resolving set in a graph [30]. Over the years a complete characterisation of graphs of order n whose metric dimension are 1, $n - 3$, $n - 2$ and $n - 1$ has been studied and determined in [11, 20, 24]. In addition to these we have many more particular classes of graphs as for example circulant graphs

[22], cycles [11], wheels [8, 9, 38], fans [9], unicycle graphs [35], hypercubes [4, 5, 14, 32], grids [34], honeycomb [33, 44], Jahangir graphs [43], classical binomial random graph [5], Sierpíski graphs[31] and trees [11, 19, 30].

The metric dimension of graphs which are structured out by some graph operations were studied as : line graph [16, 29], Cartesian product graphs [10, 30, 34, 36], corona product graphs [23, 45], joint product graphs [8, 9, 38], lexicographic product graphs [37] and hierarchical product graphs [15].

Given a tree, a vertex of degree at least 3 is called a core vertex or a core. A vertex of degree 2 is called a path vertex, and a vertex of degree 1 is called a leaf. For a core v, we often consider the subtrees created by removing v from the tree, and call them the subtrees of its neighbors or components (one subtree for each neighbour). We sometimes consider the BFS tree that is created by rooting the tree at v. If u is a vertex in T other than the root, then the parent of u is the vertex adjacent to u on the path to the root. A subtree of a neighbor of a core v, that is a path (without any cores), is called a leg (or a standard leg) of v. If a leg consists of a single vertex (that is, v is connected to a leaf), we call it a short leg, otherwise it is called a long leg. A core having at least one leg is called a stem vertex. A stem is called major if it has at least two legs otherwise is called minor stem.

In the view of T as tree rooted at w, we define a level function ℓ on $V(T)$ by $\ell w(v) = d(w, v)$. We simply denote the level function ℓ_w by ℓ when w is clear in the context. For any two vertices $u, v \in V(T)$, let $\phi(u, v) = \min\{\ell(x)$: where x is on the (u, v)-path in $T\}$.

Observation 1.1. Let u and v be two vertices of a tree T. Then the following hold:
(a) $d(u, v) = \ell(u) + \ell(v) - 2\phi(u, v)$.
(b) u and v are in different branches if and only if $\phi(u, v) = 0$.

An edge contraction is an operation that removes an edge from a graph while simultaneously merging the two vertices that it previously joined. In this article, we study the vulnerability in metric dimension of any tree under an arbitrary edge contraction.

2 Main Results

Lemma 2.1. *The number of legs and leaves in a tree T are same provided the tree has at least three leaves.*

Proposition 2.2. *For any tree T having at least three leaves, the following are true :*
(a) *For every leaf u, there exists a unique leg adjacent to some stem vertex.*
(b) *The legs, which are adjacent to two distinct stems, are disjoint.*

Lemma 2.3. *Let $R(T)$ be a resolving set of T. Then for every vertex $w \in V(T)$, $R(T)$ contains at least one vertex from each component of $T \setminus \{w\}$ with at most one exception.*

Proof. Let $N_T(w)$ be the set of all neighbours of w of T and $N_T(w)$ contains m vertices, where $m \geq 2$. Then $T \setminus \{w\}$ has m components, let they be $C_1, C_2, \ldots, C_m$. If possible, assume there to be at least two components C_i and C_j such that $R(T) \cap C_i = \emptyset$ and $R(T) \cap C_j = \emptyset$. Then $R(T) \subseteq V(T) \setminus (C_i \cup C_j)$. Let $u \in N_T(w) \cap C_i$ and $v \in N_T(w) \cap C_j$. We prove that $d_T(x, u) = d_T(x, v)$ for every vertex $x \in V(T) \setminus (C_i \cup C_j)$. Since $x \notin C_i \cup C_j$ and $u, v \in N(w)$, both the shortest paths between x to u and x to v must be via the point w because in a tree between every pair of vertices there exists a unique path. Thus $d_T(x, u) = d_T(x, w) + 1 = d_T(x, v)$ when $x \in V(T) \setminus (C_i \cup C_j)$. Therefore there exists no vertices in $R(T)$ that resolve u and v, which is a contradiction. Hence we get the result. □

Corollary 2.4. *Let s be a stem vertex of T having m legs L_1, L_2, $\ldots$, L_m. Then for every resolving set R of T the following are true :*
(a) *$R \cap L_i \neq \emptyset$ for all $i \in \{1, 2, \ldots, m\}$ with at most one exception.*
(b) *R contains at least $m - 1$ vertices from the legs adjacent to s.*

Theorem 2.5. *For any tree T, $\beta(T) \geq n_l(T) - n_s(T)$, where $n_l(T)$ and $n_s(T)$ denote the sets of all leaves and all stem vertices in T, respectively.*

Proof. Let R be an arbitrary resolving set of T. Assume T has m stem vertices and they are $s_1, s_2, \dots, s_m$, i.e., $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$. Let n_i denotes the number of legs adjacent to s_i.Then $\sum_{i=1}^{m} n_i = n_l(T)$. By using Corollary 2.4, for each $i \in \{1, 2, \dots, m\}$, R contains at least $n_i - 1$ vertices from the legs hanged on s_i.Again Proposition 2.2 2.2, gives that the legs adjacent to different stem vertices are disjoint and so R contains at least $\sum_{i=1}^{m}(n_i - 1)$ vertices. Therefore,

$$|R| \geq \sum_{i=1}^{m} n_i - m = n_l(T) - n_s(T)$$

and this is true for every resolving set R of T. Thus we obtain the result. □

Lemma 2.6. *Every tree with at least one core vertex must contains a major stem vertex.*

Proof. Let T be a tree and v be a core vertex (i.e., deg$(v) \geq 3$) in T. Root the tree at v. If every component of $T \setminus \{v\}$, does not contains any core vertex, then v itself a major stem vertex. Thus we assume at least one component, say, C, of $T \setminus \{v\}$ must contains a core vertex. Let w be a core vertex in C which is at largest distance from v. Since deg$(w) \geq 3$, so it has at least two children, say w_1 and w_2. Then the components $C^w(w_1)$ and $C^w(w_2)$ of $T \setminus \{w\}$ containing w_1 and w_2, respectively, do not contain a core vertex, otherwise it would give w is not a core vertex of maximum depth. Therefore, $C^w(w_1)$ and $C^w(w_2)$ are legs of w and consequently w is a major stem vertex of T. □

Corollary 2.7. *Let T be a tree and v be a core which is not a major stem. Each component of $T \setminus \{v\}$ contains at least one major stem vertex of T with at most one exception.*

Proof. Let deg$(v) = m$. Then $T \setminus \{v\}$ has m components, say, $T_1, T_2, \dots, T_m$. Since v is not a major stem, so each component of $T \setminus \{v\}$ contains at least one core vertex except one (say T_m). Since each T_ℓ, $1 \leq \ell \leq m-1$ contains core vertices, applying Lemma 2.6 each of these subtrees must contains major stem vertices and hence we obtain our result. □

Lemma 2.8. *Let T be a tree with at least one core vertex and $R(T)$ be a collection of vertices of T by taking exactly one from each of* leg$(s) - 1$*, where* leg(s) *is the number of legs at s. Then $R(T)$ is a resolving set of T with cardinality $n_l(T)$ and $n_s(T)$.*

Proof. To prove $R(T)$ is a resolving set, we have to show for every pair of vertices u and v in T, there exists at least one element, say x, in $R(T)$ such that $d_T(x,u) \neq d_T(x,v)$. Let $x \in R(T)$ be a fixed element and u, v be two arbitrary vertices of T that are not resolved by x. Then $d_T(x,u) = d_T(x,v)$. We consider the following two cases

Case-I: *x, u and v all lie on a same path.* Since $x \in R(T)$, so x lies on a leg L of some major stem, say s. Let T be rooted at s. Since $d_T(x,u) = d_T(x,v)$ and x, u and v are lie on a same path, it follows that at least one of u and v must be on the leg L and exactly one vertex will be at deeper level than x along the path L. With no loss of generality, we assume that u lies on a path L which is deeper level than the level of v. Then $d_T(s,u) > d_T(s,v)$. Let $w \in L$ be a vertex adjacent to s. Since T is not a path, $R(T)$ consists a vertex y from $T - \{sw\}$. In the following we calculate the distances of u and v from y by using Observation 1.1.

$$\begin{aligned} d_T(y,u) &= d_T(y,s) + d_T(s,u) \\ d_T(y,v) &= d_T(y,s) + d_T(s,v) - 2\phi(y,v). \end{aligned}$$

Since $d_T(s,u) > d_T(s,v)$, from the above equations we have $d_T(y,u) \neq d_T(y,v)$, i.e., y resolve u and v.

Case-II: *x, u and v are in different path.* Since $d_T(x,u) = d_T(x,v)$ and x, u and v are in different paths, it follows that the xu-path and xv-path intersect to a vertex (unique vertex), say w. Then w is core vertex and $d_T(w,u) = d_T(w,v)$. Let T_u and T_v be two components of $T \setminus \{w\}$

containing u and v, respectively. From the construction of R and Lemma 2.6, for every core w, R contains at least one element from each component of $T - \{v\}$ with one exception. With no loss of generality, we assume $R(T)$ contains an element z from the component T_u. Our claim, z resolve u and v. In view of Observation 1.1, the distances of u and v from z are given by

$$\begin{aligned} d_T(z,u) &= d_T(z,w) + d_T(w,u) - 2\phi(z,u) \\ d_T(z,v) &= d_T(z,w) + d_T(w,v). \end{aligned}$$

Since the vertex z lies on a leg of t and both the vertices z and u lie in the component of T_u, $\phi(z,u) \geq 1$. Also $d_T(w,u) = d_T(w,v)$. Thus the above two equations show that $d_T(z,u) \neq d_T(z,v)$, i.e., z resolve u and v. On account of Case-I and Case-II, we get $R(T)$ is a resolving set of T. □

Theorem 2.9. *For a tree T with at least one core, $\beta(T) = n_l(T) - n_s(T)$, where $n_l(T)$ and $n_s(T)$ denote the number of legs and stem vertices in T, respectively.*

Proof. The result follows immediately from Theorem 2.5 and Lemma 2.8. □

Theorem 2.10. *Let T be a tree and $e = (u,\, v)$ be an edge in T. Then*

$$\beta(T \cdot e) = \begin{cases} \beta(T \cdot e), & \text{if both } u \text{ and } v \text{ are stem vertices;} \\ \beta(T) - 1, & \text{if one of } u \text{ and } v \text{ is a leaf and other one is minor stem;} \\ \beta(T), & \text{otherwise.} \end{cases}$$

Proof. For a tree T, let $n_\ell(T)$ and $n_s(T)$ be the number of leaves and stems in T. Then applying Theorem 2.9, we have $\beta(T) = n_\ell(T) - n_s(T)$. Since $e = (u,\, v)$ be an edge in T, so both the vertices u and v can not be leaf of T, i.e., at most one of u and v can be a leaf. For $x \in \{u, v\}$, let n_x be the number of legs hanging at x. Let w be the merging vertex in $T \cdot e$. Then the total legs at w in $T \cdot e$ is $n_u + n_v$, provided no one of u and v is a leaf. Moreover, if no one of u and v is a leaf, then $T \cdot e$ and T have same number of leaves, indeed they are same leaves. Thus if both u and v are stem vertices, then $n_\ell(T \cdot e) = n_\ell(T)$ and $n_s(T \cdot e) = n_s(T) - 1$ and hence applying Theorem 2.9, we have $\beta(T \cdot e) = n_\ell(T \cdot e) - n_s(T \cdot e) = n_\ell(T) - n_s(T) + 1 = \beta(T) + 1$. Again if one of u and v, say u, is a stem and v is neither a stem nor a leaf, then $n_\ell(T \cdot e) = n_\ell(T)$ and $n_s(T \cdot e) = n_s(T)$ (here the merging vertex w will be a stem in place of v with same number of legs). Thus in this case, we have $\beta(T \cdot e) = \beta(T)$. Now we assume one of u and v, say v, is a leaf. Then the number of leafs and stem vertices in $T \cdot e$ are given by

$$n_s(T \cdot e) = \begin{cases} n_s(T) - 1, & \text{if } u \text{ is a minor stem;} \\ n_s(T), & \text{otherwise.} \end{cases}$$

and

$$n_\ell(T \cdot e) = n_\ell(T) - 1.$$

Thus if one of u and v is a leaf, then $\beta(T \cdot e) = \beta(T)$ when the other is a minor stem; and $\beta(T \cdot e) = \beta(T) - 1$ if the other is not a minor stem. This completes the proof of the result. □

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