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OPTIMIZATION APPROACH BASED ON GENETIC ALGORITHM FOR A ROBIN COEFFICIENT PROBLEM

B. Jouilik, J. Daoudi, C. Tajani and J. Abouchabaka

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Abstract This work presents numerical optimization algorithm based on genetic algorithm to solve an inverse problem to reconstruct the Robin coefficient in boundary value problem. It consists of identifying the Robin coefficient on the inaccessible part of the boundary representing the corrosion damage of some specimen material. This problem is known to be severely ill-posed in Hadamard sense. Metaheuristics are methods inspired by natural phenomena which have shown their effectiveness in solving several optimization problems in different domains. Thus, adapted genetic operators for real coded genetic algorithm is proposed by formulating the problem into an optimisation one. Numerical results are presented to illustrate and evaluate the efficiency and the robustness of the proposed algorithm.

1 Introduction

The identification of Robin coefficient constitutes a very important class of inverse problems which has interested several researchers. This kind of problem appears in many nondestructive evaluation methods where an unknown profile of material, contained in an inaccessible part of the boundary, must be recovered using a partial boundary measurement taken on an accessible part of the boundary [1, 2].

The Robin inverse problem arises in many areas of engineering and can be considered a challenge in many fields of industry. For example, in the detection of corrosion the Robin coefficient represents the corrosion damage profile [3] and in the study of MOSFET semiconductor devices, the Robin coefficient contains information about the contact resistance and location of the metal-to-silicon contact window [4].

The considered inverse problem is known to be severely ill-posed in Hadamard sense [5] since the existence or uniqueness or the continuous dependence on the data of their solutions may not be ensured. This problem has been the subject of several studies. In particular, the identifiability [6, 7] and the stability, i.e. the continuous dependence of the unknown parameter on the measured data, which is a crucial issue for numerical applications. The stability of various forms (local Lipschitz, global monotone Lipschitz and global logarithmic type) has also been extensively studied [8, 9, 10, 11].

Many performing numerical methods have been developed to overcome the ill-posed nature of this kind of problem. There are numerical methods based on the assumption that the specimen is a thin plate [3, 12, 13]. The iterative algorithm addressed by Kozlov, Mazya and Fomin since 1991 [14], also called alternating method which consists of completing Dirichlet and Neumann conditions on the inaccessible boundary and then calculate the desired coefficient [15, 16, 17]. In addition, the methods based on the minimization of the so-called Kohn and Vogelius cost function [6, 18] and those using conjugate gradient method after transforming the inverse problem into an optimisation one [19, 20].

In addition to deterministic methods, a class of stochastic methods have had great success in solving several optimization problems in different fields. The metaheuristics methods as artificial bee colony [21], genetic algorithms (GAs) [22, 23], particle swarm optimization [24], ant colony optimization [25] and bat algorithm [26] are methods inspired by natural phenomena and are part of this category of methods. This technique does not guarantee the best solution but it is to come as close as possible to the optimum value in a reasonable amount of time which is at most polynomial time. However, the application of these methods, in particular GAs, requires an adaptation of these genetic operators when solving each problem that can influence the quality of the solution (local or global optimum) and the time required to obtain the optimal solution, which explains the large number of genetic operators developed and adapted to each type of encoding.

Several researchers have used genetic algorithms (GAs) to solve different inverse problems [27, 28, 29, 30, 31, 32]. Our objective is to adapt the genetic algorithm with a good choice of genetic operators with real encoding to approach the desired solution for the Robin inverse problem. In this paper, the considered inverse problem is formulated as an optimization problem and we investigate the use of genetic algorithm with a real encoding, which has shown its efficiency in comparison to the binary encoding, with adapted crossover and mutation operators, where the obtained direct problem for Laplace's equation is discretized using the finite element method.

The remainder of this paper is organized as follows: Section 2 gives the mathematical formulation of the direct and the Robin inverse problem and its formulation on an optimisation problem. Section 3 provides a brief review of genetic algorithms and the adapted genetic algorithm for the studied inverse problem. Section 4 presents numerical results showing the effectiveness of the proposed algorithm.

2 Mathematical formulation

2.1 Problem setting

Let Ω be an open bounded domain in $\mathbb{R}^2$, representing the speciemen to be inspected, with a piecewise smooth boundary $\Gamma = \partial\Omega$.
Γ_1 and Γ_0 are two disjoint closed sub-parts of $\Gamma = \partial\Omega$, where Γ_0 is the accessible part; however, Γ_1 is the inaccessible part where the corrosion has occurred.

The problem can be mathematically modelled by the Laplace equation as follows:

$$\Delta u = 0 \quad in \quad \Omega \tag{2.1}$$

where Δ is the Laplace operator. We prescribe the Neumann boundary condition as follows:

$$\partial_n u = g \quad on \quad \Gamma_0 \tag{2.2}$$

where ∂_n denotes taking outward normal derivative.
On the inaccessible part, the boundary condition is given by the following:

$$\partial_n u + \gamma u = 0 \quad on \quad \Gamma_1 \tag{2.3}$$

The model given by equations $((2.1) - (2.2) - (2.3))$ is one of the models describing the electrostatics of a conductor Ω having an inaccessible part of its boundary, denoted by Γ_1, affected by corrosion, where u represents the electrostatic potential, g is the prescribed current density on the accessible part of the boundary Γ_0; while γ, called the corrosion coefficient, represents the characteristic of corrosion damage [33].
Direct problem : In the direct problem, γ and g would be specified in a way that leads to the Robin boundary value problem for the Laplace equation where the objective would be to find u.
Inverse problem : The aim of the Robin inverse problem is to estimate the Robin coefficient γ on the inaccessible part of the boundary $\Gamma_1 \subset \Gamma$ from the knowledge of the Neumann condition g and measuring the corresponding boundary condition $f = u_{/\Gamma 0}$ on the accessible part Γ_0.
Note that, for the inverse problem, both Dirichlet and Neumann boundary conditions are specified on the boundary Γ_0.

2.2 Formulation of the optimisation problem

The main idea behind this paper is the use of an adapted genetic algorithm to approach the solution of the inverse problem which requires a transformation of the considered problem into

an optimization problem.
Consider the following Robin boundary value problem for the Laplace equation:

$$\begin{cases} -\Delta u = 0 & in \quad \Omega \\ \partial_n u = g & on \quad \Gamma_0 \\ \partial_n u + \gamma u = 0 & on \quad \Gamma_1 \end{cases} \tag{2.4}$$

We assume that the function $g \geq 0$ does not vanish identically. Then the maximum principle [34] ensures that $u > 0$ over the boundary Γ, which consequently guarantees the identifiability of the Robin coefficient γ in the admissible set of Robin coefficients given by $A = \{\gamma \in L^\infty(\Gamma_1), c\chi_K \leq \gamma \leq c'\}$, where c and c' are positive constants, $K \subset \Gamma_1$ is an open set and χ_K is the characteristic function of K.
Knowing that, this direct problem needs to be solved by Frrefem++ Software, and its mathematical formulation should be written. The Problem (2.4) is clearly equivalent to the following weak formulation: find $u \in H^1(\Omega)$ such that for all $v \in H^1(\Omega)$

$$\int_\Omega \nabla u \cdot \nabla v dx + \int_{\Gamma_1} \gamma u v ds = \int_{\Gamma_0} g v ds$$

and the well-posedness of the last problem follows from Lax-Milgram's lemma and Poincaré-Friedrichs inequality, which implies the equivalence between the standard norm of $H^1(\Omega)$ and the norm $||.||$ defined by the following:

$$\|u\|^2 = \int_\Omega |\nabla u|^2 dx + \int_{\Gamma_1} u^2 ds$$

Since the function γ on the boundary Γ_1 is to be determined, we consider it as a control in the direct problem formulation (2.4) to fit the Cauchy data $f \in L^2(\Gamma_0)$, and we aim to find $u(\gamma, g)$ such that:

$$u(\gamma, g)_{/\Gamma_0} = f$$

In doing so, we attempt to minimise by using genetic algorithm approach the least-squares functional J or the regularized least-squares functional J_r defined as follows :

$$J(\gamma) = \frac{1}{2}\|u(\gamma, g) - f\|^2_{L^2(\Gamma_0)} = \frac{1}{2}\|u_\gamma - f\|^2_{L^2(\Gamma_0)}$$

and

$$J_r(\gamma) = \frac{1}{2}\|u(\gamma, g) - f\|^2_{L^2(\Gamma_0)} + \frac{\eta}{2}\|\gamma\|^2_{L^2(\Gamma_1)}$$

where η is the regularization parameter. The extra term in the functional J_r is the well-known Tikhonov regularization functional, and it serves to increase the numerical stability of the optimization problem [29].

3 Approach genetic for the Robin identification problem

3.1 Overview of genetic algorithm

Genetic algorithms (GAs) are the most famous Evolutionary Algorithms (EAs) which are inspired by natural evolution and selection. It is essentially a searching method based on the Darwinian principles of biological evolution. Genetic algorithms, primarily developed by Holland [21], have been successfully applied to various optimisation problems.

Genetic algorithms search from a population of possible solutions instead of a single one. It uses random operators throughout the process including reproduction, crossover, and mutation. Thus, In genetic algorithms, a population of individuals (possible solution) is randomly selected. These individuals are subject to several genetic operators (selection, crossover, mutation, insertion, ..) to produce a new population containing, in principle, a better individual. This population evolves more and more until a stopping criterion is satisfied and declared obtaining optimal best solution. The performance of a genetic algorithm depends on the choice of operators which will intervene in the production of the new populations.

The fitness or cost function used to resolve the redundancy has no requirement for continuity in the derivatives, so virtually "any" fitness function can be selected for optimizing.

Irrespective of the problems treated, genetic algorithms are based on six principles:

- Each treated problem has a specific way to encode the individuals of the genetic population. A chromosome (a particular solution) has different ways of being coded: numeric, symbolic, matrix or alphanumeric;
- Creation of an initial population formed by a finite number of solutions...;
- Definition of an evaluation function (fitness) to evaluate a solution;
- Selection mechanism to generate new solutions, used to identify individuals in a population. There are several methods in the literature, citing the method of selection by rank, roulette, by tournament, random selection, etc.;
- Reproduce the new individuals by using genetic operators:
  - Crossover operator: It is a genetic operator that combines two chromosomes (parents) to produce a new chromosome (children) with crossover probability P_c;
  - Mutation operator: It avoids establishing a uniform population that is unable to evolve. This operator used to modify the genes of a chromosome selected with a mutation probability P_m;
- Insertion mechanism: It decides what should stay and what should disappear.
- Stopping test: to make sure about the optimality of the solution obtained by the genetic Algorithm.

3.2 Choice of genetic operators for the optimisation problem

To solve the considered problem using GAs, it is necessary to adapt the genetic operators, starting with the type of encoding, the crossover operator and mutation operator.

It is known that the performance of real coded genetic algorithm is superior to binary coded genetic algorithm requiring huge computational time and memory, in particular, for high dimensional problems in which higher degree of precision is desired. Then, in this problem, a real coded GA is used where the decision variables are encoded as real numbers.

For the other operators, we have chosen the Arithmetic crossover [35] and the Non-uniform mutation [36], defined as follows:

- **Arithmetic crossover** Different types of crossover operators adapted to real coded GA have been developed by several authors that have shown their effectiveness in solving several optimization problems [37]. We opt for this proposed algorithm for the arithmetic crossover.

  In arithmetic crossover, two parents produce two offsprings. The offsprings are arithmetically represented by the following:

$$\begin{aligned} y_i^{(1)} &= \alpha_i x_i^{(1)} + (1-\alpha_i)\, x_i^{(2)} \\ y_i^{(2)} &= \alpha_i x_i^{(2)} + (1-\alpha_i)\, x_i^{(1)} \end{aligned} \tag{3.1}$$

  where α_i are uniform random numbers, say in $[-0.5, 1.5]$ [22].

- **Non-uniform mutation (NUM)** Michalewicz's non-uniform mutation is one of the most widely used mutation operators in real coded GAs. From a point $x^t = (x_1^t, x_2^t, \ldots, x_n^t)$ the muted point $x^{t+1} = \left(x_1^{t+1}, x_2^{t+1}, \ldots, x_n^{t+1}\right)$ is created as follows:

$$x_i^{t+1} = \begin{cases} x_i^t + \Delta\left(t, x_i^{\mathrm{u}} - x_i^t\right) & \text{if} \quad r \le 0.5 \\ x_i^t - \Delta\left(t, x_i^t - x_i^l\right) & \quad\text{otherwise} \end{cases} \tag{3.2}$$

  where t is current generation number and r is a uniformly distributed random number between 0 and 1. x_i^l and x_i^u are lower and upper bounds of the i the component of the decision vector, respectively. The function $\Delta(t, y)$ given below takes value in the interval $[0, y]$.

$$\Delta(t, y) = y\left(1 - u^{(1-\frac{1}{t})^b}\right),$$

where u is a uniformly distributed random number in the interval $[0, 1]$, T is the maximum number of generations and b is a parameter, determining the strength of the mutation operator. In the initial generations nonuniform mutation tends to search the space uniformly and in the later generations it tends to search the space locally, i.e. closer to its descendants.

3.3 Genetic approach for the Robin inverse problem

The approach GA used to solve the inverse problem is defined following the steps below:

- Step 1: Given an initial random population $\gamma_k^0, k = 1, 2, \ldots\ldots N,$ with N is the population size of search space, solve the problem (3.3) below for each given γ_k^0 by finite element method, given by :

$$\begin{cases} -\Delta u &= 0 \quad in \quad \Omega \\ \partial_n u &= g \quad on \quad \Gamma_0 \\ \frac{\partial u}{\partial n} + \gamma_k^0 u &= 0 \quad on \quad \Gamma_1 \end{cases} \tag{3.3}$$

- Step 2 : Evaluate $\mathbf{J}(\gamma_k^0), k = 1, 2, \ldots\ldots N$
- Step 3 : Using $\mathbf{J}(\gamma_k^0)$ as indicator of each individual, the next generation γ_k^1 is created by GA with the following rule:

$$\gamma_k^1 = M_u \cdot C_r \cdot S_e \left(\gamma_k^0\right) \tag{3.4}$$

  where :

  - S_e: Random Selection
  - C_r: The crossover operator with Crossover Probability P_c
  - M_u: The Mutation operator with Mutation Probability P_m

- Step 4 : Go to step 1 with γ_k^1 replacing γ_k^0 and continue.
- Step 5 : The process continue for γ_k^i, with $i = 1, 2, \ldots Max_{gen})$

4 Numerical results

We consider an isotropic homogeneous medium consisting of an open bounded domain Ω with a piecewise smooth boundary where $\Gamma = \Gamma_0 \cup \Gamma_1$; and $mes(\Gamma_1) \neq 0$ and $mes(\Gamma_0) \geq mes(\Gamma_1)$.

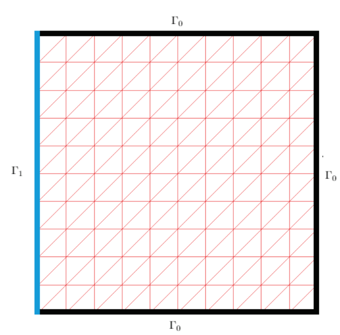

Figure 1. The considered example

We consider a typical bench-mark test example, where Γ_1 is the inaccessible part of the boundary, given by the following:

$$(P) : u_{ex}(x, y) = \cos(x)\cosh(y) + \sin(x)\sinh(y) \qquad \text{and} \qquad \gamma_{ex}(y) = -\tanh(y)$$

The genetic operators and the parameters used for the genetic algorithm for evolving each individual population are as follows:

- maximum number of generations $Max_{gen} = 200$
- population size $n_{pop} = 100$
- Arithmetic Crossover, Crossover probability $P_c = 0.9$
- Non-uniform mutation, mutation probability $P_m = 0.01$
- Insertion: Elitism

The experiments are done on an intel (R) Core(TM) i5-431OU CPU @ 2.6 GHz machine with 4.00 Go RAM.

The algorithm is implemented using the software FreeFem++ which is a free software to solve numerically partial differential equations (PDE) in $\mathbb{R}^2$ and in $\mathbb{R}^3$ with finite elements methods.

It should be noted that the software FreeFem ++ allows a rapid specification of the EDP (direct problem resulting from the considered optimization problem by writing its variational formulation).

Knowing that in most practical cases the desired solution is not known, we consider two approaches in these numerical simulations. First and foremost, we consider that the form of the function of Robin coefficient is known in a way that allows to find the function in the form $\gamma(y) = a \times \tanh(b \times y)$. For the second approach, we seek a polynomial approximation of the desired Robin coefficient.

Several parameters can be influenced on the performance of the genetic algorithm proposed to optimize the functional J and then on the complexity of the approach proposed to solve the inverse problem during the various carried out numerical simulations. Specifically, for the two forms of the function to be approximated, finding an approximation of γ amounts to approximating the coefficients a, b or the polynomial coefficients. Thus, the research domain, or interval of membership of each coefficient, can influence the speed and even the precision of the result obtained. Table 1 shows the influence of choice of membership intervals on the Robin coefficient approximation. Indeed, the carried out numerical simulations show that the more the domain of research is reduced, the more the solution found is precise and less iterations are required.

$[a, b]$	Iteration	J with Approach 1	J with Approach 2
$[-2, 2]$	50	9.66e-007	5.56e-005
$[-10, 10]$	50	1.53e-005	3.86e-004
$[-20, 20]$	50	1.08e-004	7.12e-002
$[-30, 30]$	50	1.008e-003	4.84e-002
$[-40, 40]$	50	1.95e-004	3.13e-002

Table 1. Influence of the choice of the research domain for obtained value of the functional J in the appropriate iteration

On the other hand, the number of individuals generated at each iteration, the probabilities of crossover and mutation are all important factors allowing the improvement of the performance of the proposed algorithm and having a diversity in the population after each iterations which will make it possible to overcome the convergence towards a local minimum. The presented results come from several simulations which resulted in making the best choices for these parameters, also based on the studies carried out in this direction in the work on the genetic algorithms already mentioned.

The obtained results for the first approach (known form of the solution) are presented in figure 2 which shows the evolution of the functional J during a genetic process where it has been taken into account that the sought coefficients belong to the interval [-5.5] and give the different approximations of Robin's coefficient in comparison with the exact solution showing that a good approximation is obtained after a reduced number of iterations.

For the second approach (polynomial approximation), several simulations are carried out with different choices of polynomial degrees and even by taking into consideration the possible properties of the desired coefficient (odd in this case). In this sense, by considering an approximation by a polynomial of degree 4. Figure 3 presents the differences approximations of the

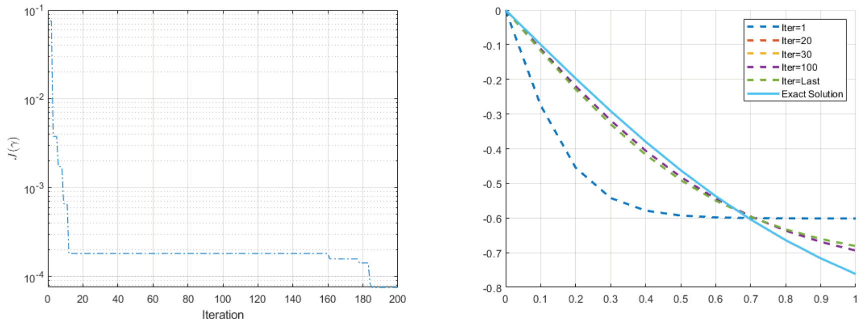

Figure 2. The fitness functional according to the number of iterations J (Left) and the numerical solution $\gamma(y)$ on the inaccessible boundary Γ_1 calculated by the GA for different iterations for the first approach (right)

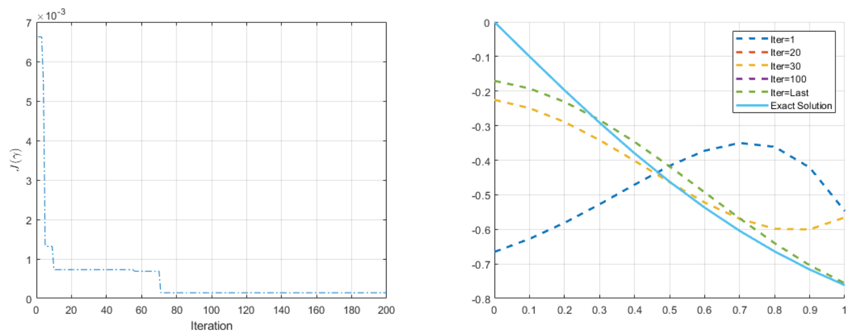

Figure 3. The fitness functional according to the number of iterations J (Left) and the numerical solution $\gamma(y)$ on the inaccessible boundary Γ_1 calculated by the GA for different iterations for the second approach (right)

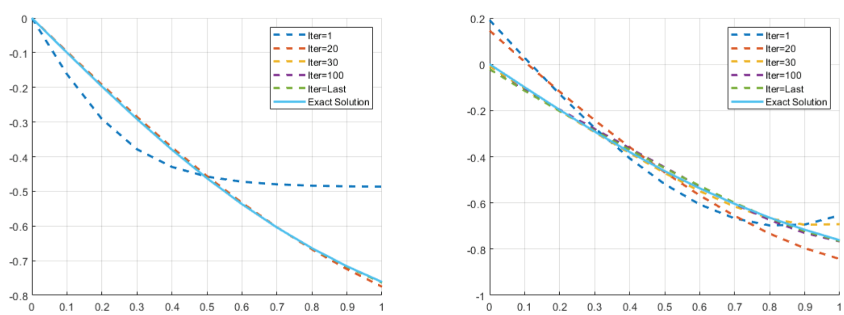

Figure 4. The numerical solution $\gamma(y)$ on the inaccessible boundary Γ_1 calculated by the GA for different iterations for the first approach (left) and the second approach (right)

requested coefficient and also shows the evolution of the functional J and the evolution of Robin coefficient during the genetic process in comparison with the exact solution.

In addition, the studied inverse problem is known to be ill-posed. In particular, the solution does not depend on Cauchy data even if it exists. To increase the numerical stability of the optimization problem, we use the functional J_r where we add a regularization term. Figure 4. gives the results obtained either for the value of J_r as a function of the iterations and the approximation of Robin's coefficient in comparison with the exact solution showing that the use of the functional with regularization term, taken here $\eta = 10^{-5}$, allows to lead to a better approximation. Indeed, with approach 1, we obtain an approximate solution with a functional cost J_r equal to 1.24×10^{-4} after 11 iterations and continues to decrease until the value 2.31×10^{-7} at the iteration 24; however, the obtained value of J is equal to 1.82×10^{-4} after 12 iteration end this value is stagnated. Similarly, with the approach 2, we obtain an approximate solution with a cost of functional J_r equal to 6.91×10^{-4} after just 7 iterations; however, almost the same value with the functional J is obtained after 70 iterations.

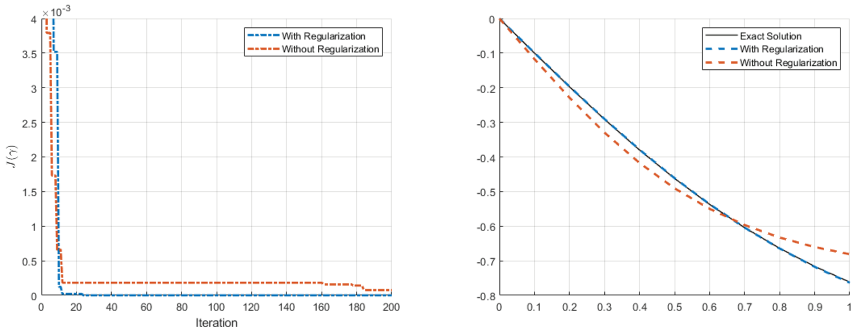

Figure 5. The cost functional with and without regularization (left) and numerical solution for $\gamma(y)$ with and without regularization in comparison with the exact solution with approach 1 (right)

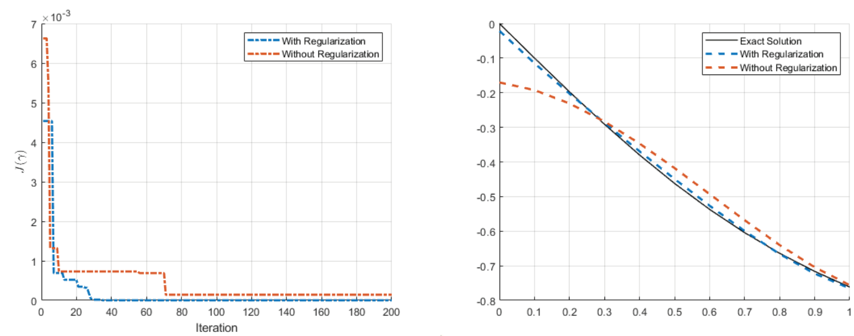

Figure 6. The cost functional with and without regularization (left) and numerical solution for $\gamma(y)$ with and without regularization in comparison with the exact solution with approach 2 (right)

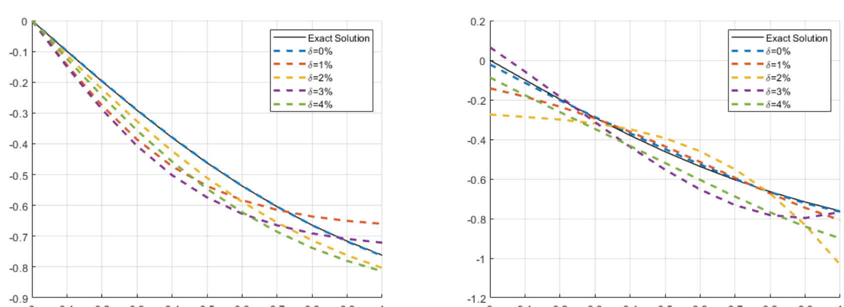

Figure 7. The numerical solution $\gamma(y)$ calculated by the GA, for various levels of noise 1%, 2%, 3% and 4% , in comparison with the exact solution for the first (left) and second approach (right)

Figure 5. and figure 6. present a comparison between the numerical results obtained for the approximation of Robin's coefficient by the first and second approach showing that the proposed

algorithm gives better results if we consider the functional to be optimized with regularization term.

For the study of the stability aspect, the limit data were disturbed with different noise levels in order to simulate the measurement errors. We see that for a noise of 1% for example, there is a good agreement between the two numerical solutions (with noise and without noise), and they are both good approximations of the exact solution. Similar results are obtained for various noise levels added to the input data (see figure 7.). Moreover, it can be seen that as the noise level decreases, the numerical solution gets closer to the exact solution.

5 Conclusion

In this work, a Robin inverse problem is considered. This problem is ill-posed in the sense of Hadamard, which requires regularizing methods to solve it. An optimization approach based on genetic algorithms is proposed. To implement it, a formulation into an optimization problem is carried out giving rise to a functional to be optimized by applying a genetic algorithm with an adequate choice of the genetic operators. The direct problem found is solved by the finite element method. Numerical results are proposed, for an irregular domain, to approach the Robin coefficient as being a solution of a typical bench-mark test example of the inverse Robin problem showing the efficiency of the proposed approach.

The proposed approach is an alternative to existing methods requiring a certain regularity (differentiability for example) to be developed and shows the efficiency of metaheuristic methods in solving this type of inverse problem by adapting the parameters of genetic algorithms. It should be noted that work is in progress to extend this strategy and apply other metaheuristics to other classes of inverse problems.

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Author information

B. Jouilik, Department of Mathematics, Faculty of sciences, Ibn Tofail University,, Morocco.
E-mail: jouilik-bouchta@hotmail.com

J. Daoudi, Department of Mathematics, Polydisciplinary faculty of Larache, Abdelmalek Essaadi University,, Morocco.
E-mail: jamaldaouid5@gmail.com

C. Tajani, Department of Mathematics, Polydisciplinary faculty of Larache, Abdelmalek Essaadi University,, Morocco.
E-mail: chakir_tajani@hotmail.fr

J. Abouchabaka, Department of Mathematics, Faculty of sciences, Ibn Tofail University,, Morocco.
E-mail: abouchabaka3@yahoo.fr

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