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DEVELOPMENT AND ANALYSIS OF SEXTIC POLYNOMIAL EXPLICIT METHOD FOR LOGISTIC MODELS

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Abstract: This paper presents the development and analysis of Sextic Polynomial Explicit Method (SPEM) for the solution of logistic models. The proposed method is derived by means of interpolating function of polynomial form. The properties of SPEM were analysed and investigated. Three numerical examples were solved to measure the performance of SPEM in terms of applicability, accuracy and suitability. The comparative study of the results generated via SPEM and the well-known Classical Runge-Kutta Method (RK4) in the context of the exact solution is presented. The results show that SPEM outperforms RK4. Hence, SPEM is found to be accurate and suitable for the solution of logistic models emanating from real life situation.

1 Introduction

Ordinary differential equations (ODEs) occur in the fields of science and engineering. In real world applications, many differential equations cannot be solved using the standard analytical methods. In such situations, approximation to the solution is needed which are obtained using various numerical algorithms. A great number of numerical methods for determining approximations to the solution of ODEs have been proposed by researchers. There are two main categories of numerical integrators, namely one-step methods and multi-step methods. Fatunla [1] proposed one type of numerical technique by representing the theoretical solution of the initial value problem (IVP) by (linear or nonlinear) interpolating function. In [2], the authors studied the numerical accuracy of the Runge-Kutta method of second, third and fourth order for the numerical solution of differential equations. Fadugba and Falodun [3] developed a new one-step scheme for the solution of IVPs in ODEs. Analysis of composite Runge-Kutta methods and new one-step technique for stiff delay differential equations was considered by [4]. Abolarin and Akingbade [5] derived the fourth stage inverse polynomial scheme for solving initial value problems. Shaalini and Emimal [6] studied the numerical solutions of stiff and non-stiff delay differential equations using Lagrange interpolation. Fadugba [7] developed an improved numerical integration method via the transcendental function of exponential form for IVPs in ODEs. Ref. [8 - 13] also studied the numerical solutions of IVPs in ODEs via several developed methods. In this present work, a new numerical scheme has been developed by representing the theoretical solution of the IVP by an interpolating polynomial of degree six. It is termed here as Sextic Polynomial Explicit Method (SPEM). The stability, convergence and consistency of the proposed method have been discussed. The applicability of this method has been demonstrated by considering three logistic models. The rest of the paper has been organised as follows: Section 2 describes SPEM and its properties. Section 3 provides numerical examples of three nonlinear logistic models. Section 4 explains the concluding remarks.

2 A Proposed Sextic Polynomial Explicit Method

This section presents the problem formulation, derivation of the method and its properties.

2.1 Problem Formulation

Consider an initial value problem of first order ordinary differential equation of the form

$$y' = f(x, y),\ y(a) = y_0, \quad x \in [a, b],\ \ y \in (-\infty, \infty) \tag{2.1}$$

The existence and uniqueness of solution of (2.1) has been guaranteed via the Lipschitz condition on the interval *I* = [*a, b]*. The analytical solution of (2.1) at $x = x_n$ is given by $y(x_n)$.

2.2 Derivation of the Method

Consider the interpolating polynomial of the form

$$F(x) = \sum_{j=0}^{6} \beta_j x^j \tag{2.2}$$

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$are undetermined constants. The integration interval of $[a, b]$is defined as $a = x_0 \le x \le x_n = b$. The step length is defined as

$$h = \frac{b-a}{N} \tag{2.3}$$

The mesh point is defined as

$$x_{n+1} = x_0 + (n+1)h, n = 0, 1, 2, ...N-1 \tag{2.4}$$

or

$$x_n = x_0 + nh, n = 1, 2, ...N \tag{2.5}$$

Using (2.4) and (2.5), with $x_0 = 0$, yields

$$x_n = nh \tag{2.6}$$

$$x_{n+1} = (n+1)h \tag{2.7}$$

$$x_{n+1} - x_n = h \tag{2.8}$$

$$x_{n+1}^2 - x_n^2 = (2n+1)h^2 \tag{2.9}$$

$$x_{n+1}^3 - x_n^3 = (3n^2 + 3n + 1)h^3 \tag{2.10}$$

$$x_{n+1}^4 - x_n^4 = (4n^3 + 6n^2 + 4n + 1)h^4 \tag{2.11}$$

$$x_{n+1}^5 - x_n^5 = (5n^4 + 10n^3 + 10n^2 + 5n + 1)h^5 \tag{2.12}$$

$$x_{n+1}^6 - x_n^6 = (6n^5 + 15n^4 + 20n^3 + 15n^2 + 6n + 1)h^6 \tag{2.13}$$

Expanding (2.2) at the points $x_n$and x_{n+1} yields

$$F(x_n) = \sum_{j=0}^{6} \beta_j x_n^j \tag{2.14}$$

and

$$F(x_{n+1}) = \sum_{j=0}^{6} \beta_j x_{n+1}^j \tag{2.15}$$

respectively. Setting (2.14) and (2.15) into $Y(x_n)$ and $Y(x_{n+1})$, we have that

$$Y(x_n) = \sum_{j=0}^{6} \beta_j x_n^j \tag{2.16}$$

and

$$Y(x_{n+1}) = \sum_{j=0}^{6} \beta_j x_{n+1}^j \tag{2.17}$$

Suppose that

$$Y(x_{n+1}) - Y(x_n) \equiv y_{n+1} - y_n \tag{2.18}$$

where

$$\begin{aligned} Y(x_{n+1}) - Y(x_n) = \textstyle\sum_{j=1}^{6} \beta_j \left(x_{n+1}^j - x_n^j\right) &= \beta_1 \left(x_{n+1} - x_n\right) + \beta_2 \left(x_{n+1}^2 - x_n^2\right) \\ &+\beta_3 \left(x_{n+1}^3 - x_n^3\right) + \beta_4 \left(x_{n+1}^4 - x_n^4\right) + \beta_5 \left(x_{n+1}^5 - x_n^5\right) \\ &+\beta_6 \left(x_{n+1}^6 - x_n^6\right) \end{aligned} \tag{2.19}$$

This implies that

$$\begin{aligned} y_{n+1} - y_n = \beta_1 \left(x_{n+1} - x_n\right) + \beta_2 \left(x_{n+1}^2 - x_n^2\right) + \beta_3 \left(x_{n+1}^3 - x_n^3\right) \\ +\beta_4 \left(x_{n+1}^4 - x_n^4\right) + \beta_5 \left(x_{n+1}^5 - x_n^5\right) + \beta_6 \left(x_{n+1}^6 - x_n^6\right) \end{aligned} \tag{2.20}$$

Differentiating (2.16), yields

$$f_n = \sum_{j=1}^{6} j\beta_j x_n^{j-1} \tag{2.21}$$

$$f_n^{(1)} = \sum_{j=2}^{6} j(j-1)\beta_j x_n^{j-2} \tag{2.22}$$

$$f_n^{(2)} = \sum_{j=3}^{6} j(j-1)(j-2)\beta_j x_n^{j-3} \tag{2.23}$$

$$f_n^{(3)} = \sum_{j=4}^{6} j(j-1)(j-2)(j-3)\beta_j x_n^{j-4} \tag{2.24}$$

$$f_n^{(4)} = \sum_{j=5}^{6} j(j-1)(j-2)(j-3)(j-4)\beta_j x_n^{j-5} \tag{2.25}$$

$$f_n^{(5)} = \sum_{j=6}^{6} j(j-1)(j-2)(j-3)(j-4)(j-5)\beta_j x_n^{j-6} \tag{2.26}$$

Solving (2.21)-(2.26) and using (2.6), we obtain

$$\beta_1 = \frac{1}{720}\left(720 f_n - 720nh f_n^{(1)} + 360(nh)^2 f_n^{(2)} - 120(nh)^3 f_n^{(3)} + 30(nh)^4 f_n^{(4)} - 6(nh)^5 f_n^{(5)}\right) \tag{2.27}$$

$$\beta_2 = \frac{1}{720}\left(360 f_n^{(1)} - 360nh f_n^{(2)} + 180(nh)^2 f_n^{(3)} - 60(nh)^3 f_n^{(4)} + 15(nh)^4 f_n^{(5)}\right) \tag{2.28}$$

$$\beta_3 = \frac{1}{720}\left(120 f_n^{(2)} - 120nh f_n^{(3)} + 60(nh)^2 f_n^{(4)} - 20(nh)^3 f_n^{(5)}\right) \tag{2.29}$$

$$\beta_4 = \frac{1}{720}\left(30 f_n^{(3)} - 30nh f_n^{(4)} + 15(nh)^2 f_n^{(5)}\right) \tag{2.30}$$

$$\beta_5 = \frac{1}{720}\left(6f_n^{(4)} - 6nhf_n^{(5)}\right) \tag{2.31}$$

$$\beta_6 = \frac{f_n^{(5)}}{720} \tag{2.32}$$

Using (2.6)-(2.13), (2.27)-(2.32) in (2.20), yields

$$y_{n+1} - y_n = \frac{h}{720}\left(B_1 + B_2 + B_3 + B_4 + B_5 + B_6\right) \tag{2.33}$$

with

$$B_1 = \left(720f_n - 720nhf_n^{(1)} + 360(nh)^2 f_n^{(2)} - 120(nh)^3 f_n^{(3)} + 30(nh)^4 f_n^{(4)} - 6(nh)^5 f_n^{(5)}\right) \tag{2.34}$$

$$B_2 = h(2n+1)\left(360f_n^{(1)} - 360nhf_n^{(2)} + 180(nh)^2 f_n^{(3)} - 60(nh)^3 f_n^{(4)} + 15(nh)^4 f_n^{(5)}\right) \tag{2.35}$$

$$B_3 = h^2(3n^2+3n+1)\left(120f_n^{(2)} - 120nhf_n^{(3)} + 60(nh)^2 f_n^{(4)} - 20(nh)^3 f_n^{(5)}\right) \tag{2.36}$$

$$B_4 = h^3(4n^3+6n^2+4n+1)\left(30f_n^{(3)} - 30nhf_n^{(4)} + 15(nh)^2 f_n^{(5)}\right) \tag{2.37}$$

$$B_5 = h^4(5n^4+10n^3+10n^2+5n+1)\left(6f_n^{(4)} - 6nhf_n^{(5)}\right) \tag{2.38}$$

$$B_6 = h^5(6n^5+15n^4+20n^3+15^2+6n+1)f_n^{(5)} \tag{2.39}$$

Equation (2.33) is the newly proposed Sextic Polynomial Explicit Method for the solution of initial value problems of ordinary differential equations.

2.3 Properties of the scheme

The properties of the method are discussed as follows:

2.3.1 Order and Consistency of the Method

According to [14], a numerical method is said to be consistent if it has at least order p =1. To determine the order of the method and to show the consistency property of the method, we follow the procedures of [14] and [15]. Substituting (2.34)-(2.39) into (2.33) and simplifying further, yields

$$\frac{y_{n+1} - y_n}{h} = f_n + \frac{h}{2}f_n^{(1)} + \frac{h^2}{6}f_n^{(2)} + \frac{h^3}{24}f_n^{(3)} + \frac{h^4}{120}f_n^{(4)} + \frac{h^5}{720}f_n^{(5)} \tag{2.40}$$

Taking the limit as $h \to 0$, we get

$$\frac{y_{n+1} - y_n}{h} = f_n = f(x_n, y_n) \tag{2.41}$$

Hence, the method given by (2.33) is consistent. By virtue of the Taylor series, it is found that the method is of order 6. Also, the local truncation error for this method is obtained as $O(h^7)$.

2.3.2 Linear Stability Analysis of the Method

Consider the linear test equation of the form

$$y' = \lambda y, y(x_0) = y_0 \tag{2.42}$$

where λ is a constant. Then

$$f_n = \lambda y_n, f_n^{(1)} = \lambda^2 y_n, f_n^{(2)} = \lambda^3 y_n, f_n^{(3)} = \lambda^4 y_n, f_n^{(4)} = \lambda^5 y_n, f_n^{(5)} = \lambda^6 y_n \tag{2.43}$$

Thus, (2.33) becomes

$$\frac{y_{n+1}}{y_n} = \left(1 + \lambda h + \frac{(\lambda h)^2}{2} + \frac{(\lambda h)^3}{6} + \frac{(\lambda h)^4}{24} + \frac{(\lambda h)^5}{120} + \frac{(\lambda h)^6}{720}\right) \tag{2.44}$$

Setting z = λh, then (2.44) becomes

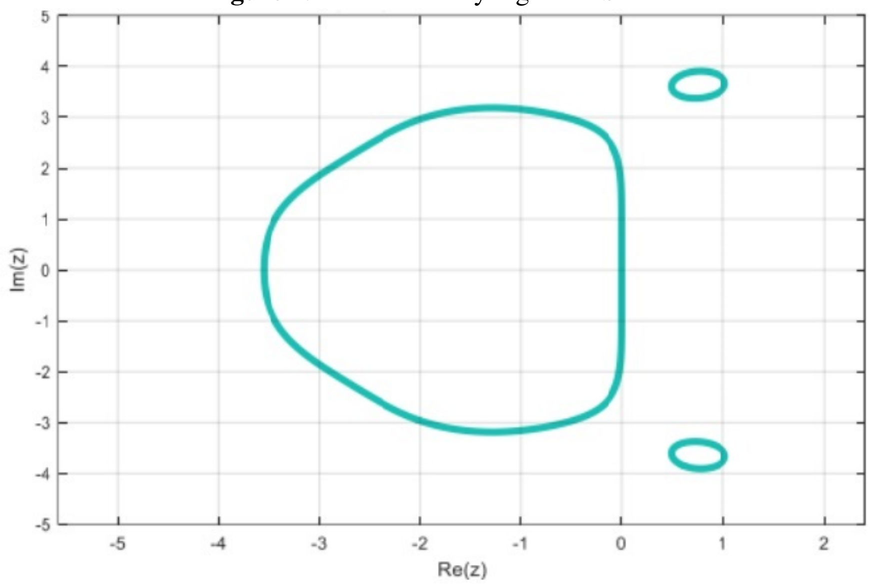
$$\frac{y_{n+1}}{y_n} = \left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{120} + \frac{z^6}{720}\right) \tag{2.45}$$

The stability polynomial of this new method is given by

$$p(z) = \sum_{k=0}^{6} \frac{z^k}{\Gamma(k+1)} \tag{2.46}$$

The linear stability region of SPEM is obtained and given in the following figure.

Figure 1. Linear stability region of SPEM



2.3.3 Zero Stability of the Method

A linear explicit multistep method of k = 1 is said to be zero stable if the zeros of the first characteristic polynomial

$$p(r) = \sum_{j=1}^{1} \alpha_j r^j \tag{2.47}$$

satisfy the Dahlquist root conditions:

(i) all zeros r satisfy $|r| \leq 1$

(ii) multiple zeros satisfy $|r| < 1$

The characteristic polynomial for SPEM is given by

$$p(r) = r - 1 \tag{2.48}$$

To get the zero(s), setting $p(r) = 0$
This implies that

$$r - 1 = 0, r = 1 \tag{2.49}$$

Since the zero of the first characteristic polynomial of SPEM satisfies the above root conditions, hence, it is concluded that SPEM is zero stable.

2.3.4 Convergence of the Method
From the order of accuracy of the method, it is clearly seen that the method is of order six. Also the method is zero stable and consistent. The necessary and sufficient conditions for a numerical method to be convergent are zero stability and consistency. Since these conditions are satisfied, we can conclude that SPEM is convergent

3 Numerical Examples and Results

The performance of the SPEM is tested on the following logistics models.
Example 1: Consider a non-linear logistic model for the bacteria growth rate of the form

$$\frac{dc}{dt} = kc\left(1 - 0.0125c\right), c(0) = 4, k = 5 \tag{3.1}$$

The exact solution is obtained as

$$c(t) = \frac{80}{1 + 19\exp(-5t)} \tag{3.2}$$

The results generated via SPEM and RK4 are presented in Tables 1 and 2.

Example 2: Consider a non-linear logistic model

$$\frac{du}{dt} = u(1 - u), u(0) = 0.4 \tag{3.3}$$

The exact solution is obtained as

$$u(t) = \frac{0.4\exp(t)}{1 + 0.4(\exp(t) - 1)} \tag{3.4}$$

The results generated via SPEM and RK4 are shown in Tables 3 and 4.

Example 3: Consider a non-linear logistic model

$$\frac{du}{dt} = u(a - bu), u(0) = u_0 \tag{3.5}$$

The exact solution is obtained as

$$u(t) = \frac{au_0\exp(at)}{a + bu_0(\exp(at) - 1)} \tag{3.6}$$

where a is the coefficient for the virus transmission mechanism and b is the coefficient for the effectiveness of the government restrictions (quarantine rule). The results generated via SPEM and RK4 are displayed in Tables 5, 6 and 7.

Table 1. Final absolute relative error generated via SPEM and RK4 for Problem 1

h	SPEM	RK4
0.1	0.000050703451	0.006096554432
0.01	0.000000000046	0.000000668329
0.001	0.000000000000	0.000000000068
0.0001	0.000000000000	0.000000000000

Table 2. Comparative results analyses of SPEM, RK4 and exact solution for Problem 1 with h = 0.1

‘TN’	‘CN’$_{SPEM}$	‘CN’$_{RK4}$	‘CTN’$_{EXACT}$	‘EN’$_{SPEM}$	‘EN’$_{RK4}$
0.0	4.0000000000000	4.0000000000000	4.0000000000000	0.000000000000	0.000000000000
0.1	6.3877041904300	6.3869308532960	6.3876934521450	0.000010738285	0.000762598848
0.2	10.012905523178	10.010782619961	10.012879839867	0.000025683311	0.002097219906
0.3	15.268732237921	15.264702526237	15.268711060481	0.000021177440	0.004008534244
0.4	22.400329045712	22.394184281537	22.400364973206	0.000035927494	0.006180691669
0.5	31.254602812292	31.246730899601	31.254700733190	0.000097920898	0.007969833589
0.6	41.110901769627	41.102181767458	41.110934640935	0.000032871308	0.008752873477
0.7	50.834079405603	50.825520453662	50.833986027699	0.000093377904	0.008465574037
0.8	59.347386587158	59.339647124272	59.347306972859	0.000079614299	0.007659848587
0.9	66.057222691939	66.050387768979	66.057237226231	0.000014534292	0.006849457252
1.0	70.920615221808	70.914569370827	70.920665925259	0.000050703451	0.006096554432

Table 3. Final absolute relative error generated via SPEM and RK4 for Problem 2

h	SPEM	RK4
0.1	0.000000000143	0.000000014610
0.01	0.000000000000	0.000000000001
0.001	0.000000000000	0.000000000000
0.0001	0.000000000000	0.000000000000

Table 4. Comparative results analyses of SPEM, RK4 and exact solution for Problem 2 with h = 0.1

‘TN’	‘UN’$_{SPEM}$	‘UN’$_{RK4}$	‘UTN’$_{EXACT}$	‘EN’$_{SPEM}$	‘EN’$_{RK4}$
0.0	0.400000000000	0.400000000000	0.400000000000	0.000000000000	0.000000000000
0.1	0.424222038729	0.424222037019	0.424222038718	0.000000000011	0.000000001700
0.2	0.448813669556	0.448813666287	0.448813669530	0.000000000026	0.000000003244
0.3	0.473658134962	0.473658130255	0.473658134918	0.000000000044	0.000000004663
0.4	0.498633726438	0.498633720377	0.498633726374	0.000000000065	0.000000005996
0.5	0.523616137863	0.523616130491	0.523616137777	0.000000000086	0.000000007286
0.6	0.548480927451	0.548480918768	0.548480927346	0.000000000105	0.000000008577
0.7	0.573105985409	0.573105975371	0.573105985287	0.000000000122	0.000000009917
0.8	0.597373904008	0.597373892527	0.597373903873	0.000000000134	0.000000011346
0.9	0.621174153740	0.621174140697	0.621174153598	0.000000000142	0.000000012901
1.0	0.644404982788	0.644404968035	0.644404982645	0.000000000143	0.000000014610

Table 5. Comparative results analyses of SPEM, RK4 and exact solution for Problem 3 with h = 0.1

‘TN’	‘UN’$_{SPEM}$	‘UN’$_{RK4}$	‘UTN’$_{EXACT}$	‘EN’$_{SPEM}$	‘EN’$_{RK4}$
0.0	571.000000000000	571.000000000000	571.000000000000	0.000000000000	0.000000000000
0.1	587.094946807399	587.094946728635	587.094946807399	0.000000000000	0.000000078764
0.2	603.640118867513	603.640118705663	603.640118867514	0.000000000000	0.000000161851
0.3	620.647913364301	620.647913114868	620.647913364302	0.000000000001	0.000000249433
0.4	638.131057592256	638.131057250567	638.131057592257	0.000000000001	0.000000341690
0.5	656.102617101494	656.102616662689	656.102617101495	0.000000000001	0.000000438806
0.6	674.576004006246	674.576003465277	674.576004006247	0.000000000002	0.000000540970
0.7	693.564985457754	693.564984809376	693.564985457756	0.000000000002	0.000000648380
0.8	713.083692282365	713.083691521129	713.083692282367	0.000000000001	0.000000761237
0.9	733.146627785476	733.146626905724	733.146627785477	0.000000000002	0.000000879754
1.0	753.768676721770	753.768675717628	753.768676721772	0.000000000002	0.000001004144

Table 6. Comparative results analyses of SPEM, RK4 and exact solution for Problem 3 with h = 0.1 and different values of time, t (days)

t	‘UN’$_{SPEM}$	‘UN’$_{RK4}$	‘UTN’$_{EXACT}$
0	571.0000000000000	571.0000000000000	571.000000000000
5	2266.103420036719	2266.103405749799	2266.103420036581
10	8457.563886923883	8457.563798505764	8457.563886922031
15	25923.854096541858	25923.853848460403	25923.854096542524
20	52826.493984558489	52826.493719219063	52826.493984553577
25	70994.527830096296	70994.527692920543	70994.527830096500
30	77573.486837016710	77573.486784128749	77573.486837016419
35	79387.635604725554	79387.635586962890	79387.635604725525
40	79848.116894757739	79848.116889236670	79848.116894757666
45	79962.492437872614	79962.492436233428	79962.492437872585
50	79990.747480797407	79990.747480325139	79990.747480797450
55	79997.718158025498	79997.718157892130	79997.718158025382
60	79999.437292604693	79999.437292567658	79999.437292604678

Table 7. Absolute relative error generated via SPEM and RK4 for Problem 3 with h = 0.1 and different values of time, t (days)

t	'EN'$_{SPEM}$	'EN'$_{RK4}$
0	0.000000000000	0.000000000000
5	0.000000000139	0.000014286782
10	0.000000001852	0.000088416267
15	0.000000000666	0.000248082120
20	0.000000004911	0.000265334515
25	0.000000000204	0.000137175957
30	0.000000000291	0.000052887670
35	0.000000000029	0.000017762635
40	0.000000000073	0.000005520997
45	0.000000000029	0.000001639157
50	0.000000000044	0.000000472312
55	0.000000000116	0.000000133252
60	0.000000000015	0.000000037020

4 Concluding Remarks

In this paper, Sextic Polynomial Explicit Method (SPEM) for the solution of logistic models has been developed. The properties of SPEM in terms of order of accuracy, consistency, linear stability, zero stability and convergence were analysed and investigated. To measure the performance of SPEM, three numerical examples have been solved and the results were compared with the Classical Runge-Kutta Method (RK4) in the context of the Exact Solution (ES). Furthermore, by varying the step length, there are six-order decrease in the values of the final absolute relative errors generated via SPEM as shown in Tables 1 and 3. Moreover**,** it is also observed from Tables 2, 4, 5 and 6 that SPEM outperformed the well-known RK4. In addition, it is clearly seen from Table 6 that the results of SPEM followed that of exact solution elegantly for different values of time, *t* as this is evident in Table 7. Hence, SPEM is found to be accurate, consistent, stable, zero stable, convergence and a good sixth order explicit method for the numerical solutions of IVPs of different characteristics in ODEs.

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