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Corrigendum to "SOME PROPERTIES OF MEROMORPHICALLY UNIFORMLY CONVEX FUNCTIONS DEFINED BY HURWITZ -LERCH ZETA FUNCTION

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The authors received a message from the editor-in-chief of the PJM, Ayman Badawi, who kindly informed us that in our article [1], pages number 624-625, we have taken the below paragraph from the Hacettepe journal of mathematics and statistics, Volume 45 (5) (2016), 1449 -1460 authored by Zhi-Gang Wang and Lei Shi [12].

This is the part that was taken from the paper [12]

The following we recall a general Hurwitz-Lerch Zeta function $\phi(z,s,a)$ defined by (see [10], p. 121).

$$\phi(z,s,a)=\sum_{m=0}^{\infty}\frac{z^m}{(m+a)^s}$$

for $a\in\mathbb{C}\setminus\mathbb{Z}_0^-, s\in\mathbb{C}$ when $|z|<1$; $\Re(s)>1$ when $|z|=1$, where $\mathbb{Z}_0^-=\mathbb{Z}\setminus\{\mathbb{N}\}$, $\mathbb{Z}=\{0,\pm1,\pm2,\cdots\}$, $\mathbb{N}=\{1,2,3,\cdots\}$.

Several interesting properties and characteristics of the Hurwitz-Lerch Zeta function $\phi(z,s,a)$ can be found in the recent investigation by (for example) Choi and Srivastava [2], Ferreira and Lopez [3], Garg et al. [4], Lin and Srivastava [6], Luo and Srivastava [7], Srivastava et al. [11], Ghanim [5] and others.

By making use of Hurwitz-Lerch Zeta function $\phi(z,s,a)$, Srivastava and Attiya [9] recently introduced and investigated the integral operator

$$\mathcal{J}_{s,b}f(z)=z+\sum_{m=2}^{\infty}\left(\frac{1+b}{k+b}\right)^s a_m z^m,(b\in\mathbb{C}\setminus\mathbb{Z}_0^-,s\in\mathbb{C},z\in U).$$

Motivated essentially by the above mentioned Srivastava-Atiya operator $\mathcal{J}_{s,b}$, the linear operator

$$\mathcal{W}_{s,b}:\Sigma\to\Sigma$$

defined, in terms of the Hardmard product (or convolution), by

$$\mathcal{W}_{s,b}f(z)=\Theta_{s,b}(z)*f(z),\quad(b\in\mathbb{C}\setminus\mathbb{Z}_0^-\cup\{1\},s\in\mathbb{C},f\in\Sigma,z\in U^*),\tag{0.1}$$

where for convenience,

$$\Theta_{s,b}(z)=(b-1)^s\left[\phi(z,s,b)-b^{-s}+\frac{1}{z(b-1)^s}\right],\quad z\in U^*.$$

It can be easily be seen from (0.1) that

$$\mathcal{W}_{s,b}f(z)=\frac{1}{z}+\sum_{m=1}^{\infty}L(m,s,b)a_m z^m,\tag{0.2}$$

$$\text{where} \quad L(m,s,b) = \left(\frac{b-1}{b+m}\right)^s.$$

Indeed, the operator $\mathcal{W}_{s,b}$ can be defined for $b \in \mathbb{C} \setminus \mathbb{Z}_0^- \cup \{1\}$, where

$$\mathcal{W}_{s,0} f(z) = \lim_{b \to 0} \{\mathcal{W}_{s,b} f(z)\}.$$

We observe that

$$\mathcal{W}_{\{0,b\}} f(z) = f(z),$$

and

$$\mathcal{W}_{1,\gamma} = \frac{\gamma - 1}{z^\gamma} \int_0^z t^{\gamma-1} f(t) dt, \quad \Re(\gamma) > 1.$$

Furthermore, from the definition (0.2), we find that

$$\mathcal{W}_{s+1,b} f(z) = \frac{b-1}{z^b} \int_0^z t^{b-1} \mathcal{W}_{s,b} f(t) dt, \quad \Re(b) > 1. \tag{0.3}$$

Differentiating both sides of (0.3) with respect to z, we get the following useful relationship:

$$z\left(\mathcal{W}_{s+1,b} f\right)'(z) = (b-1)\mathcal{W}_{s,b} f(z) - b\mathcal{W}_{s+1,b} f(z).$$

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