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NEW METHOD FOR SOLVING FUZZY SEQUENCING PROBLEM

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Abstract This paper deals with sequencing problems for 'n' jobs on two machines, 'n' jobs on three machines and 'n' jobs on m machines. We have solved the sequencing problem by converting the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut . An algorithm is provided for obtaining an optimal sequence and also for determining the minimum total elapsed time and idle time for each machine. To illustrate this, numerical examples are provided.

1 Introduction

Operations Research is relatively a new discipline which originated during World War II, and later became very popular throughout the world. It is used successfully in almost all the fields. Operations Research helps us to make better decisions in complex scenarios. It also includes the application of scientific tools for finding the optimum solution to a problem involving the operations of a system. Sequencing problem is considered to be one of the important applications of Operations research. A series, in which a few jobs or tasks are to be performed following an order, is called sequencing. An algorithm was proposed by Johnson [2] for scheduling jobs in two machines. Its primary objective is to find the optimal sequence of jobs and to reduce the total amount of time it takes to complete all the jobs. It also reduces the amount of idle time between the two machines. Furthermore, Johnson's method has been extended to 'm' machines problem with an objective to complete all the jobs in a minimum duration. Generally, in sequencing problems, the processing times are valued precisely. But in reality, it is perceived that the processing times during the performance of the job are imprecise and uncertain. In order to handle this uncertainties, we use intervals and fuzzy numbers. Interval computation was first suggested by Dwyer [1]. The concept of fuzzy sets was proposed by Zadeh [4]. Suparna Das et al. [3] have proposed the new methods for solving the linear simultaneous equations with interval and fuzzy parameters (triangular and trapezoidal).

The rest of this paper is organized as follows:
In section 2, basic preliminaries of interval and its arithmetic, types of intervals, ordering of intervals, fuzzy number, trapezoidal fuzzy number, α-cut of a fuzzy number and its arithmetic are given. In section 3, basic terminologies of sequencing and an algorithm for solving sequencing problem is provided. In section 4, numerical examples illustrating the algorithm are given. Finally, the conclusion.

2 Preliminaries

2.1. Interval Number

An interval number A is defined as $A = [\beta_1, \beta_2] = \{x : \beta_1 \le x \le \beta_2, x \in R\}$. Here, $\beta_1, \beta_2 \in R$ are the lower and upper bounds of the interval.

Arithmetic operations of interval

Let $A = [\beta_1, \beta_2]$and $B = [\gamma_1, \gamma_2]$ be two intervals. Then

Addition: $A + B = [\beta_1 + \gamma_1, \beta_2 + \gamma_2]$

Subtraction: $A - B = [\beta_1 - \gamma_2, \beta_2 - \gamma_1]$

Multiplication : $A \times B = [min(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2), max(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2)]$

Division : $\frac{A}{B} = \frac{[\beta_1,\beta_2]}{[\gamma_1,\gamma_2]} = [\beta_1, \beta_2].\frac{1}{[\gamma_1,\gamma_2]}$
where $\frac{1}{[\gamma_1,\gamma_2]} = [\frac{1}{\gamma_2}, \frac{1}{\gamma_1}], 0 \notin [\gamma_1, \gamma_2]$
$\frac{1}{[\gamma_1,0]} = [-\infty, \frac{1}{\gamma_1}]$
$\frac{1}{[0,\gamma_2]} = [\frac{1}{\gamma_2}, \infty]$ and
$\frac{1}{[\gamma_1,\gamma_2]} = [-\infty, \frac{1}{\gamma_1}] \cup [\frac{1}{\gamma_2}, \infty] = [-\infty, \infty], 0 \in [\gamma_1, \gamma_2]$

Scalar Multiplication: Let $A = [\beta_1, \beta_2]$ then $uA = [u\beta_1, u\beta_2], u \geq 0$ and
$uA = [u\beta_2, u\beta_1], u \leq 0$.

2.2. Types of Intervals

Let $A = [\beta_1, \beta_2]$ and $B = [\gamma_1, \gamma_2]$ be two intervals. Therefore these can be classified into three types as follows :

Type I - Non overlapping intervals :
If two intervals are disjoint then they are known as non overlapping intervals.

Type II - Partially overlapping intervals :
If one interval contains the other interval partially then they are known as partially overlapping intervals.

Type III- Completely overlapping intervals : If one interval is completely contained in the other interval then they are known as completely overlapping intervals.

These three types of intervals are shown in the figures below

Figure 1. Type – I intervals

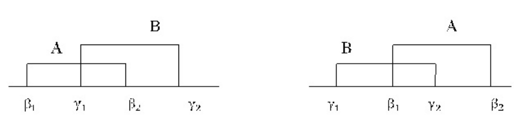

Figure 2. Type – II intervals

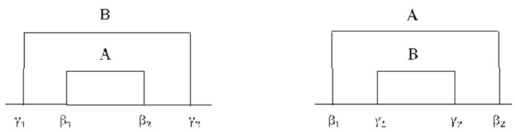

Figure 3. Type – III intervals

2.3. Ordering of Intervals

Let $A = [\beta_1, \beta_2]$ be the interval number. It can also be expressed by its centre and radius and is denoted by $\langle a_c, a_w \rangle$, where $a_c = \frac{\beta_1+\beta_2}{2}$ and $a_w = \frac{\beta_2-\beta_1}{2}$ and they are known as centre and radius of the interval respectively.
Let $A = [\beta_1, \beta_2] = \langle a_c, a_w \rangle$ and $B = [\gamma_1, \gamma_2] = \langle b_c, b_w \rangle$. Then the relation on interval number is defined as
i. $A < B$ iff $a_c < b_c$ whenever $a_c \neq b_c$.
ii. $A > B$ iff $a_c > b_c$ whenever $a_c \neq b_c$.
iii. $A < B$ iff $a_w < b_w$ whenever $a_c = b_c$.
iv. $A > B$ iff $a_w > b_w$ whenever $a_c = b_c$.

2.4. Definition

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\bar{A}}$ such that the value assigned to the element of the universal set X will fall within a specified range i.e $\mu_{\bar{A}} \colon X \to [0, 1]$. The assigned value indicates the membership grade or degree of the element in the set A. The function $\mu_{\bar{A}}$ is called the membership function and the set $\bar{A} = \{(x, \mu_{\bar{A}}(x)) \colon x \in X\}$ defined by $\mu_{\bar{A}}$ for each $x \in X$ is called a fuzzy set.

2.5. Fuzzy Number

A fuzzy set $\bar{A}$ defined on a set of real number R is said to be a fuzzy number, if its membership function $\mu_{\bar{A}}(x) \colon R \to [0, 1]$ satisfies the following properties .
i. $\tilde{A}$ is convex.
that is $\mu_{\tilde{A}}\{\lambda x_1 + (1-\lambda)x_2\} \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ for all $x_1, x_2 \in R$ and $\lambda \in [0, 1]$
ii. $\tilde{A}$ is normal there exists an element $x_0 \in \tilde{A}$ such that $\mu_{\tilde{A}}(x_0) = 1$
iii. $\mu_{\tilde{A}}(x)$ is piece wise continuous.

2.6. Trapezoidal Fuzzy number

A fuzzy number $\tilde{A} = (\beta_1, \beta_2, \beta_3, \beta_4)$ is said to be trapezoidal fuzzy number if its membership function is given by $\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < \beta_1 \\ \frac{x-\beta_1}{\beta_2-\beta_1} & \text{if } \beta_1 \leq x \leq \beta_2 \\ 1 & \text{if } \beta_2 \leq x \leq \beta_3 \\ \frac{\beta_4-x}{\beta_4-\beta_3} & \text{if } \beta_3 \leq x \leq \beta_4 \\ 0 & \text{if } x > \beta_4 \end{cases}$
where $\beta_1, \beta_2, \beta_3$ and β_4 are real numbers.

2.7. Definition of Alpha Cut

The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set. $\tilde{A}_\alpha = \{x \in X \colon \mu_{\tilde{A}}(x) \geq \alpha\}$ where $\alpha \in [0, 1]$.

2.8. Conversion of Trapezoidal Fuzzy Number into Interval using Alpha Cut

Let $\tilde{A} = (\beta_1, \beta_2, \beta_3, \beta_4)$be the trapezoidal fuzzy number then to find α -cut of $\tilde{A}$. We first set α equal to the left and right membership function of $\tilde{A}$.
That is $\alpha = \frac{x-\beta_2}{\beta_2-\beta_1}$ and $\alpha = \frac{\beta_4-x}{\beta_4-\beta_3}$.
Expressing x in terms of α we have , $x = \alpha(\beta_2 - \beta_1) + \beta_1$ and $x = -\alpha(\beta_4 - \beta_3) + \beta_4$
Therefore we can write the fuzzy interval in terms of $\alpha-$ cut interval :
$\tilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$

2.9. Fuzzy Arithmetic

As $\tilde{A}_\alpha$ is now interval, fuzzy addition, subtraction,multiplication and division are the same as interval arithmetic.

3 Sequencing Problem

3.1. Principal assumptions

While solving a sequencing problem, the following assumptions are made:
i. The processing time on different machines, are not dependent of the order, of the job in which they are to be performed.
ii. No machine can process more than one job at a time.
iii. The time required in transferring a job from one machine to another machine is negligible and it can be considered as zero.
iv. Each operation as well as job once started must be completed.
v. Processing time are known and fixed.
vi. An operation must be completed before its succeeding operation commence.

3.2. Basic Terminologies

Number of Machines:
Number of service facilities through which the job is to be passed.

Processing Time:
The time required for a job to process on a particular machine.

Processing Order:
Sequence in which various machines are needed for the completion of a job.

Total elapsed time:
The total time required to complete all the jobs from the first to the last in a sequence.

Idle Time:
Idle time on a machine is the time for which the machine remains idle during the total elapsed time.

No passing rule:
If each of the n jobs are to be processed through two machines M_1 and M_2 then the order should be maintained that is , it must go on to machineM_1 first and then to Machine M_2.

3.3. Algorithm for solving sequencing problem

3.3.A. Processing 'n' jobs on two machines

Let $A'_1, A'_2 \ldots A'_n$be the processing time of 'n' jobs on Machine 1 and $B'_1, B'_2 \ldots B'_n$ be the processing times of 'n' jobs on Machine 2. The problem is to find the order in which the 'n' jobs are to be processed through two machines with the minimum total elapsed time.

Procedure:

Step 1 : Convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut.

Step 2 : Use ordering of intervals to identify the minimum processing time from the given list of processing times $A'_1, A'_2...A'_n$ and $B'_1, B'_2 \ldots B'_n$.

Step 3 : If the minimum processing time is A'_p (i.e., job number p on machine 1) then do the p^{th} job first in the sequence. If the minimum processing time is B'_q (i.e., job number q on machine 2) then do the q^{th} job last in the sequence.

Step 4:

i. If there is a tie in the minimum processing time of Machine 1 and Machine 2 (i.e., $A'_p = B'_q$), process the p^{th} job first and q^{th} job last in the sequence.

ii. If the tie for the minimum occurs among the processing time on Machine 1, select the job corresponding to the minimum of processing time on Machine 2 and process it first.

iii. If the tie for the minimum occurs among the processing time on Machine 2, select the job corresponding to the minimum of processing time on Machine 1 and process it last.

Step 5: Cancel the jobs already assigned and repeat steps 2 to 4 until all the jobs have been assigned. The resulting order will minimise the total elapsed time and it is known as optimal sequence.

Step 6: After obtaining an optimal sequence as stated above, the total elapsed time and also the idle time on machines 1 and 2 are calculated as follows :

Total elapsed time = Time out of the last job on machine 2.

Idle time for machine 1 = Total elapsed time - time when the last job is out of machine 1

Idle time for machine 2 = Time at which the first job on machine 1 finishes in a sequence $+\Sigma_{i=2}^{n}$ [(time when the i^{th} job starts on machine 2)$-$(time when $(i-1)^{th}$ job finishes on machine 2)]

3.3.B. Processing 'n' jobs on three machines

Let $A'_1, A'_2 \ldots A'_n$ be the processing time of 'n' jobs on Machine 1, $B'_1, B'_2 \ldots B'_n$ be the processing time of 'n' jobs on Machine 2 and $C'_1, C'_2 \ldots C'_n$ be the processing times of 'n' jobs on Machine 3.

First, convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut . There is no standard procedure to obtain an optimal sequence for processing 'n' jobs on 3 Machines. So, we have to convert the three machine problem into a two machine problem by satisfying any one or both of the following conditions.
1. $Min(A'_i) \geq Max(B'_i)$, for $i = 1, 2...n$
2. $Min(C'_i) \geq Max(B'_i)$, for $i = 1, 2, ...n$

To determine the minimum or maximum of processing time on machines, we use ordering of intervals.

If atleast one of the above condition is satisfied, we introduce two fictious machines G and H such that the processing times on G and H are given by
$G = A'_i + B'_i$, for $i = 1, 2, ...n$

$H = B'_i + C'_i$, for $i = 1, 2...n$

Now we can proceed to determine the optimal sequence using 3.3.A.

After obtaining an optimal sequence, the total elapsed time and also the idle time on machines 1,2 and 3 are calculated as follows :

Total elapsed time = Time out of the last job on machine 3

Idle time of machine 1 = Total elapsed time- time when the last job is out of machine 1

Idle time of machine 2 = (Total elapsed time- time when the last job is out of machine 2) $+\Sigma_{i=2}^{n}$ [(time when the i^{th} job starts on machine 2)$-$(time when $(i-1)^{th}$ job finishes on machine 2)]

Idle time of machine 3 = Time at which the first job in a sequence finishes on machine 2 $+\Sigma_{i=2}^{n}$[(time when the i^{th}job starts on machine 3)$-$(time when $(i-1)^{th}$ job finishes on machine 3)]

3.3.C. Processing 'n' jobs on m machines

Let there be 'n' jobs which are to be processed through 'm' machines $M_1, M_2 \ldots M_m$ in the order $M_1, M_2 \ldots M_m$ and T_{ik} be the time taken by the i^{th} job on k^{th} machine.

Procedure

Step 1: Convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut.

Step 2: Use ordering of intervals to identify Min(T_{i1}) (Minimum time for the first machine), Min (T_{im}) (Minimum time on the last machine) and Max(T_{ik}) for $k = 2, 3 \ldots m-1$ and $i = 1, 2 \ldots n$ (Maximum time on intermediate machines).

Step 3 : Check the following conditions :

i. Minimum Time T_{i1} for the first machine $(M_1) \geq$ Maximum time (T_{ik}) on intermediate machines $(M_2$ to $M_{m-1})$

ii. Minimum time T_{im} for the last machine $(M_m) \geq$ Maximum Time (T_{ik}) on intermediate machines (M_2 to M_{m-1}). (i.e., the minimum processing time on the machines M_1 and M_m (First and last machines) should be greater than or equal to maximum time on any of the 2 to m-1 machines).

Step 4 : If the conditions in step 3 are not satisfied, then the problem cannot be solved using this method, hence go to the next step.

Step 5: Convert the 'n' job 'm' machine problem into 'n' job two machine problem by considering two machines G and H such that
$G_{ij} = T_{i1} + T_{i2} + \ldots + T_{i(m-1)}$
$H_{ij} = T_{i2} + T_{i3} + \ldots + T_{im}$

Step 6 : Now we can proceed to determine the optimal sequence using 3.3.A. After obtaining an optimal sequence, the total elapsed time and also the idle time on machines are determined.

4 Numerical Examples

4.1. There are five jobs, each of which is to be processed through two machines M_1 and M_2 in the order M_1, M_2. Processing time (in hours) are given below.

Job	A	B	C	D	E
Machine (M_1)	(1,2,4,5)	(4,7,9,12)	(3,4,6,7)	(1,6,8,13)	(0,2,6,8)
Machine (M_2)	(0,2,6,8)	(5,9,11,15)	(4,5,7,8)	(3,4,6,7)	(4,7,9,12)

Obtain the optimal sequence and also determine the minimum total elapsed time and idle time for each of the machines.

Convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut. $\tilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$

Job	Machine(M_1)	Machine (M_2)	Order of Cancellation
A	$[\alpha + 1, -\alpha + 5]$	$[2\alpha + 0, -2\alpha + 8]$	(1)
B	$[3\alpha + 4, -3\alpha + 12]$	$[4\alpha + 5, -4\alpha + 15]$	(5)
C	$[\alpha + 3, -\alpha + 7]$	$[\alpha + 4, -\alpha + 8]$	(3)
D	$[5\alpha + 1, -5\alpha + 13]$	$[\alpha + 3, -\alpha + 7]$	(4)
E	$[2\alpha + 0, -2\alpha + 8]$	$[3\alpha + 4, -3\alpha + 12]$	(2)

Optimal Sequence : A-E-C-B-D

To obtain minimum total elapsed time :

Job	Machine(M_1)		Machine(M_2)		Idle Time(M_2)
	Time in	Time out	Time in	Time out	
A	[0,0]	$[\alpha + 1, -\alpha + 5]$	$[\alpha + 1, -\alpha + 5]$	$[3\alpha + 1, -3\alpha + 13]$	$[\alpha + 1, -\alpha + 5]$
E	$[\alpha + 1, -\alpha + 5]$	$[3\alpha + 1, -3\alpha + 13]$	$[3\alpha + 1, -3\alpha + 13]$	$[6\alpha + 5, -6\alpha + 25]$	$[6\alpha - 12, -6\alpha + 12]$
C	$[3\alpha + 1, -3\alpha + 13]$	$[4\alpha + 4, -4\alpha + 20]$	$[6\alpha + 5, -6\alpha + 25]$	$[7\alpha + 9, -7\alpha + 33]$	$[12\alpha - 20, -12\alpha + 20]$
B	$[4\alpha + 4, -4\alpha + 20]$	$[7\alpha + 8, -7\alpha + 32]$	$[7\alpha + 9, -7\alpha + 33]$	$[11\alpha + 14, -11\alpha + 48]$	$[14\alpha - 24, -14\alpha + 24]$
D	$[7\alpha + 8, -7\alpha + 32]$	$[12\alpha + 9, -12\alpha + 45]$	$[11\alpha + 14, -11\alpha + 48]$	$[12\alpha + 17, -12\alpha + 55]$	$[22\alpha - 34, -22\alpha + 34]$
			Total		$[55\alpha - 89, -55\alpha + 95]$

Minimum Total Elapsed time = $[12\alpha + 17, -12\alpha + 55]$ hours
Idle time for Machines are respectively ;
$M_1 = [12\alpha + 17, -12\alpha + 55] - [12\alpha + 9, -12\alpha + 45] = [24\alpha - 28, -24\alpha + 46]$ hours
$M_2 = [55\alpha - 89, -55\alpha + 95]$ hours
Corresponding plot for the minimum total elapsed time and idle time for each machine are given in the following figures : .

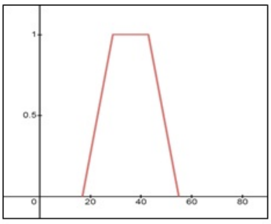

Figure 4. Minimum total Elapsed time

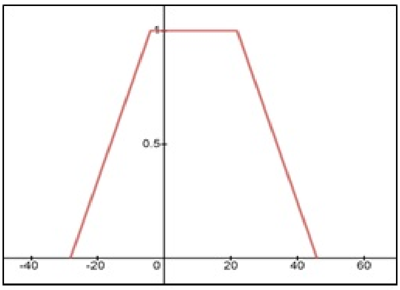

Figure 5. Idle Time for Machine.1

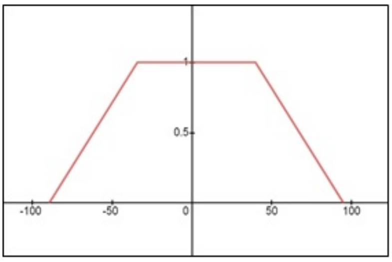

Figure 6. Idle Time for Machine.2

The minimum total elapsed time and idle time for each machine in terms of trapezoidal fuzzy number are given below :
Minimum Total elapsed time $= (17, 29, 43, 55)$ hours.
Idle time for Machine $M_1 = (-28, -4, 22, 46)$ hours.
Idle time for Machine$M_2 = (-89, -34, 40, 95)$ hours.

4.2. There are seven jobs, each of which is to be processed through three machines M_1, M_2 and M_3 in the order $M_1 M_2$ and M_3. Processing time (in hours) are given below.

Job	Machine (M_1)	Machine(M_2)	Machine (M_3)
A	(1,2,4,5)	(0,2,6,8)	(4,5,7,8)
B	(4,7,9,12)	(1,2,4,5)	(1,6,8,13)
C	(1,6,8,13)	(-2,0,4,6)	(3,4,6,7)
D	(0,2,6,8)	(3,4,6,7)	(7,9,13,15)
E	(4,8,10,14)	(-1,0,2,3)	(3,4,6,7)
F	(4,7,9,12)	(0,2,6,8)	(4,5,7,8)
G	(1,6,8,13)	(1,2,4,5)	(10,11,13,14)

Obtain the optimal sequence and also determine the minimum total elapsed time and idle time for each of the machine.

Convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut. $\tilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$

Job	Machine (M_1)	Machine(M_2)	Machine (M_3)
A	$[\alpha+1, -\alpha+5]$	$[2\alpha+0, -2\alpha+8]$	$[\alpha+4, -\alpha+8]$
B	$[3\alpha+4, -3\alpha+12]$	$[\alpha+1, -\alpha+5]$	$[5\alpha+1, -5\alpha+13]$
C	$[5\alpha+1, -5\alpha+13]$	$[2\alpha-2, -2\alpha+6]$	$[\alpha+3, -\alpha+7]$
D	$[2\alpha+0, -2\alpha+8]$	$[\alpha+3, -\alpha+7]$	$[2\alpha+7, -2\alpha+15]$
E	$[4\alpha+4, -4\alpha+14]$	$[\alpha-1, -\alpha+3]$	$[\alpha+3, -\alpha+7]$
F	$[3\alpha+4, -3\alpha+12]$	$[2\alpha+0, -2\alpha+8]$	$[\alpha+4, -\alpha+8]$
G	$[5\alpha+1, -5\alpha+13]$	$[\alpha+1, -\alpha+5]$	$[\alpha+10, -\alpha+14]$

Since the problem is a three machine problem, we convert this into a two machine problem. For that it has to satisfy any one or both of the following conditions
i) Min(M_1) $\geq$Max(M_2)
ii) Min(M_3) $\geq$ Max(M_2)

Here Min $(M_1) = [\alpha+1, -\alpha+5]$ and Max $(M_2) = [\alpha+3, -\alpha+7] = Min(M_3)$.
i) Min $(M_1) \ngeq$ Max (M_2)
ii) Min $(M_3) = Max(M_2)$

Therefore, the second condition is satisfied. We convert the problem into a two machine problem as H and K. The processing time of the two machines H and K for 7 jobs are as follows :
H = $M_1 + M_2$ and K = $M_2 + M_3$

Job	H	K	Order of Cancellation
A	$[3\alpha+1, -3\alpha+13]$	$[3\alpha+4, -3\alpha+16]$	(2)
B	$[4\alpha+5, -4\alpha+17]$	$[6\alpha+2, -6\alpha+18]$	(6)
C	$[7\alpha-1, -7\alpha+19]$	$[3\alpha+1, -3\alpha+13]$	(3)
D	$[3\alpha+3, -3\alpha+15]$	$[3\alpha+10, -3\alpha+22]$	(4)
E	$[5\alpha+3, -5\alpha+17]$	$[2\alpha+2, -2\alpha+10]$	(1)
F	$[5\alpha+4, -5\alpha+20]$	$[3\alpha+4, -3\alpha+16]$	(7)
E	$[6\alpha+2, -6\alpha+18]$	$[2\alpha+11, -2\alpha+19]$	(5)

Optimal Sequence : A-D-G-F-B-C-E

Job	Machine (M_1)		
	Timein	Time Out	Idle Time
A	$[0, 0]$	$[\alpha+1, -\alpha+5]$	—
D	$[\alpha+1, -\alpha+5]$	$[3\alpha+1, -3\alpha+13]$	—
G	$[3\alpha+1, -3\alpha+13]$	$[8\alpha+2, -8\alpha+26]$	—
F	$[8\alpha+2, -8\alpha+26]$	$[11\alpha+6, -11\alpha+38]$	—
B	$[11\alpha+6, -11\alpha+38]$	$[14\alpha+10, -14\alpha+50]$	—
C	$[14\alpha+10, -14\alpha+50]$	$[19\alpha+11, -19\alpha+63]$	—
E	$[19\alpha+11, -19\alpha+63]$	$[23\alpha+15, -23\alpha+77]$	—
	Total		$[38\alpha-44, -38\alpha+70]$

Job	Machine (M_2)		
	Timein	Time Out	Idle Time
A	$[\alpha+1,-\alpha+5]$	$[3\alpha+1,-3\alpha+13]$	$[\alpha+1,-\alpha+5]$
D	$[3\alpha+1,-3\alpha+13]$	$[4\alpha+4,-4\alpha+20]$	$[6\alpha-12,-6\alpha+12]$
G	$[8\alpha+2,-8\alpha+26]$	$[9\alpha+3,-9\alpha+31]$	$[12\alpha-18,-12\alpha+22]$
F	$[11\alpha+6,-11\alpha+38]$	$[13\alpha+6,-13\alpha+46]$	$[20\alpha-25,-20\alpha+35]$
B	$[14\alpha+10,-14\alpha+50]$	$[15\alpha+11,-15\alpha+55]$	$[27\alpha-36,-27\alpha+44]$
C	$[19\alpha+11,-19\alpha+63]$	$[21\alpha+9,-21\alpha+69]$	$[34\alpha-44,-34\alpha+52]$
E	$[23\alpha+15,-23\alpha+77]$	$[24\alpha+14,-24\alpha+80]$	$[44\alpha-54,-44\alpha+68]$
	Total		$[144\alpha-188,-144\alpha+238]$

Job	Machine (M_3)		
	Timein	Time Out	Idle Time
A	$[3\alpha+1,-3\alpha+13]$	$[4\alpha+5,-4\alpha+21]$	$[3\alpha+1,-3\alpha+13]$
D	$[4\alpha+5,-4\alpha+21]$	$[6\alpha+12,-6\alpha+36]$	$[8\alpha-16,-8\alpha+16]$
G	$[6\alpha+12,-6\alpha+36]$	$[7\alpha+22,-7\alpha+50]$	$[12\alpha-24,-12\alpha+24]$
F	$[7\alpha+22,-7\alpha+50]$	$[8\alpha+26,-8\alpha+58]$	$[14\alpha-28,-14\alpha+28]$
B	$[8\alpha+26,-8\alpha+58]$	$[13\alpha+27,-13\alpha+71]$	$[16\alpha-32,-16\alpha+32]$
C	$[13\alpha+27,-13\alpha+71]$	$[14\alpha+30,-14\alpha+78]$	$[26\alpha-44,-26\alpha+44]$
E	$[14\alpha+30,-14\alpha+78]$	$[15\alpha+33,-15\alpha+85]$	$[28\alpha-48,-28\alpha+48]$
	Total		$[107\alpha-191,-107\alpha+205]$

Total elapsed time= $[15\alpha+33,-15\alpha+85]$hours

Idle time of Machines M_1, M_2 and M_3 are given below respectively ;

$$M_1 = [15\alpha+33,-15\alpha+85]-[23\alpha+15,-23\alpha+77] = [38\alpha-44,-38\alpha+70]\text{hours}$$
$$M_2 = [15\alpha+33,-15\alpha+85]-[24\alpha+14,-24\alpha+80]+[144\alpha-188,-144\alpha+238]$$
$$= [183\alpha-235,-183\alpha+309]\text{hours}$$
$$M_3 = [107\alpha-191,-107\alpha+205]\text{hours}$$

Corresponding plot for the minimum total elapsed time and idle time for each machine are given in the following figures :

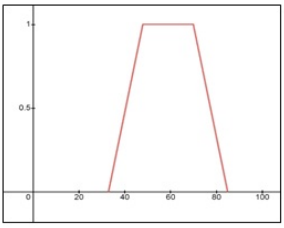

Figure 7. Minimum total Elapsed time

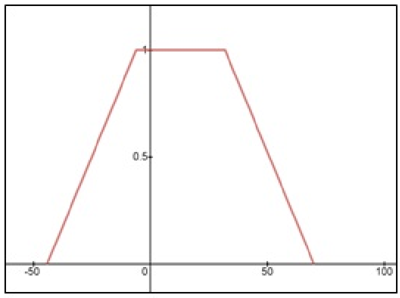

Figure 8. Idle Time for Machine.1

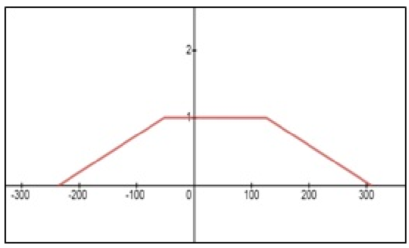

Figure 9. Idle Time for Machine.2

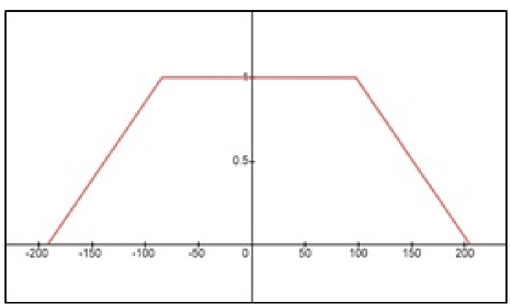

Figure 10. Idle Time for Machine.3

The minimum total elapsed time and idle time for each machine in terms of trapezoidal fuzzy number are given below:
Minimum Total elapsed time= $(33, 48, 70, 85)$hours.
Idle time for Machine$M_1 = (-44, -6, 32, 70)$ hours.
Idle time for Machine $M_2 = (-235, -52, 126, 309)$hours.
Idle time for Machine$M_3 = (-191, -84, 98, 205)$ hours.

4.3. Obtain an optimal sequence for the following sequencing problem of four jobs and four machines when passing is not allowed, of which processing time (in hours) are given below. Also calculate the minimum total elapsed time and idle time for each of the machines.

Job	Machine (M_1)	Machine(M_2)	Machine (M_3)	Machine (M_4)
A	(11,12,14,15)	(4,7,9,12)	(1,6,8,13)	(11,13,15,17)
B	(6,11,13,18)	(2,4,8,10)	(4,7,9,12)	(17,18,20,21)
C	(4,8,10,14)	(1,6,8,13)	(4,7,9,12)	(13,14,16,17)
D	(4,7,9,12)	(3,4,6,7)	(2,4,8,10)	(13,14,16,17)

Convert the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut $\tilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$

Job	Machine (M_1)	Machine(M_2)	Machine (M_3)	Machine (M_4)
A	$[\alpha + 11, -\alpha + 15]$	$[3\alpha + 4, -3\alpha + 12]$	$[5\alpha + 1, -5\alpha + 13]$	$[2\alpha + 11, -2\alpha + 17]$
B	$[5\alpha + 6, -5\alpha + 18]$	$[2\alpha + 2, -2\alpha + 10]$	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 17, -\alpha + 21]$
C	$[4\alpha + 4, -4\alpha + 14]$	$[5\alpha + 1, -5\alpha + 13]$	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 13, -\alpha + 17]$
D	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 3, -\alpha + 7]$	$[2\alpha + 2, -2\alpha + 10]$	$[\alpha + 13, -\alpha + 17]$

To find an Optimal Sequence, we convert the 4 Machine problem into a 2 Machine problem. For this it has to satisfy any one of the following condition.
i)$Min(M_1) \geq Max(M_2, M_3)$
ii) $Min(M_4) \geq Max(M_2, M_3)$
where $Min(M_1) = [3\alpha + 4, -3\alpha + 12], Min(M_4) = [2\alpha + 11, -2\alpha + 17]$
$Max(M_2) = [3\alpha + 4, -3\alpha + 12] = Max(M_3)$
$Max(M_2, M_3) = [3\alpha + 4, -3\alpha + 12]$

Here both the conditions are satisfied. We convert this problem into 2 machine problem as G and H as follows:
$G = M_1 + M_2 + M_3$ and $H = M_2 + M_3 + M_4$

Job	G	H	Order of Cancellation
A	$[9\alpha + 16, -9\alpha + 40]$	$[10\alpha + 16, -10\alpha + 42]$	(4)
B	$[10\alpha + 12, -10\alpha + 40]$	$[6\alpha + 23, -6\alpha + 43]$	(3)
C	$[12\alpha + 9, -12\alpha + 39]$	$[9\alpha + 18, -9\alpha + 42]$	(2)
D	$[6\alpha + 9, -6\alpha + 29]$	$[4\alpha + 18, -4\alpha + 34]$	(1)

Optimal Sequence: D-C-B-A

The processing time of 4 jobs on 4 machines in the order of optimal sequence is given by

Job	Machine (M_1)	Machine(M_2)	Machine (M_3)	Machine (M_4)
D	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 3, -\alpha + 7]$	$[2\alpha + 2, -2\alpha + 10]$	$[\alpha + 13, -\alpha + 17]$
C	$[4\alpha + 4, -4\alpha + 14]$	$[5\alpha + 1, -5\alpha + 13]$	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 13, -\alpha + 17]$
B	$[5\alpha + 6, -5\alpha + 18]$	$[2\alpha + 2, -2\alpha + 10]$	$[3\alpha + 4, -3\alpha + 12]$	$[\alpha + 17, -\alpha + 21]$
A	$[\alpha + 11, -\alpha + 15]$	$[3\alpha + 4, -3\alpha + 12]$	$[5\alpha + 1, -5\alpha + 13]$	$[2\alpha + 11, -2\alpha + 17]$

To obtain minimum total elapsed time

Job	Machine (M_1)		Machine (M_2)	
	Time in	Time out	Time in	TIme out
D	[0,0]	$[3\alpha+4,-3\alpha+12]$	$[3\alpha+4,-3\alpha+12]$	$[4\alpha+7,-4\alpha+19]$
C	$[3\alpha+4,-3\alpha+12]$	$[7\alpha+8,-7\alpha+26]$	$[7\alpha+8,-7\alpha+26]$	$[12\alpha+9,-12\alpha+39]$
B	$[7\alpha+8,-7\alpha+26]$	$[12\alpha+14,-12\alpha+44]$	$[12\alpha+14,-12\alpha+44]$	$[14\alpha+16,-14\alpha+54]$
A	$[12\alpha+14,-12\alpha+44]$	$[13\alpha+25,-13\alpha+59]$	$[13\alpha+25,-13\alpha+59]$	$[16\alpha+29,-16\alpha+71]$

Job	Machine (M_3)		Machine (M_4)	
	Time in	Time out	Time in	TIme out
D	$[4\alpha+7,-4\alpha+10]$	$[6\alpha+9,-6\alpha+29]$	$[6\alpha+9,-6\alpha+29]$	$[7\alpha+22,-7\alpha+46]$
C	$[12\alpha+9,-12\alpha+39]$	$[15\alpha+13,-15\alpha+51]$	$[7\alpha+22,-7\alpha+46]$	$[8\alpha+35,-8\alpha+63]$
B	$[14\alpha+16,-14\alpha+54]$	$[17\alpha+20,-17\alpha+66]$	$[8\alpha+35,-8\alpha+63]$	$[9\alpha+52,-9\alpha+84]$
A	$[16\alpha+29,-16\alpha+71]$	$[21\alpha+30,-21\alpha+84]$	$[9\alpha+52,-9\alpha+84]$	$[11\alpha+63,-11\alpha+101]$

Job	Idle time (M_1)	Idle time (M_2)	Idle time (M_3)	Idle time(M_4)
D	—	$[3\alpha+4,-3\alpha+12]$	$[4\alpha+7,-4\alpha+19]$	$[6\alpha+9,-6\alpha+29]$
C	—	$[11\alpha-11,-11\alpha+19]$	$[18\alpha-20,-18\alpha+30]$	$[14\alpha-24,-14\alpha+24]$
B	—	$[24\alpha-25,-24\alpha+35]$	$[29\alpha-35,-29\alpha+41]$	$[16\alpha+28,-16\alpha+28]$
A	—	$[27\alpha-29,-27\alpha+43]$	$[33\alpha-37,-33\alpha+51]$	$[18\alpha-32,-18\alpha+32]$
Total	$[24\alpha+4,-24\alpha+76]$	$[65\alpha-61,-65\alpha+109]$	$[84\alpha-85,-84\alpha+141]$	$[54\alpha-75,-54\alpha+113]$

Minimum Total elapsed time $=[11\alpha+63,-11\alpha+101]$ hours
Idle time for Machines M_1 , M_2, M_3 and M_4 are given below respectively;

$$M_1=[11\alpha+63,-11\alpha+101]-[13\alpha+25,-13\alpha+59]=[24\alpha+4,-24\alpha+76]\text{hours}$$
$$M_2=[11\alpha+63,-11\alpha+101]-[16\alpha+29,-16\alpha+71]+[65\alpha-61,-65\alpha+109]$$
$$=[92\alpha-69,-92\alpha+181]\text{hours}$$
$$M_3=[11\alpha+63,-11\alpha+101]-[21\alpha+30,-21\alpha+84]+[84\alpha-85,-84\alpha+141]$$
$$=[116\alpha-106,-116\alpha+212]\text{hours}$$
$$M_4=[54\alpha-75,-54\alpha+113]\text{hours}$$

Corresponding plot for the minimum total elapsed time and idle time for each machines are given in the following figures :

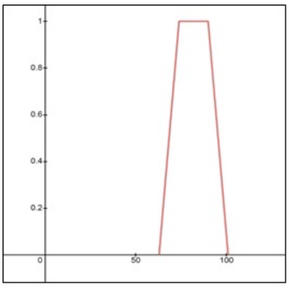

Figure 11. Minimum total Elapsed time

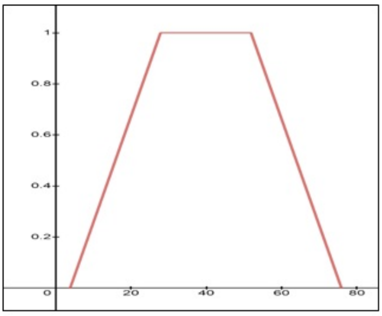

Figure 12. Idle Time for Machine 1

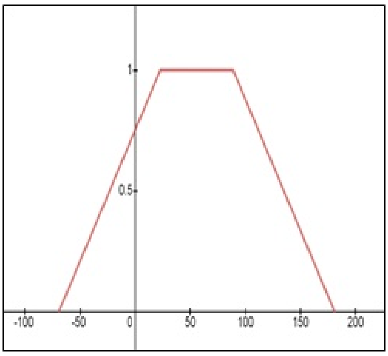

Figure 13. Idle Time for Machine 2

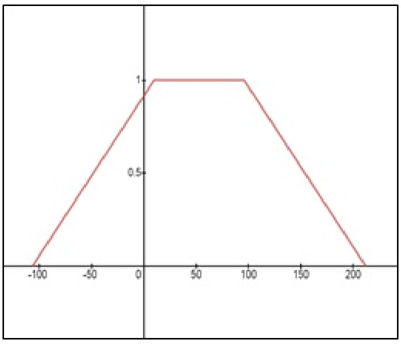

Figure 14. Idle Time for Machine 3

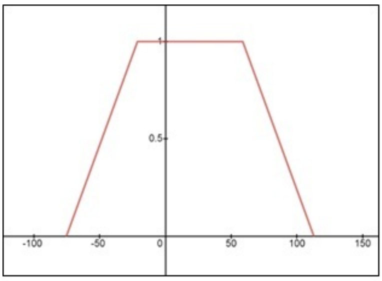

Figure 15. Idle Time for Machine 4

The minimum total elapsed time and idle time for each machines in terms of trapezoidal fuzzy number are given below:
Minimum Total elapsed time $= (63, 74, 90, 101)$ hours.
Idle time for Machine$M_1 = (4, 28, 52, 76)$ hours.
Idle time for Machine$M_2 = (-69, 23, 89, 181)$ hours.
Idle time for Machine $M_3 = (-106, 10, 96, 212)$ hours.
Idle time for Machine $M_4 = (-75, -21, 59, 113)$ hours.

5 Conclusion

In this paper, we have solved sequencing problem for 'n' jobs on 2 machines, 'n' jobs on 3 machines and 'n' jobs on m machines by converting the processing time which is given in terms of trapezoidal fuzzy numbers into intervals using α-cut. After which, we obtained the optimal sequence and determined the minimum total elapsed time and idle time for each machine. The concept of sequencing problem provides an efficient framework in solving real-life problems.

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