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# On $\mathcal{I}^{\mathcal{K}-st}$ convergence of sequence of real numbers

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**Abstract**. In this paper, we extend the notion of $\mathcal{I}^{\mathcal{K}}$ convergence of sequence of real numbers to $\mathcal{I}^{\mathcal{K}-st}$ convergence by using the notion of $\mathcal{K}-$statistical convergence. We investigate some of its properties and study its relation with some other known types of convergence.

## 1 Introduction

The notion of statistical convergence was first introduced in the year 1951 independently by Fast [9] and Steinhaus [23] in connection with summability theory. Following them, the concept was investigated by Fridy ([10],[11]), Salat [19], and many others from the sequence space point of view. In 2000, the concept of $\mathcal{I}$ and $\mathcal{I}^*-$convergence was introduced by Kostrkyo and Salat [15] mainly as an extension of statistical convergence. Since then rapid development occurred in this direction due to Kostorkyo [14], Demicri[8], Gogola[12], Gurdal[18], Tripathy[20], and many others. $\mathcal{I}^{\mathcal{K}}-$convergence was one of the tremendous extensions of $\mathcal{I}^*-$convergence by M. Macaj and M. Sleziak in [16], where two ideals $\mathcal{I}$ and $\mathcal{K}$ got involved. In the case of $\mathcal{I}^*-$convergence, the type of convergence along a set from the associated filter $\mathcal{F}(\mathcal{I})$ was the usual notion of convergence. But in the case of $\mathcal{I}^{\mathcal{K}}-$convergence that usual convergence was replaced by ideal convergence for some ideal $\mathcal{K}$ and eventually $\mathcal{I}^*-$convergence becomes a particular kind of $\mathcal{I}^{\mathcal{K}}-$convergence for $\mathcal{K} = \mathcal{I}_f$. For more on $\mathcal{I}^{\mathcal{K}}-$convergence, one may refer ([1],[2],[3],[5],[6],[13]).

On the other hand in 2011, Das et. al.[4] introduced the notion of $\mathcal{I}-$statistical convergence as a generalization of statistical convergence. Later on, several investigations in this direction have been made by Debnath [7], Mursaleen [17], Savas ([21],[22]), and many others.

In this paper, using the notion of $\mathcal{I}^{\mathcal{K}}-$convergence and $\mathcal{I}-$statistical convergence, we introduce the notion of $\mathcal{I}^{\mathcal{K}-st}$ convergence. We study various properties of the newly introduced convergence concept and relations with some existing notions of convergence.

## 2 Definitions and Preliminaries

**Definition 2.1.** [11] If $K$ is a subset of the positive integers $\mathbb{N}$, then $K_n$ denotes the set $\{k \in K : k \leq n\}$. The natural density of $K$ is given by $d(K) = \lim\limits_{n\to\infty} \frac{|K_n|}{n}$.

**Definition 2.2.** [11] A sequence $x = (x_n)$ is said to be statistically convergent to $l$ if for every $\varepsilon > 0$, the set $A(\varepsilon) = \{k \in N :| x_k - l |\geq \varepsilon\}$ has natural density zero. $l$ is called the statistical limit of the sequence $(x_n)$ and symbolically $st - lim\, x = l$.

**Definition 2.3.** [15] A family $\mathcal{I} \subset 2^X$ of subsets of a nonempty set $X$ is said to be an ideal in $X$ if and only if (i) $\emptyset \in \mathcal{I}$ (ii) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$ (Additive) and (iii) $A \in \mathcal{I}, B \subset A$ implies $B \in \mathcal{I}$ (Hereditary).

If $\forall x \in X$, $\{x\} \in \mathcal{I}$ then $\mathcal{I}$ is said to be admissible. Also, $\mathcal{I}$ is said to be non-trivial if $X \notin \mathcal{I}$ and $\mathcal{I} \neq \{\emptyset\}$.

Some standard examples of ideal are given below:
(i) The set $\mathcal{I}_f$ of all finite subsets of $\mathbb{N}$ is an admissible ideal in $\mathbb{N}$.
(ii) The set $\mathcal{I}_d$ of all subsets of natural numbers having natural density 0 is an admissible ideal in $\mathbb{N}$.
(iii) The set $\mathcal{I}_c = \{A \subseteq \mathbb{N} : \sum_{a\in A} a^{-1} < \infty\}$ is an admissible ideal in $\mathbb{N}$.
(iv) Suppose $\mathbb{N} = \bigcup\limits_{p=1}^{\infty} D_p$ be a decomposition of $\mathbb{N}$ (for $i \neq j$, $D_i \cap D_j = \emptyset$). Then the set $\mathcal{I}$ of all subsets of $\mathbb{N}$, which intersects finitely many $D_p$'s forms an ideal in $\mathbb{N}$.

More important examples can be found in [12] and [14].

**Definition 2.4.** [15] A family $\mathcal{F} \subset 2^X$ of subsets of a nonempty set $X$ is said to be a filter in $X$ if and only if (i) $\emptyset \notin \mathcal{F}$ (ii) $M, N \in \mathcal{F}$ implies $M \cap N \in \mathcal{F}$ and (iii) $M \in \mathcal{F}, N \supset M$ implies $N \in \mathcal{F}$.

If $\mathcal{I}$ is a proper non-trivial ideal in $X$, then $\mathcal{F}(\mathcal{I}) = \{M \subset X : \exists\, A \in \mathcal{I}\; s.t\; M = X \setminus A\}$ is a filter in $X$. It is called the filter associated with the ideal $\mathcal{I}$.

**Definition 2.5.** [15] A sequence $x = (x_k)$ is said to be $\mathcal{I}-$convergent to $l$ if and only if for every $\varepsilon > 0$, the set $\{k \in \mathbb{N} : |x_k - l| \geq \varepsilon\}$ belongs to $\mathcal{I}$. The real number $l$ is called the $\mathcal{I}-$limit of the sequence $x = (x_k)$. Symbolically, $\mathcal{I} - \lim x = l$.

**Definition 2.6.** [15] Let $\mathcal{I}$ be an admissible ideal in $\mathbb{N}$. A sequence $x = (x_k)$ is said to be $\mathcal{I}^*-$convergent to $l$, if there exixts a set $M = \{m_1 < m_2 < ... < m_k < ...\}$ in the associated filter $\mathcal{F}(\mathcal{I})$ such that $\lim\limits_{k} x_{m_k} = l$.

**Definition 2.7.** [4] A sequence $x = (x_k)$ is said to be $\mathcal{I}-$statistically convergent to $l$ if and only if for every $\varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k}|\{n \leq k : |x_n - l| \geq \varepsilon\}| \geq \delta\} \in \mathcal{I}.$$

If a sequence $x = (x_k)$ is $\mathcal{I}-$statistically convergent to $l$, then it is denoted by $\mathcal{I}-st-\lim x = l$.

**Theorem 2.8.** *([7], Theorem 3.2) For any sequence $x = (x_k)$, $st - \lim x = l$ implies $\mathcal{I} - st - \lim x = l$.*

**Theorem 2.9.** *([7], Theorem 3.4) For any sequence $x = (x_k)$, $\mathcal{I} - \lim x = l$ implies $\mathcal{I} - st - \lim x = l$.*

**Definition 2.10.** [16] Let $\mathcal{I}$ and $\mathcal{K}$ be two ideals in $\mathbb{N}$. A sequence $x = (x_k)$ is said to be $\mathcal{I}^{\mathcal{K}}-$convergent to $l$ if, there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $y = (y_k)$ defined by

$$y_k = \begin{cases} x_k, & k \in M \\ l, & k \notin M \end{cases}$$

is $\mathcal{K}-$convergent to $l$.

If we consider $\mathcal{K} = \mathcal{I}_f$, then the $\mathcal{I}^{\mathcal{K}}-$convergence concept coincides with $\mathcal{I}^*-$convergence [15]. Further, if we take $\mathcal{K} = \mathcal{I}_d$, then we get $\mathcal{I}^*-$statistical convergence which was introduced by Debnath and Rakshit in [7]. Note that $\mathcal{I}^{\mathcal{I}_d}-$convergence implies $\mathcal{I}-$statistical convergence.

Throughout the paper, unless stated, the symbols $\mathcal{I}, \mathcal{K}, \mathcal{I}_1, \mathcal{I}_2, \mathcal{K}_1$, and $\mathcal{K}_2$ stands for non-trivial admissible ideal in $\mathbb{N}$, and the sequences that we have considered are real sequences.

## 3 Main Results

**Definition 3.1.** A sequence $x = (x_k)$ is said to be $\mathcal{I}^{\mathcal{K}-statistical}$ convergent (in short $\mathcal{I}^{\mathcal{K}-st}$ convergent) to a real number $l$, if there exists a set $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $y = (y_k)$ defined as,

$$y_k = \begin{cases} x_k, & k \in M \\ l, & k \notin M \end{cases}$$

is $\mathcal{K}-$statistical convergent to $l$. Symbolically we write, $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$.

**Example 3.2.** Consider the decomposition of $\mathbb{N}$ given by $\mathbb{N} = \bigcup_{p=1}^{\infty} D_p$, where $D_p = \{2^{p-1}(2q-1) : q = 1, 2, 3, ..\}$. Let $\mathcal{I}$ be the ideal consisting of all subsets of $\mathbb{N}$ which intersects a finite number of $D_p$'s. Consider the sequence $x = (x_k)$ defined by $x_k = \frac{1}{p}$ if $k \in D_p$. Then the sequence is $\mathcal{I}^{\mathcal{K}-st}$convergent to 0, for $\mathcal{I} = \mathcal{K}$.

**Theorem 3.3.** *For any sequence $x = (x_k)$, if $\mathcal{I}^{\mathcal{K}} - \lim x = l$, then $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$.*

*Proof.* The proof follows from the fact that $\mathcal{K} - \lim x = l$ implies $\mathcal{K} - st \lim x = l$, using Theorem 2.9. □

But the converse of the above theorem is not necessarily true.

**Example 3.4.** Consider $\mathcal{I} = \mathcal{K} = \mathcal{I}_f$. Then the sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} 0, & k = m^2, m \in \mathbb{N} \\ 1, & otherwise \end{cases}$$

is $\mathcal{I}^{\mathcal{K}-st}$convergent to 1 but not $\mathcal{I}^{\mathcal{K}}-$convergent to 1.

**Theorem 3.5.** *For any sequence $x = (x_k)$, if $\mathcal{I}^* - st - \lim x = l$ then $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$.*

*Proof.* The proof follows from the fact that $st - \lim x = l$ implies $\mathcal{K} - st \lim x = l$, by Theorem 2.8. □

It is clear from Theorem 3.3 and Theorem 3.5 that $\mathcal{I}^{\mathcal{K}-st}$ convergence of a sequence is a generalization of $\mathcal{I}^{\mathcal{K}}$ convergence as well as $\mathcal{I}^* - st-$convergence.

**Theorem 3.6.** *Suppose $x = (x_k)$ be a sequence such that $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$. Then, $l$ is unique.*

*Proof.* If possible suppose $\mathcal{I}^{\mathcal{K}-st} - \lim x = l_1$ and $\mathcal{I}^{\mathcal{K}-st} - \lim x = l_2$ for some $l_1 \neq l_2$. Then by Definition 3.1, there exists $M, N \in \mathcal{F}(\mathcal{I})$ such that the sequences $y = (y_k)$ and $z = (z_k)$ defined by,

$$y_k = \begin{cases} x_k, & k \in M \\ l_1, & k \notin M \end{cases} \text{ and } z_k = \begin{cases} x_k, & k \in N \\ l_2, & k \notin N \end{cases}$$

are $\mathcal{K} - st$ convergent to $l_1$ and $l_2$ respectively. In other words, for every $\varepsilon > 0$, $\delta > 0$, the two sets $A_1, A_2 \in \mathcal{F}(\mathcal{K})$ where

$$A_1 = \{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |y_n - l_1| \geq \varepsilon\} \mid < \delta\}$$

and

$$A_2 = \{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |z_n - l_2| \geq \varepsilon\} \mid < \delta\}.$$

Eventually, $A_1 \cap A_2 \in \mathcal{F}(\mathcal{K})$ and is an infinite set. Choose $\varepsilon = \frac{|l_1 - l_2|}{3}$ and a natural number $p \in A_1 \cap A_2$ large enough to gurantee the existence of an element $\xi \in [1, p] \cap M \cap N$ and a $\delta$ small enough to agree $|y_\xi - l_1| < \varepsilon$ as well as $|z_\xi - l_2| < \varepsilon$. Thus we have,

$$\begin{aligned} 3\varepsilon = |l_1 - l_2| &\leq |x_\xi - l_1| + |x_\xi - l_2| \\ &= |y_\xi - l_1| + |z_\xi - l_2| \\ &< \varepsilon + \varepsilon = 2\varepsilon, \text{ which is a contradiction.} \end{aligned}$$

Hence we must have $l_1 = l_2$. This completes the proof. □

**Theorem 3.7.** *Let $\mathcal{I}$, $\mathcal{K}$ be non-trivial ideal in $\mathbb{N}$ such that $\mathcal{I}^{\mathcal{K}-st}-\lim x = l_1$ and $\mathcal{I}^{\mathcal{K}-st}-\lim y = l_2$. Then, (i) $\mathcal{I}^{\mathcal{K}-st}-\lim(x+y) = l_1+l_2$ and (ii) $\mathcal{I}^{\mathcal{K}-st}-\lim(xy) = l_1 l_2$.*

*Proof.* Let $\mathcal{I}^{\mathcal{K}-st}-\lim x = l_1$ and $\mathcal{I}^{\mathcal{K}-st}-\lim y = l_2$. Then, by Definition 3.1, there exists $M, N \in \mathcal{F}(\mathcal{I})$ such that the sequences $u=(u_k)$ and $v=(v_k)$ defined by

$$u_k = \begin{cases} x_k, & k \in M \\ l_1, & k \notin M \end{cases} \text{ and } v_k = \begin{cases} y_k, & k \in N \\ l_2, & k \notin N \end{cases}$$

are respectively $\mathcal{K}-st$ convergent to $l_1$ and $l_2$. Consequently the sequence $u+v=(u_k+v_k)$ is also $\mathcal{K}-st$ convergent to $l_1+l_2$. Therefore $\forall \varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |(u_n+v_n)-(l_1+l_2)| \ge \varepsilon\} \mid \ge \delta\} \in \mathcal{K}. \tag{3.1}$$

Now consider $M_1 = M \cap N \in \mathcal{F}(\mathcal{I})$ and construct the sequence $w=(w_k)$ defined by

$$w_k = \begin{cases} x_k+y_k, & k \in M_1 \\ l_1+l_2, & k \notin M_1 \end{cases}.$$

Then the following inclusion holds good

$$\begin{aligned} &\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |(u_n+v_n)-(l_1+l_2)| \ge \varepsilon\} \mid \ge \delta\} \\ &\supseteq \{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |w_n-(l_1+l_2)| \ge \varepsilon\} \mid \ge \delta\}. \end{aligned} \tag{3.2}$$

From (3.1) and (3.2) we can conclude that

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |w_n-(l_1+l_2)| \ge \varepsilon\} \mid \ge \delta\} \in \mathcal{K};$$

i.e., $w=(w_k)$ is $\mathcal{K}-st$ convergent to $l_1+l_2$. Hence we have $\mathcal{I}^{\mathcal{K}-st}-\lim(x+y) = l_1+l_2$.
(ii) The proof is similar to that of (i), so omitted. □

**Theorem 3.8.** *Suppose $x=(x_k)$ be a sequence such that $\mathcal{K}-st\lim x = l$. Then $\mathcal{I}^{\mathcal{K}-st}-\lim x = l$.*

*Proof.* Since $\mathcal{K}-st-\lim x = l$, so for every $\varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |x_n - l| \ge \varepsilon\} \mid \ge \delta\} \in \mathcal{K}. \tag{3.3}$$

Choose $M = \mathbb{N}$ from $\mathcal{F}(\mathcal{I})$. Consider the sequence $y=(y_k)$ defined by $y_k = x_k, k \in M$. Then using (3.3) we get,

$$\forall \varepsilon > 0, \delta > 0, \{k \in \mathbb{N} : \frac{1}{k} \mid \{n \le k : |y_n - l| \ge \varepsilon\} \mid \ge \delta\} \in \mathcal{K};$$

i.e., $y=(y_k)$ is $\mathcal{K}-st-$convergent to $l$. Hence $\mathcal{I}^{\mathcal{K}-st}-\lim x = l$. □

**Theorem 3.9.** *Suppose $x=(x_k)$ be a sequence such that $st-\lim x = l$. Then $\mathcal{I}^{\mathcal{K}-st}-\lim x = l$.*

*Proof.* The proof follows from the fact that $st-\lim x = l$ implies $\mathcal{K}-st\lim x = l$ (by Theorem 2.8) and applying Theorem 3.8. □

But the converse of the above theorem is not necessarily true. Example 3.2 serves as the counterexample.

**Theorem 3.10.** *Suppose $x=(x_k)$ be a sequence such that $\mathcal{K}-\lim x = l$. Then $\mathcal{I}^{\mathcal{K}-st}-\lim x = l$.*

*Proof.* The proof follows from the fact that $\mathcal{K}-\lim x = l$ implies $\mathcal{K}-st\lim x = l$ (by Theorem 2.9) and applying Theorem 3.8. □

But the converse of the above theorem is not true.

**Example 3.11.** Consider the ideals $\mathcal{I}_d = \{A \subseteq \mathbb{N} : d(A) = 0\}$ and $\mathcal{I}_c = \{A \subseteq \mathbb{N} : \sum_{a\in A} a^{-1} < \infty\}$. Let $x = (x_k)$ be the sequence defined as

$$x_k = \begin{cases} 1, & k \text{ is prime} \\ 0, & k \text{ is not prime} \end{cases}.$$

Then $x = (x_k)$ is ${\mathcal{I}_d}^{\mathcal{I}_c-st}$ convergent to 0.
But we claim that $x = (x_k)$ is not $\mathcal{I}_c-$convergent to 0. For if $\mathcal{I}_c - \lim x = 0$, then for $\varepsilon = \frac{1}{2}$, the set $\{k \in \mathbb{N} : |x_k - 0| \geq \frac{1}{2}\}$ = set of all prime numbers $\in \mathcal{I}_c$, a contradiction.

**Theorem 3.12.** *Let $\mathcal{I}, \mathcal{I}_1, \mathcal{I}_2, \mathcal{K}, \mathcal{K}_1, \mathcal{K}_2$ be admissible ideals in $\mathbb{N}$ satisfying $\mathcal{I}_1 \subseteq \mathcal{I}_2$ and $\mathcal{K}_1 \subseteq \mathcal{K}_2$. Then,*
*(i) $\mathcal{I}_1^{\mathcal{K}-st} - \lim x = l$ implies $\mathcal{I}_2^{\mathcal{K}-st} - \lim x = l$;*
*(ii) $\mathcal{I}^{\mathcal{K}_1-st} - \lim x = l$ implies $\mathcal{I}^{\mathcal{K}_2-st} - \lim x = l$.*

*Proof.* (i) Suppose $\mathcal{I}_1^{\mathcal{K}-st} - \lim x = l$. Then by Definition 3.1, there exists $M \in \mathcal{F}(\mathcal{I}_1)$ such that the sequence $y = (y_k)$ defined as, $y_k = \begin{cases} x_k, & k \in M \\ l, & k \notin M \end{cases}$ is $\mathcal{K}-$statistical convergent to $l$. Now since $M \in \mathcal{F}(\mathcal{I}_1)$, so $\mathbb{N} \setminus M \in \mathcal{I}_1$ and therefore by hypothesis $\mathbb{N} \setminus M \in \mathcal{I}_2$ which again implies $M \in \mathcal{F}(\mathcal{I}_2)$. Hence we must have $\mathcal{I}_2^{\mathcal{K}-st} - \lim x = l$.

(ii) Suppose $\mathcal{I}^{\mathcal{K}_1-st} - \lim x = l$. Then, by Definition 3.1, there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $y = (y_k)$ defined as, $y_k = \begin{cases} x_k, & k \in M \\ l, & k \notin M \end{cases}$ satisfies the following property $\forall \varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |y_n - l| \geq \varepsilon\} \mid \geq \delta\} \in \mathcal{K}_1.$$

Now by hypothesis the inclusion $\mathcal{K}_1 \subseteq \mathcal{K}_2$ holds, so we must have $\forall \varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |y_n - l| \geq \varepsilon\} \mid \geq \delta\} \in \mathcal{K}_2.$$

Hence $\mathcal{I}^{\mathcal{K}_2-st} - \lim x = l$. □

**Theorem 3.13.** *Let $\mathcal{I}$ and $\mathcal{K}$ be two admissible ideal in $\mathbb{N}$ such that $\mathcal{K} \subseteq \mathcal{I}$. Then, $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$ implies $\mathcal{I} - st - \lim x = l$.*

*Proof.* Suppose $\mathcal{K} \subseteq \mathcal{I}$ and $\mathcal{I}^{\mathcal{K}-st} - \lim x = l$. Then, there exists a set $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $y = (y_k)$ defined as, $y_k = \begin{cases} x_k, & k \in M \\ l, & k \notin M \end{cases}$ has the following property $\forall \varepsilon > 0, \delta > 0$,

$$A = \{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |y_n - l| \geq \varepsilon\} \mid \geq \delta\} \in \mathcal{K} \subseteq \mathcal{I}.$$

Therefore we must have $\forall \varepsilon > 0, \delta > 0$,

$$\{k \in \mathbb{N} : \frac{1}{k} \mid \{n \leq k : |x_n - l| \geq \varepsilon\} \mid \geq \delta\} \subseteq (\mathbb{N} \setminus M) \cup A \in \mathcal{I}.$$

Hence, $\mathcal{I} - st - \lim x = l$. □

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