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SOME RESULTS ON GROUP S_3 CORDIAL REMAINDER LABELING

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Abstract Let $G = (V(G), E(G))$ be a graph and let $g : V(G) \to S_3$ be a function. For each edge xy assign the label r where r is the remainder when $o(g(x))$ is divided by $o(g(y))$ or $o(g(y))$ is divided by $o(g(x))$ according as $o(g(x)) \geq o(g(y))$ or $o(g(y)) \geq o(g(x))$. The function g is called a group S_3 cordial remainder labeling of G if $|v_g(x) - v_g(y)| \leq 1$ and $|e_g(1) - e_g(0)| \leq 1$, where $v_g(x)$ denotes the number of vertices labeled with x and $e_g(i)$ denotes the number of edges labeled with i $(i = 0, 1)$. A graph G which admits a group S_3 cordial remainder labeling is called a group S_3 cordial remainder graph. In this paper, we prove that lotus inside a circle, double fan, ladder, slanting ladder and triangular ladder graphs admit a group S_3 cordial remainder labeling.

1 Introduction

Graphs considered here are finite, undirected and simple. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$. Terms not defined here are taken from Harary [3]. Graph labeling was first introduced in 1960's. Most of the graph labeling trace their origins in the paper presented by Alex Rosa in 1967 [9]. The complete survey of graph labeling is in [2]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by I. Cahit in [1]. Lourdusamy et al. [5] introduced the concept of the group S_3 cordial remainder labeling and they proved that path, cycle, star, bistar, complete bipartite graph, wheel, fan, comb and crown graph admit a group S_3 cordial remainder labeling. In [4], they proved that shadow graph of cycle and path, splitting graph of cycle, armed crown, umbrella graph and dumbbell graph admit a group S_3 cordial remainder labeling. Also they proved that snake related graphs are a group S_3 cordial remainder graphs. In [6, 7, 8], they investigated the behaviour of group S_3 cordial remainder labeling of subdivision of star, subdivision of bistar, subdivision of wheel, subdivision of comb, subdivision of crown, subdivision of fan, subdivision of ladder, helm graph, flower graph, closed helm graph, gear graph, sunflower graph, triangular snake, quadrilateral snake, square of the path, duplication of a vertex by a new edge in path and cycle graphs, duplication of an edge by a new vertex in path and cycle graph, total graph of cycle and path graph.

The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. The double fan DF_n is defined as $P_n + 2K_1$.

The lotus inside a circle LC_n is obtained from the cycle $C_n : v_1 v_2 \cdots v_n v_1$ and a star $K_{1,n}$ with central vertex u and the end vertices $u_1 u_2 \cdots u_n$ by joining each u_i to v_i and $v_{i+1} (mod\, n)$.

The Cartesian product $G_1 \times G_2$ of two graphs is defined to be the graph with vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \times G_2$ if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1. The ladder L_n is defined as $P_n \times P_2$. The slanting ladder SL_n is the graph obtained from two paths $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ by joining each u_i with v_{i+1} for $1 \leq i \leq n-1$. The triangular ladder TL_n is the graph obtained from L_n by adding the edges $u_i v_{i+1}, 1 \leq i \leq n-1$, where u_i and $v_i, 1 \leq i \leq n$ are the vertices of L_n such that $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ are two paths of length n in the graph L_n.

2 Group S_3 Cordial Remainder Graphs

Definition 2.1. Consider the symmetric group S_3. Let the elements of S_3 be e, a, b, c, d, f where

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$
$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

We have $o(e) = 1$, $o(a) = o(b) = o(c) = 2$, $o(d) = o(f) = 3$.

Definition 2.2. Let $G = (V(G), E(G))$ be a graph and let $g : V(G) \to S_3$ be a function. For each edge xy assign the label r where r is the remainder when $o(g(x))$ is divided by $o(g(y))$ or $o(g(y))$ is divided by $o(g(x))$ according as $o(g(x)) \geq o(g(y))$ or $o(g(y)) \geq o(g(x))$. The function g is called a group S_3 cordial remainder labeling of G if $|v_g(x) - v_g(y)| \leq 1$ and $|e_g(1) - e_g(0)| \leq 1$, where $v_g(x)$ denotes the number of vertices labeled with x and $e_g(i)$ denotes the number of edges labeled with i $(i = 0, 1)$. A graph G which admits a group S_3 cordial remainder labeling is called a group S_3 cordial remainder graph.

Example 2.3. A group S_3 cordial remainder labeling of graph is given in Figure 1.

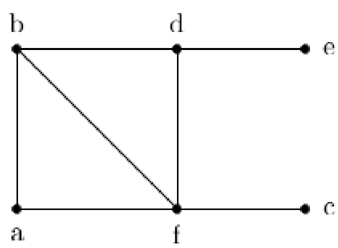

Figure 1.

Theorem 2.4. *Lotus inside a circle LC_n is a group S_3 cordial remainder graph for $n \geq 3$.*

Proof. Let $V(LC_n) = \{u, u_i, v_i : 1 \leq i \leq n\}$ and $E(LC_n) = \{uu_i, u_iv_i : 1 \leq i \leq n\} \bigcup \{v_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_nu_1, v_nv_1\}$. Therefore, $|V(LC_n)| = 2n + 1$ and $|E(LC_n)| = 4n$. Define $g : V(LC_n) \to S_3$ as follows:

Case 1. $n = 3$.

$g(u) = d$;

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ d & \text{if } i = 2 \\ e & \text{if } i = 3 \, ; \end{cases} \qquad g(v_i) = \begin{cases} c & \text{if } i = 1 \\ b & \text{if } i = 2 \\ f & \text{if } i = 3 \, . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = v_g(f) = 1, v_g(d) = 2$ and $e_g(0) = e_g(1) = 6$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 2. $n = 4$.

$g(u) = d$;

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ c & \text{if } i = 2 \\ f & \text{if } i = 3 \\ d & \text{if } i = 4 \, ; \end{cases} \qquad g(v_i) = \begin{cases} f & \text{if } i = 1 \\ b & \text{if } i = 2 \\ e & \text{if } i = 3 \\ c & \text{if } i = 4 \, . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(e) = 1, v_g(c) = v_g(d) = v_g(f) = 2$ and $e_g(0) = e_g(1) = 8$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 3. $n = 5$.

$g(u) = d$;

$$g(u_i)=\begin{cases} a & \text{if } i=1\\ d & \text{if } i=2\\ b & \text{if } i=3\\ c & \text{if } i=4\\ f & \text{if } i=5\,; \end{cases} \qquad g(v_i)=\begin{cases} a & \text{if } i=1\\ b & \text{if } i=2\\ f & \text{if } i=3\\ c & \text{if } i=4\\ e & \text{if } i=5\,. \end{cases}$$

Here we have $v_g(a)=v_g(b)=v_g(c)=v_g(d)=v_g(f)=2, v_g(e)=1$ and $e_g(0)=e_g(1)=10$. Therefore $|v_g(i)-v_g(j)|\leq 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\leq 1$.

Case 4. $n\geq 6$.

Subcase 4.1. $n\equiv 0\ (mod\, 6)$.

Let $n=6k$ and $k\geq 1$.

$$g(u)=d;$$

$$g(u_i)=\begin{cases} a & \text{if } i\equiv 1\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ d & \text{if } i\equiv 2\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ b & \text{if } i\equiv 3\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ c & \text{if } i\equiv 4\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ f & \text{if } i\equiv 5\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ e & \text{if } i\equiv 0\ (mod\, 6) \text{ and } 1\leq i\leq 6k\,; \end{cases}$$

$$g(v_i)=\begin{cases} a & \text{if } i\equiv 1\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ b & \text{if } i\equiv 2\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ d & \text{if } i\equiv 3\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ e & \text{if } i\equiv 4\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ c & \text{if } i\equiv 5\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ f & \text{if } i\equiv 0\ (mod\, 6) \text{ and } 1\leq i\leq 6k\,. \end{cases}$$

Here we have $v_g(a)=v_g(b)=v_g(c)=v_g(e)=v_g(f)=2k, v_g(d)=2k+1$ and $e_g(0)=e_g(1)=12k$. Therefore $|v_g(i)-v_g(j)|\leq 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\leq 1$.

Subcase 4.2. $n\equiv 5\ (mod\, 6)$.

Let $n=6k+5$ and $k\geq 1$. Assign the labels to the vertices u,u_i,v_i for $1\leq i\leq 6k$ as in Subcase 4.1 and for the remaining vertices assign the following labels:

$$g(u_i)=\begin{cases} a & \text{if } i=6k+1\\ d & \text{if } i=6k+2\\ b & \text{if } i=6k+3\\ c & \text{if } i=6k+4\\ f & \text{if } i=6k+5\,; \end{cases} \qquad g(v_i)=\begin{cases} a & \text{if } i=6k+1\\ b & \text{if } i=6k+2\\ f & \text{if } i=6k+3\\ c & \text{if } i=6k+4\\ e & \text{if } i=6k+5\,. \end{cases}$$

Here we have $v_g(a)=v_g(b)=v_g(c)=v_g(d)=v_g(f)=2k+2, v_g(e)=2k+1$ and $e_g(0)=e_g(1)=12k+10$. Therefore $|v_g(i)-v_g(j)|\leq 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\leq 1$.

Subcase 4.3. $n\equiv 4\ (mod\, 6)$.

Let $n=6k+4$ and $k\geq 1$.

$$g(u)=d;$$

$$g(u_i)=\begin{cases} a & \text{if } i\equiv 1\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ b & \text{if } i\equiv 2\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ d & \text{if } i\equiv 3\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ c & \text{if } i\equiv 4\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ f & \text{if } i\equiv 5\ (mod\, 6) \text{ and } 1\leq i\leq 6k\\ e & \text{if } i\equiv 0\ (mod\, 6) \text{ and } 1\leq i\leq 6k\,; \end{cases}$$

$$g(v_i) = \begin{cases} d & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \le i \le 6k \\ a & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \le i \le 6k \\ b & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \le i \le 6k \\ f & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \le i \le 6k \\ c & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \le i \le 6k \\ e & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \le i \le 6k\ ; \end{cases}$$

and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4\ ; \end{cases} \qquad g(v_i) = \begin{cases} a & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4\ . \end{cases}$$

Here we have $v_g(b) = v_g(c) = v_g(e) = 2k+1, v_g(a) = v_g(d) = v_g(f) = 2k+2$ and $e_g(0) = e_g(1) = 12k+8$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$.

Subcase 4.4. $n \equiv 3\ (mod\ 6)$.

Let $n = 6k+3$ and $k \ge 1$. Assign the labels to the vertices u, u_i, v_i for $1 \le i \le 6k$ as in Subcase 4.1 and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2 \\ e & \text{if } i = 6k+3\ ; \end{cases} \qquad g(v_i) = \begin{cases} c & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = v_g(f) = 2k+1, v_g(d) = 2k+2$ and $e_g(0) = e_g(1) = 12k+6$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$.

Subcase 4.5. $n \equiv 2\ (mod\ 6)$.

Let $n = 6k+2$ and $k \ge 1$. We assign the labels to the vertices u, u_i, v_i for $1 \le i \le 6k$ as in Subcase 4.3 and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2\ ; \end{cases} \qquad g(v_i) = \begin{cases} f & \text{if } i = 6k+1 \\ c & \text{if } i = 6k+2\ . \end{cases}$$

Here we have $v_g(a) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 2k+1, v_g(b) = 2k$ and $e_g(0) = e_g(1) = 12k+4$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$.

Subcase 4.6. $n \equiv 1\ (mod\ 6)$.

Let $n = 6k+1$ and $k \ge 1$. Assign the labels to the vertices u, u_i, v_i for $1 \le i \le 6k$ as in Subcase 4.3, except for the vertices u_{6k+1}, v_{6k+1} which are labeled by f, b respectively. Here we have $v_g(a) = v_g(c) = v_g(e) = 2k, v_g(b) = v_g(d) = v_g(f) = 2k+1$ and $e_g(0) = e_g(1) = 12k+2$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$.

Thus g is a group S_3 cordial remainder labeling. Hence, lotus inside a circle LC_n is a group S_3 cordial remainder graph for $n \ge 3$. □

Example 2.5. A group S_3 cordial remainder labeling of LC_7 is given in Figure 2.

Theorem 2.6. *Double fan DF_n is a group S_3 cordial remainder graph for $n \ge 2$.*

Proof. Let $V(DF_n) = \{v, w, u_i : 1 \le i \le n\}$ and $E(DF_n) = \{vu_i, wu_i : 1 \le i \le n\} \bigcup \{u_iu_{i+1} : 1 \le i \le n-1\}$. Therefore DF_n is of order $n+2$ and size $3n-1$. Table 1 gives group S_3 cordial remainder labeling of DF_n for $2 \le n \le 5$.

Assume $n \ge 6$. Define $g : V(DF_n) \to S_3$ as follows:

Case 1. $n \equiv 0\ (mod\ 6)$.

Let $n = 6k$ and $k \ge 1$.

$g(v) = d;\ g(w) = a;$

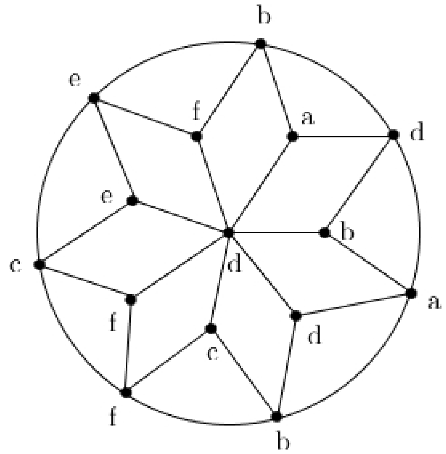

Figure 2.

Nature of n	v	w	u_1	u_2	u_3	u_4	u_5
$n=2$	d	a	b	f			
$n=3$	d	a	b	c	f		
$n=4$	d	a	e	b	f	c	
$n=5$	d	a	e	b	d	c	f

Table 1.

$$g(u_i) = \begin{cases} e & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ a & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ . \end{cases}$$

Here we have $v_g(b) = v_g(c) = v_g(e) = v_g(f) = k, v_g(a) = v_g(d) = k+1$ and $e_g(0) = 9k-1, e_g(1) = 9k$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 2. $n \equiv 5\ (mod\ 6)$.

Let $n = 6k+5$ and $k \geq 1$. Assign the labels to the vertices v, w, u_i for $1 \leq i \leq 6k$ as in Case 1 and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} e & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \\ f & \text{if } i = 6k+5\ ; \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = v_g(f) = k+1, v_g(d) = k+2$ and $e_g(0) = 9k+7, e_g(1) = 9k+7$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 3. $n \equiv 4\ (mod\ 6)$.

Let $n = 6k+4$ and $k \geq 1$. Assign the labels to the vertices v, w, u_i for $1 \leq i \leq 6k$ as in Case 1 and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} e & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = k+1$ and $e_g(0) =$

$9k+6, e_g(1) = 9k+5$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 4. $n \equiv 3\ (mod\ 6)$.

Let $n = 6k+3$ and $k \geq 1$. Assign the labels to the vertices v, w, u_i for $1 \leq i \leq 6k$ as in Case 1 and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} b & \text{if } i = 6k+1 \\ c & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(f) = k+1, v_g(e) = k$ and $e_g(0) = 9k+4, e_g(1) = 9k+4$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 5. $n \equiv 2\ (mod\ 6)$.

Let $n = 6k+2$ and $k \geq 1$. We assign the labels to the vertices v, w, u_i for $1 \leq i \leq 6k$ as in Case 1, except that the last two vertices u_{6k+1}, u_{6k+2} which is labeled by b, f respectively. Here we have $v_g(a) = v_g(b) = v_g(d) = v_g(f) = k+1, v_g(c) = v_g(e) = k$ and $e_g(0) = 9k+2, e_g(1) = 9k+3$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 6. $n \equiv 1\ (mod\ 6)$.

Let $n = 6k+1$ and $k \geq 1$. We assign the labels to the vertices v, w, u_i for $1 \leq i \leq 6k$ as in Case 1, except that the last vertex u_{6k+1} which is labeled by b. Here we have $v_g(a) = v_g(b) = v_g(d) = k+1, v_g(c) = v_g(e) = v_g(f) = k$ and $e_g(0) = 9k+1, e_g(1) = 9k+1$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Thus g is a group S_3 cordial remainder labeling. Hence, double fan DF_n is a group S_3 cordial remainder graph for $n \geq 2$. □

Example 2.7. A group S_3 cordial remainder labeling of DF_5 is given in Figure 3.

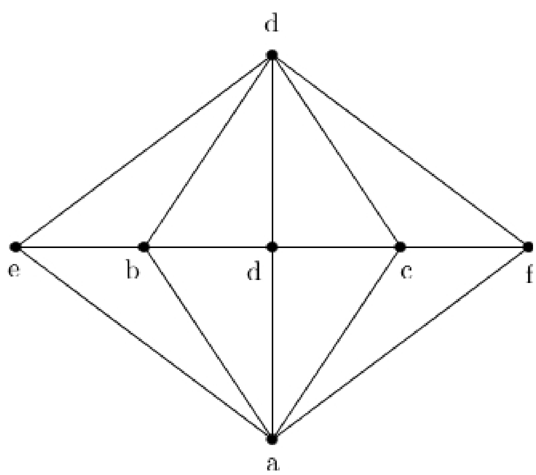

Figure 3.

Theorem 2.8. *Ladder L_n is a group S_3 cordial remainder graph for every n.*

Proof. Let $u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n$ be the vertices of the ladder L_n. Let $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Then L_n is of order $2n$ and size $3n-2$. Define $g : V(L_n) \to S_3$ as follows:

Case 1. n is odd.

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \leq i \leq n\ ; \end{cases}$$

$$g(v_i) = \begin{cases} f & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n. \end{cases}$$

Case 2. n is even.

$$g(u_i) = \begin{cases} d & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n\,; \end{cases}$$

$$g(v_i) = \begin{cases} b & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n. \end{cases}$$

From Table 2, it is easy to verify that $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Therefore g is a group S_3 cordial remainder labeling. Hence, L_n is a group S_3 cordial remainder graph for every n.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$	$e_g(0)$	$e_g(1)$
$6k-5(k \geq 1)$	$2k-1$	$2k-2$	$2k-2$	$2k-2$	$2k-2$	$2k-1$	$9k-9$	$9k-8$
$6k-4(k \geq 1)$	$2k-2$	$2k-1$	$2k-2$	$2k-1$	$2k-1$	$2k-1$	$9k-7$	$9k-7$
$6k-3(k \geq 1)$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$9k-6$	$9k-5$
$6k-2(k \geq 1)$	$2k$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k$	$9k-4$	$9k-4$
$6k-1(k \geq 1)$	$2k-1$	$2k-1$	$2k$	$2k$	$2k$	$2k$	$9k-3$	$9k-2$
$6k(k \geq 1)$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$9k-1$	$9k-1$

Table 2.

□

Example 2.9. A group S_3 cordial remainder labeling of L_5 is given in Figure 4.

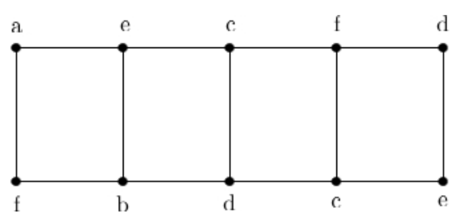

Figure 4.

Corollary 2.10. *$C_n \times P_2$ is a group S_3 cordial remainder graph for $n \geq 3$.*

Proof. The same labeling pattern as in Theorem 2.8 is followed, except that the label 'a' is replaced by the label 'd' for the last vertex v_n if $n \equiv 4 \ (mod\ 6)$. Hence it is easy to verify that $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Therefore g is a group S_3 cordial remainder labeling of $C_n \times P_2$ for $n \geq 3$. □

Theorem 2.11. *The slanting ladder SL_n is a group S_3 cordial remainder graph.*

Proof. Let $V(SL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(SL_n) = \{v_iv_{i+1}, u_iu_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\}$. Then SL_n is of order $2n$ and size $3n-3$. Define $g : V(SL_n) \to S_3$ as follows:

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n\,; \end{cases}$$

$$g(v_i) = \begin{cases} f & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n. \end{cases}$$

From Table 3, it is easy to verify that $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Therefore g is a group S_3 cordial remainder labeling.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$	$e_g(0)$	$e_g(1)$
$6k-5(k \geq 1)$	$2k-1$	$2k-2$	$2k-2$	$2k-2$	$2k-2$	$2k-1$	$9k-9$	$9k-9$
$6k-4(k \geq 1)$	$2k-1$	$2k-2$	$2k-2$	$2k-1$	$2k-1$	$2k-1$	$9k-7$	$9k-8$
$6k-3(k \geq 1)$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$9k-6$	$9k-6$
$6k-2(k \geq 1)$	$2k-1$	$2k-1$	$2k-1$	$2k$	$2k$	$2k-1$	$9k-5$	$9k-4$
$6k-1(k \geq 1)$	$2k-1$	$2k$	$2k$	$2k$	$2k$	$2k-1$	$9k-3$	$9k-3$
$6k(k \geq 1)$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$9k-1$	$9k-2$

Table 3.

□

Example 2.12. A group S_3 cordial remainder labeling of SL_8 is given in Figure 5.

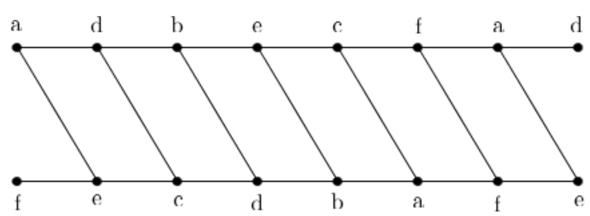

Figure 5.

Theorem 2.13. *The triangular ladder TL_n is a group S_3 cordial remainder graph.*

Proof. Let $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_iv_i : 1 \leq i \leq n\}$. Then TL_n is of order $2n$ and size $4n-3$. Define $g : V(TL_n) \to S_3$ as follows:

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n\,; \end{cases}$$

$$g(v_i)=\begin{cases} f & \text{if } i\equiv 1\ (mod\ 6) \text{ and } 1\le i\le n\\ c & \text{if } i\equiv 2\ (mod\ 6) \text{ and } 1\le i\le n\\ d & \text{if } i\equiv 3\ (mod\ 6) \text{ and } 1\le i\le n\\ a & \text{if } i\equiv 4\ (mod\ 6) \text{ and } 1\le i\le n\\ e & \text{if } i\equiv 5\ (mod\ 6) \text{ and } 1\le i\le n\\ b & \text{if } i\equiv 0\ (mod\ 6) \text{ and } 1\le i\le n. \end{cases}$$

From Table 4, it is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Therefore g is a group S_3 cordial remainder labeling.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$	$e_g(0)$	$e_g(1)$
$n=3k-2(k\ge 1)$	k	$k-1$	$k-1$	$k-1$	$k-1$	k	$6k-6$	$6k-5$
$n=3k-1(k\ge 1)$	k	$k-1$	k	$k-1$	k	k	$6k-3$	$6k-4$
$n=3k(k\ge 1)$	k	k	k	k	k	k	$6k-1$	$6k-2$

Table 4.

□

Example 2.14. A group S_3 cordial remainder labeling of TL_7 is given in Figure 6.

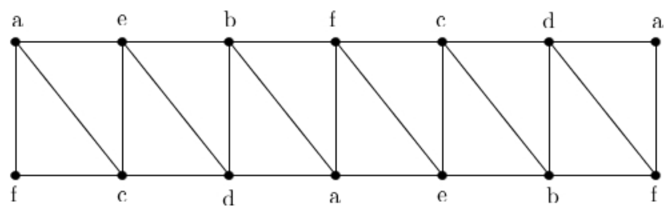

Figure 6.

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