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ON CHARACTERIZATIONS OF CURVES IN THE GALILEAN PLANE

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Abstract In this paper, we study Smarandache curves and curves of constant breadth by using the Galilean plane frame in G_2. Moreover, we get some characterizations of these types of special curves in G_2.

1 Introduction

The theory of curves is the usual starting point and interesting subject of researchers at differential geometry with the use of the concept of derivatives in calculus. One of the interesting problems on the curve theory is how to characterize special curves in different types of planes and spaces. There are also many important studies of curves in differential geometry within this context. In consideration of the existing studies, researchers have introduced new types of curves. Especially, Smarandache curves and curves of constant breadth are both of them. Some geometric properties of Smarandache curve, which a regular curve whose position vector is composed by Frenet frame vectors on another regular curve, were given in the different planes and spaces in [1, 2, 4]. Moreover, curves of constant breadth have been studied by many researchers in [6, 8, 9, 10, 11, 14].

The main aim of this work is to study the special curves due to the Galilean frame in the Galilean plane G_2.

This manuscript is organized as follows; in Section 2, we give a brief review of the Galilean plane. In Section 3, we investigate Smarandache curves, and curves of constant breadth into consideration in the Galilean plane G_2. Also, we obtain some characterizations of these types of special curves in the Galilean plane G_2.

2 Preliminaries

To use later, we give a brief review of Galilean geometry from [7, 12, 13].

We consider R^2 with the bilinear form

$$\langle \mathbf{x}, \mathbf{y} \rangle = \begin{cases} x_1 y_1, & \text{if } x_1 \neq 0 \text{ or } y_1 \neq 0 \\ x_2 y_2, & \text{if } x_1 = 0 \text{ and } y_1 = 0 \end{cases}, \tag{2.1}$$

where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$. Then, we have the Galilean plane G_2. This is one of the three Cayley-Klein plane geometries with a parabolic measure of distance. Denote R^2 with the bilinear form (2.1).

Hence, the norm of the vector $\mathbf{x} = (x_1, x_2)$ on the Galilean plane is given by

$$\|\mathbf{x}\| = \begin{cases} |x_1|, & \text{if } x_1 \neq 0 \\ |x_2|, & \text{if } x_1 = 0 \end{cases}. \tag{2.2}$$

Let $\alpha : I \subseteq R \to G_2$ be a curve in the Galilean plane given by $\alpha = (s, x(s))$, where s is a

Galilean invariant the arc-length on α. Then, we have

$$\begin{aligned} T(s) &= (1, x'(s)), \\ N(s) &= \frac{1}{\kappa(s)}(0, x''(s)) = (0, 1), \end{aligned}$$

where $\kappa(s) = x''(s)$.

Then the following moving bihedron of α is written by

$$\begin{aligned} T'(s) &= \kappa(s)N(s), \\ N'(s) &= 0, \end{aligned} \tag{2.3}$$

where T and N is said to be the tangent and the principal normal of α in G_2.

3 Special Curve in the Galilean plane G_2

In this section, our aim is to give some special curves such as Smarandache curves and curves of constant breadth in the Galilean plane .

3.1 Smarandache Curves

Definition 3.1. A curve in G_2, whose position vector is obtained by Frenet frame vectors on another curve, is called Smarandache curve via the Galilean frame. Now, we will study $TN-$ Smarandache curve as the only Smarandache curve in the Galilean plane.

Definition 3.2. Let $\alpha = \alpha(s)$ be a curve in the Galilean plane and T_α and N_α are the tangent and principal normal vectors of the Smarandache curve of the curve α. Then, $TN-$Smarandache curves are defined by

$$\beta(s_\beta) = \frac{T_\alpha + N_\alpha}{\|T_\alpha + N_\alpha\|}. \tag{3.1}$$

Let us investigate the Frenet invariants of $TN-$ Smarandache curve according to $\alpha = \alpha(s)$. Differentiating (3.1), we have

$$\dot{\beta} = \frac{d\beta}{ds_\beta}.\frac{ds_\beta}{ds} = k_\alpha N_\alpha, \tag{3.2}$$

and hence

$$T_\beta = N_\alpha, \tag{3.3}$$

where

$$\frac{ds_\beta}{ds} = k_\alpha. \tag{3.4}$$

By differentiating (3.3) with respect to s, we have

$$N_\beta = 0. \tag{3.5}$$

Corollary 3.3. *$TN-$ Smarandache curve is also involute of the curve of α, for further information see [3].*

3.2 Curves of Constant Breadth

Let $\varphi = \varphi(s)$ and $\varphi^* = \varphi^*(s^*)$ be simple closed curves of constant breadth in the Galilean plane. These curves will be given by C and C^*. The normal plane at every point P on the curve meets the curve at a single point Q other than P. The point Q is called as the opposite point of P. The normal plane at every point P on the curve meets the curve in the class Γ as in [5] having parallel tangents T and T^* in opposite directions at the opposite points φ and φ^* of the curve.

A simple closed curve of constant breadth having parallel tangents in opposite directions at opposite points can be given in terms of the Galilean frame as:

$$\varphi^* = \varphi + m_1 T + m_2 N, \tag{3.6}$$

where m_1 and m_2 are arbitrary functions of s, φ and φ^* which are opposite points.

By differentiating both sides of (3.6) using (2.3), we get

$$\frac{d\varphi^*}{ds} = T^* \frac{ds^*}{ds} = (1 + m_1')T + (m_1 k + m_2')N. \tag{3.7}$$

Since $T^* = -T$ and using (3.7), we get the following system,

$$\begin{aligned} 1 + m_1' + \frac{ds^*}{ds} &= 0, \\ m_1 k + m_2' &= 0. \end{aligned} \tag{3.8}$$

Let θ be the angle between the tangent of the curve C at point φ with a given fixed direction. Then we have

$$\frac{d\theta}{ds} = k = \frac{1}{\rho} \text{ and } \frac{d\theta}{ds^*} = k^* = \frac{1}{\rho^*}. \tag{3.9}$$

By using (3.9) into (3.8), (3.8) becomes as

$$\begin{aligned} \frac{dm_1}{d\theta} &= -f(\theta), \\ m_1 + \frac{dm_2}{d\theta} &= 0, \end{aligned} \tag{3.10}$$

where $f(\theta) = \rho + \rho^*$, and ρ and ρ^*denote the radius of curvature at φ and φ^*.

In the above equation, if we eliminate m_1, then we get the linear differential equation of the second order as follows

$$\frac{d^2 m_2}{d\theta^2} = f(\theta). \tag{3.11}$$

If we solve the equation (3.11), then we get

$$m_2 = H(\theta), \tag{3.12}$$

$$m_1 = -F(\theta), \tag{3.13}$$

where $H(\theta) = \int F(\theta)d\theta + c_2$ and $F(\theta) = \int f(\theta)d\theta + c_1$ and c_1, c_2 are constants.

If the distance between the opposite points of C and C^* is constant, then we have

$$\|\varphi^* - \varphi\|^2 = \begin{cases} m_1^2, & \text{if } m_1 \neq 0 \\ m_2^2, & \text{if } m_1 = 0 \end{cases} = \text{constant}. \tag{3.14}$$

Taking the account of the conditions (3.14), we give the following cases:

Case 3.4. If $m_1 \neq 0$, then from(3.14), we get

$$m_1 \frac{dm_1}{d\theta} = 0, \tag{3.15}$$

and $m_1 = \lambda$ =constant, by using (3.10), we get $m_2 = -\lambda\theta + c_2$ where $H(\theta) = -\lambda\theta + c_2$, $F(\theta) = c_1 = -\lambda$ and $f(\theta) = 0$.

Thus, the equation (3.6) turns into

$$\varphi^* = \varphi + \lambda T + (-\lambda\theta + c_2)N. \tag{3.16}$$

Case 3.5. If $m_1 = 0$, then from (3.14), we get

$$m_2 \frac{dm_2}{d\theta} = 0, \tag{3.17}$$

and $m_2 = \lambda = constant \neq 0$, by using (3.10), we get $H(\theta) = c_2 = \lambda$ and $F(\theta) = f(\theta) = c_1 = 0$.

Thus, the equation (3.6) turns into

$$\varphi^* = \varphi + \lambda N. \tag{3.18}$$

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