Palestine Journal of Mathematics

Vol. 10(1)(2021) , 340–348

n$\mathcal{I}_{g\mu}$-CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

Selvaraj Ganesan

Communicated by Thabet Abdeljawad

MSC 2010 Classifications: Primary 20M99, 13F10; Secondary 13A15, 13M05.

Keywords and phrases: n$\mathcal{I}_{g\mu}$-closed sets, n$\mathcal{I}_{g\mu}$-continuous, n$\mathcal{I}_{g}$-continuous, μ-n$\mathcal{I}$-LC-continuous and ζ_{μ}-n$\mathcal{I}$-continuous.

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and suggestions which have helped immensely in improving the quality of the paper.

Abstract Characterizations and properties of n$\mathcal{I}_{g\mu}$-closed sets and n$\mathcal{I}_{g\mu}$-open sets are given. The main purpose of this paper is to introduce the concepts of μ-n$\mathcal{I}$-locally closed sets, n$\wedge_{\mu}$-sets, ζ_{μ}-n$\mathcal{I}$-closed sets, n$\mathcal{I}_{g\mu}$-continuous, μ-n$\mathcal{I}$-LC-continuous, ζ_{μ}-n$\mathcal{I}$-continuous and to obtain decompositions of n$\star$-continuity in nano ideal topological spaces.

1 Introduction and Preliminaries

Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on U, where $\mathcal{N}$ = $\tau_R(X)$ and $(.)^*_n$: $\wp$(U)$\rightarrow\wp$(U) ($\wp$(U) is the set of all subsets of U) [6, 7]. For a subset A $\subseteq$U, $A^*_n(\mathcal{I},\mathcal{N})$={x$\in$U : $G_n\cap$ A $\notin\mathcal{I}$, for every $G_n\in G_n$(x)}, where G_n={G_n | x$\in$ G_n, G_n $\in\mathcal{N}$}is called the nano local function(brielfy n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for A^*_n ($\mathcal{I}$, $\mathcal{N}$)

Theorem 1.1. *[6, 7] Let (U, $\mathcal{N}$) be a nano topological space with ideal $\mathcal{I}$, $\mathcal{I}'$ on U and A, B be subsets of U. Then*
(1) A $\subseteq$ B $\Rightarrow A^*_n \subseteq B^*_n$.
(2) $\mathcal{I}\subseteq\mathcal{I}' \Rightarrow A^*_n$ ($\mathcal{I}'$) $\subseteq A^*_n$ ($\mathcal{I}$).
(3) A^*_n=n-cl(A^*_n) $\subseteq$ n-cl(A) (A^*_n is a nano closed subset of n-cl(A)).
(4) $(A^*_n)^*_n \subseteq A^*_n$.
(5) $A^*_n\cup B^*_n$ =$(A\cup B)^*_n$.
(6)$A^*_n - B^*_n$ = $(A - B)^*_n - B^*_n \subseteq (A - B)^*_n$.
(7) V $\in\mathcal{N}\Rightarrow$ V $\cap A^*_n$ = V $\cap$ $(V\cap A)^*_n \subseteq (V\cap A)^*_n$ and
(8) J $\in\mathcal{I}\Rightarrow$ $(A\cup J)^*_n$ = A^*_n = $(A -J)^*_n$.

Lemma 1.2. *[6, 7] Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano topological space with an ideal $\mathcal{I}$ and A$\subseteq A^*_n$, then A^*_n=n-cl(A^*_n)=n-cl(A).*

Definition 1.3. [6, 7] Let (U, $\mathcal{N}$) be an nano topological space eith an ideal $\mathcal{I}$ on U. The set operator n-cl* is called a nano $\star$ -closure and is defined as n-cl*(A)= A $\cup$ A^*_n for A $\subseteq$X.

Theorem 1.4. *[6, 7] The set operator n-cl* satisfies the following conditions:*
(1) A $\subseteq$ n-cl*(A).
(2) n-cl*(ϕ) = ϕ and n-cl*(U) = U.
(3) If A $\subseteq$B, then n-cl*(A) $\subseteq$ n-cl*(B).
(4) n-cl*(A) $\cup$ n-cl*(B) = n-cl*(A $\cup$ B).
(5) n-cl*(n-cl*(A)) =n-cl*(A).

Definition 1.5. [6, 7] An ideal $\mathcal{I}$ ina space (U, $\mathcal{N}$, $\mathcal{I}$) is called $\mathcal{N}$-codense ideal if $\mathcal{N}\cap\mathcal{I}$ =ϕ.

Definition 1.6. [6, 7] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is n$\star$-dense in itself (resp. n$\star$-perfect and n$\star$-closed) if A $\subseteq A^*_n$ (resp. A= A^*_n, $A^*_n\subseteq$ A).

Lemma 1.7. *[6, 7] Let (U, $\mathcal{N}$, $\mathcal{I}$) be a nano ideal topological space and A $\subseteq$ U. If A is n$\star$-dense in itself A_n^* =n-cl(A_n^*) =n-cl(A_n^*)=n-cl(A)=n-cl*(A).*

Definition 1.8. [6, 7] A subset A of an nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be

(1) nano-$\mathcal{I}$-generalized closed (briefly, n$\mathcal{I}$g-closed if $A_n^* \subseteq$ V whenever A$\subseteq$ V and V is n-open.

(2) n$\mathcal{I}$g-open if its complement is n$\mathcal{I}$g-closed.

Definition 1.9. [5] A subset M of a space (U, $\tau_R(X)$) is said to be

(1) Nano α-open set if M $\subseteq$ Nint(Ncl(Nint(M))).

(2) Nano semi-open set if M $\subseteq$ Ncl(Nint(M)).

The complement of the above mentioned Nano open sets are called their respective Nano closed sets.

The Nano α-closure [2] of a subset M of U, denoted by Nαcl(M) is defined to be the intersection of all Nano α-closed sets of (U, $\tau_R(X)$) containing M.

Definition 1.10. [3] A subset M of a space (U, $\tau_R(X)$) is called

(1) a Nano gα*-closed set if Nαcl(A) $\subseteq$ Nano int(U) whenever A $\subseteq$ U and U is Nano α-open in (U, $\tau_R(X)$). The complement of Nano gα*-closed set is called Nano gα*-open set.

(2) a Nano μ-closed set if Ncl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Nano gα*-open in (U, $\tau_R(X)$). The complement of Nano μ-closed set is called Nano μ-open set.

(3) a Nano gμ-closed set if Ncl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Nano μ-open in (U, $\tau_R(X)$). The complement of Nano gμ-closed set is called Nano gμ-open set.

Definition 1.11. [1] A subset a of of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called an lightly nano $\mathcal{I}$ -locally closed (briefly $\mathcal{L}$-n$\mathcal{I}$-LC) if A = M $\cap$ N where M is n-open and N is n$\star$-closed.

2 n$\mathcal{I}_{g\mu}$-closed sets

Definition 2.1. A subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is said to be

(1) n$\mathcal{I}_{g\mu}$-closed if $A_n^* \subseteq$ U whenever A$\subseteq$ U and U is nμ-open,

(2) n$\mathcal{I}_{g\mu}$-open if its complement is n$\mathcal{I}_{g\mu}$-closed.

Theorem 2.2. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then every n$\mathcal{I}_{g\mu}$-closed set is n$\mathcal{I}_g$-closed but not conversely.*

Proof. It follows from the fact that every n-open set is nμ-open. □

Example 2.3. Let K = {4, 5, 6}, with K/ R= {{4}, {5, 6}} and X= {4}. Then the Nano topology $\mathcal{N}$ = {ϕ, {4}, K} and $\mathcal{I}$={$\emptyset$, {1}}. Then n$\mathcal{I}_{g\mu}$-closed sets are ϕ, K, {4}, {5, 6} and n$\mathcal{I}_g$-closed sets are ϕ, K, {4}, {5}, {6}, {4, 5}, {4, 6}, {5, 6}. It is clear that {5} is n$\mathcal{I}_g$-closed but it is not n$\mathcal{I}_{g\mu}$-closed.

The following theorem gives characterizations of n$\mathcal{I}_{g\mu}$-closed sets.

Theorem 2.4. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space and A $\subseteq$ K, then the following are equivalent.*

(1) A is n$\mathcal{I}_{g\mu}$-closed,

(2) n-cl(A)$\subseteq$U whenever A $\subseteq$ U and U is nμ-open in K,*

(3) For all k$\in$n-cl(A), n-μcl({k})$\cap$A$\neq\emptyset$.*

(4) n-cl(A)$-$A contains no nonempty nμ-closed set,*

(5) A_n^*-A contains no nonempty nμ-closed set.

Proof. (1)$\Rightarrow$(2) If A is n$\mathcal{I}_{g\mu}$-closed, then $A_n^*\subseteq$U whenever A$\subseteq$U and U is nμ-open in K and so n-cl*(A)=A$\cup A_n^*\subseteq$U whenever A$\subseteq$U and U is nμ-open in K. This proves (2).

(2)$\Rightarrow$(3) Suppose k$\in$n-cl*(A). If n-μcl({k})$\cap$A=$\emptyset$, then A$\subseteq$K$-$n-μcl({k}). By (2), n-cl*(A)$\subseteq$ K$-$n-μcl({k}), a contradiction, since k$\in$n-cl*(A).

(3)⇒(4) Suppose F⊆n-cl*(A)−A, F is nμ-closed and k∈F. Since F⊆K−A and F is nμ-closed, then A⊆K−F and F is nμ-closed, n-μcl({k})∩A=∅. Since k∈n-cl*(A) by (3), n-μcl({x})∩A≠∅. Therefore n-cl*(A)−A contains no nonempty nμ-closed set.

(4)⇒(5) Since n-cl*(A)−A=(A∪A$^*_n$)−A=(A∪A$^*_n$)∩A^c=(A∩A^c)∪ (A$^*_n$∩A^c)=A$^*_n$∩A^c= A$^*_n$−A. Therefore A$^*_n$−A contains no nonempty nμ-closed set.

(5)⇒(1) Let A⊆U where U is nμ-open set. Therefore K−U⊆K−A and so A$^*_n$∩(K−U) ⊆A$^*_n$∩(K−A)=A$^*_n$−A. Therefore A$^*_n$∩(K−U)⊆A$^*_n$−A. Since A$^*_n$ is always n-closed set, so A$^*_n$ is nμ-closed set and so A$^*_n$∩(K−U) is a nμ-closed set contained in A$^*_n$−A. Therefore A$^*_n$∩(K−U)=∅ and hence A$^*_n$⊆U. Therefore A is n$\mathcal{I}_{g\mu}$-closed. □

Theorem 2.5. *Every n⋆-closed set is n$\mathcal{I}_{g\mu}$-closed but not conversely.*

Proof. Let A be a n⋆-closed, then A$^*_n$⊆A. Let A⊂U where U is nμ-open. Hence A$^*_n$⊆U whenever A⊆U and U is nμ-open. Therefore A is n$\mathcal{I}_{g\mu}$-closed. □

Example 2.6. Let K = {4, 5, 6} with K/ R= {{6}, {4, 5}, {5, 4}} and X= {4, 5}. Then Nano topology $\mathcal{N}$ = {ϕ, {4, 5}, K} and $\mathcal{I}$={∅, {4}}. Then n$\mathcal{I}_{g\mu}$-closed sets are ϕ, K, {4}, {6}, {4, 6}, {5, 6} and n⋆-closed sets are ϕ, K, {4}, {6}, {4, 6}. It is clear that {5, 6} is n$\mathcal{I}_{g\mu}$-closed set but it is not n⋆-closed.

Theorem 2.7. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. For every A∈$\mathcal{I}$, A is n$\mathcal{I}_{g\mu}$-closed.*

Proof. Let A⊆U where U is nμ-open set. Since A$^*_n$=∅ for every A∈$\mathcal{I}$, then n-cl*(A)=A∪A$^*_n$ =A⊆U. Therefore, by Theorem 2.4, A is n$\mathcal{I}_{g\mu}$-closed. □

Theorem 2.8. *If (K, $\mathcal{N}$, $\mathcal{I}$) is an nano ideal topological space, then A$^*_n$ is always n$\mathcal{I}_{g\mu}$-closed for every subset A of K.*

Proof. Let A$^*{}_n$⊆U where U is nμ-open. Since $(A^*_n)^*_n$⊆A$^*_n$ Theorem 1.1 (4), we have $(A^*_n)^*_n$ ⊆U whenever A$^*_n$⊆U and U is nμ-open. Hence A$^*_n$ is n$\mathcal{I}_{g\mu}$-closed. □

Theorem 2.9. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every n$\mathcal{I}_{g\mu}$-closed, nμ-open set is n⋆-closed set.*

Proof. Since A is n$\mathcal{I}_{g\mu}$-closed and nμ-open. Then A$^*_n$⊆A whenever A⊆A and A is nμ-open. Hence A is n⋆-closed. □

Definition 2.10. An nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is said to be a nT$_{\mathcal{I}}$-space if every n$\mathcal{I}_g$-closed subset of K is a n⋆-closed.

Theorem 2.11. *If (K, $\mathcal{N}$, $\mathcal{I}$) is a nT$_{\mathcal{I}}$ nano ideal space and A is an n$\mathcal{I}_g$-closed set, then A is n⋆-closed set.*

Proof. It is follows from Definition 2.10. □

Corollary 2.12. *If (K, $\mathcal{N}$, $\mathcal{I}$) is a nT$_{\mathcal{I}}$ nano ideal space and A is an n$\mathcal{I}_{g\mu}$-closed set, then A is n⋆-closed set.*
Proof. *By assumption A is n$\mathcal{I}_{g\mu}$-closed in (K, $\mathcal{N}$, $\mathcal{I}$) and so by Theorem 2.2, A is n$\mathcal{I}_g$-closed. Since (K, $\mathcal{N}$, $\mathcal{I}$) is an nT$_{\mathcal{I}}$-space by Definition 2.10, A is n⋆-closed.* □

Corollary 2.13. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A be an n$\mathcal{I}_{g\mu}$-closed set. Then the following are equivalent.*
(1) A is a n⋆-closed set,
(2) n-cl(A)−A is a nμ-closed set,*
(3) A$^*_n$−A is a nμ-closed set.

Proof. *(1)⇒(2) If A is n⋆-closed, then A^*_n⊆A and so n-cl*(A)−A=(A∪$A^*{}_n$)-A =∅. Hence n-cl*(A)−A is nμ-closed set.*

(2)⇒(3) Since n-cl(A)−A=A^*_n−A and so A^*_n−A is nμ-closed set.*

(3)⇒(1) If A^*_n−A is a nμ-closed set, since A is n$\mathcal{I}_{g\mu}$-closed set, by Theorem **??** *(5), A^*_n−A=∅ and so A is n⋆-closed.* □

Theorem 2.14. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every ngμ-closed set is an n$\mathcal{I}_{g\mu}$-closed set but not conversely.*

Proof. *Let A be a ngμ-closed set. Then n-cl(A)⊆U whenever A⊆U and U is nμ-open. So by Theorem 1.1 (3), A^*_n⊆n-cl(A)⊆U whenever A⊆U and U is nμ-open. Hence A is n$\mathcal{I}_{g\mu}$-closed.* □

Example 2.15. Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 2.6. Then ngμ-closed sets are ϕ, K, $\{6\}$, $\{4, 6\}$, $\{5, 6\}$. It is clear that $\{4\}$ is n$\mathcal{I}_{g\mu}$-closed set but it is not ngμ-closed.

Theorem 2.16. *If (K, $\mathcal{N}$, $\mathcal{I}$) is an nano ideal topological space and A is a n⋆-dense in itself, n$\mathcal{I}_{g\mu}$-closed subset of K, then A is ngμ-closed.*

Proof. *Suppose A is a n⋆-dense in itself, n$\mathcal{I}_{g\mu}$-closed subset of K. Let A⊆U where U is nμ-open. Then by Theorem 2.4 (2), n-cl*(A)⊆U whenever A⊆U and U is nμ-open. Since A is n⋆-dense in itself, by Lemma 1.7, n-cl(A)=n-cl*(A). Therefore n-cl(A)⊆U whenever A⊆U and U is nμ-open. Hence A is ngμ-closed.* □

Corollary 2.17. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space where $\mathcal{I}$={∅}, then A is n$\mathcal{I}_{g\mu}$-closed if and only if A is ngμ-closed.*

Proof. *The proof follows from the fact that for $\mathcal{I}$={∅}, A^*_n=n-cl(A)⊇A. Therefore A is n⋆-dense in itself. Since A is n$\mathcal{I}_{g\mu}$-closed, by Theorem 2.16, A is ngμ-closed.*

Conversely, by Theorem 2.14, every ngμ-closed set is n$\mathcal{I}_{g\mu}$-closed set. □

Lemma 2.18. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then the following are equivalent*

(1) K = K^*_n
(2) $\mathcal{N} \cap \mathcal{I} = \phi$.
(3) If I ∈ $\mathcal{I}$ then n-int(I) = ϕ.*
(4) for every G ∈ $\mathcal{N}$, G ⊆ G^*_n.

Theorem 2.19. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then the following are equivalent*

(1) K = K^*_n.
(2) for every A ∈ Nano open, A ⊆ A^*_n.
(3) for every A ∈ Nano semi open, A ⊆ A^*_n.

Proof. *(1) and (2) are equivalent by Lemma 2.18.*
(2) ⇒ (3). Suppose A ∈ Nano semi open (K, $\mathcal{N}$). Then there exists an n-open set M such that M ⊆ A ⊆ n-cl(M). Since M is n-open, M ⊆ M^*_n and so by Lemma 1.2, A ⊆ n-cl(M) ⊆n-cl(M^*_n) = M^*_n ⊆ A^*_n. Hence A⊆ A^*_n.
(3) ⇒ (1). It is clear. □

Corollary 2.20. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space where $\mathcal{I}$ is $\mathcal{N}$-codense and A is a Nano semi-open, n$\mathcal{I}_{g\mu}$-closed subset of K, then A is ngμ-closed.*

Proof. *The proof follows Theorem 2.19, A is n⋆-dense in itself. By Theorem 2.16, A is ngμ-closed.* □

Theorem 2.21. *Every n-closed set is $n\mathcal{I}_{g\mu}$-closed but not conversely.*

Proof. *Let A be a n-closed, then $A^*_n \subseteq A$. Let $A \subset U$ where U is $n\mu$-open. Hence $A^*_n \subseteq U$ whenever $A \subseteq U$ and U is $n\mu$-open. Therefore A is $n\mathcal{I}_{g\mu}$-closed.* □

Example 2.22. Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 2.3. Then n-closed sets are ϕ, K, {5, 6}. It is clear that {4} is $n\mathcal{I}_{g\mu}$-closed set but it is not n-closed.

Remark 2.23. remark 2.23 ng-closed sets and $n\mathcal{I}_{g\mu}$-closed sets are independent.

Example 2.24. Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 2.3. Then ng-closed sets are ϕ, K, {5}, {6}, {4, 5}, {4, 6}, {5, 6}. It is clear that {5} is ng-closed set but it is not $n\mathcal{I}_{g\mu}$-closed. Also it is clear that {4} is $n\mathcal{I}_{g\mu}$-closed set but it is not ng-closed.

Remark 2.25. (1) Every n-closed is n$\star$-closed set but not conversely. [1]
(2) Every n-closed set is ngμ-closed but not conversely. [3]
(3) Every ngμ-closed set is ng-closed but not conversely. [3]
(4) Every ng-closed set is $n\mathcal{I}_g$-closed but not conversely. [7]

Remark 2.26. We have the following implications for the subsets stated above.

$n - closed@ >>> ng\mu - closed@ >>> ng - closed$
$@VVV@VVV@VVV$
$n\star -closed@ >>> n\mathcal{I}_{g\mu} - closed@ >>> n\mathcal{I}_g - closed$

Theorem 2.27. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and $A \subseteq K$. Then A is $n\mathcal{I}_{g\mu}$-closed if and only if $A=F-M$ where F is n$\star$-closed and M contains no nonempty $n\mu$-closed set.*

Proof. *If A is $n\mathcal{I}_{g\mu}$-closed, then by Theorem 2.4 (5), $M=A^*_n-A$ contains no nonempty $n\mu$-closed set. If $F=ncl^*(A)$, then F is n$\star$-closed such that $F-M=(A\cup A^*_n)-(A^*_n-A)$ $=(A\cup A^*_n)\cap (A^*_n\cap A^c)^c=(A\cup A^*_n)\cap ((A^*_n)^c\cup A)=(A\cup A^*_n)\cap(A\cup(A^*_n)^c)=$ $A\cup(A^*_n\cap(A^*_n)^c)=A$.*

Conversely, suppose $A=F-M$ where F is n$\star$-closed and M contains no nonempty $n\mu$-closed set. Let U be an $n\mu$-open set such that $A\subseteq U$. Then $F-M\subseteq U$ which implies that $F\cap(K-U)\subseteq M$. Now $A\subseteq F$ and $F^*_n\subseteq F$ then $A^*_n\subseteq F^*_n$ and so $A^*_n\cap(K-U)\subseteq F^*_n\cap(K-U)\subseteq F\cap (K-U)\subseteq M$. By hypothesis, since $A^*_n\cap(K-U)$ is $n\mu$-closed, $A^*_n\cap(K-U)=\emptyset$ and so $A^*_n\subseteq U$. Hence A is $n\mathcal{I}_{g\mu}$-closed. □

Theorem 2.28. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and $A\subseteq K$. If $A\subseteq B\subseteq A^*_n$, then $A^*_n=B^*_n$ and B is n$\star$-dense in itself.*

Proof. *Since $A\subseteq B$, then $A^*_n\subseteq B^*_n$ and since $B\subseteq A^*_n$, then $B^*_n\subseteq(A^*_n)^*_n\subseteq A^*_n$ Theorem 1.1 (4). Therefore $A^*_n=B^*_n$ and $B\subseteq A^*_n\subseteq B^*_n$. Hence proved.* □

Theorem 2.29. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A and B are subsets of K such that $A\subseteq B\subseteq n\text{-}cl^*_n(A)$ and A is $n\mathcal{I}_{g\mu}$-closed, then B is $n\mathcal{I}_{g\mu}$-closed.*

Proof. *Since A is $n\mathcal{I}_{g\mu}$-closed, then by Theorem 2.4 (1), $n\text{-}cl^*_n(A)-A$ contains no nonempty $n\mu$-closed set. Since $n\text{-}cl^*(B)-B\subseteq n\text{-}cl^*(A)-A$ and so $n\text{-}cl^*(B)-B$ contains no nonempty $n\mu$-closed set and so by Theorem 2.4 (4), B is $n\mathcal{I}_{g\mu}$-closed.* □

Corollary 2.30. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A and B are subsets of K such that $A\subseteq B\subseteq A^*_n$ and A is $n\mathcal{I}_{g\mu}$-closed, then A and B are ngμ-closed sets.*

Proof. *Let A and B be subsets of K such that A⊆B⊆A_n^* which implies that A⊆B ⊆A_n^*⊆n-cl*(A) and A is n$\mathcal{I}_{g\mu}$-closed. By Theorem 2.29, B is n$\mathcal{I}_{g\mu}$-closed. Since A⊆B⊆A_n^*, then A_n^*=B_n^* and so A and B are n⋆-dense in itself. By Theorem 2.16, A and B are ngμ-closed.* □

The following theorem gives a characterization of n$\mathcal{I}_{g\mu}$-open sets.

Theorem 2.31. *Let (K, $\mathcal{N}$, $\mathcal{I}$)be an nano ideal topological space and A⊆K. Then A is n$\mathcal{I}_{g\mu}$-open if and only if F⊆n-int*(A) whenever F is nμ-closed and F⊆A.*

Proof. *Suppose A is n$\mathcal{I}_{g\mu}$-open. If F is nμ-closed and F⊆A, then K−A⊆K−F and so n-cl*(K−A)⊆K−F by Theorem 2.4 (2). Therefore F⊆K−n-cl*(K−A)=n-int*(A). Hence F⊆n-int*(A).*

Conversely, suppose the condition holds. Let U be a nμ-open set such that K−A⊆U. Then K−U⊆A and so K−U⊆n-int(A). Therefore n-cl*(K−A)⊆U. By Theorem 2.4 (2), K−A is n$\mathcal{I}_{g\mu}$-closed. Hence A is n$\mathcal{I}_{g\mu}$-open.* □

Corollary 2.32. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. If A is n$\mathcal{I}_{g\mu}$-open, then F⊆n-int*(A) whenever F is n-closed and F⊆A.*

The following theorem gives a property of n$\mathcal{I}_{g\mu}$-closed.

Theorem 2.33. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. If A is n$\mathcal{I}_{g\mu}$-open and n-int*(A)⊆B⊆A, then B is n$\mathcal{I}_{g\mu}$-open.*

Proof. *Since A is n$\mathcal{I}_{g\mu}$-open, then K−A is n$\mathcal{I}_{g\mu}$-closed. By Theorem 2.4 (4), n-cl*(K−A)−(K−A) contains no nonempty nμ-closed set. Since n-int*(A)⊆n-int*(B) which implies that n-cl*(K−B)⊆n-cl*(K−A) and so n-cl*(K−B)−(K−B)⊆
n-cl*(K−A)−(K−A). Hence B is n$\mathcal{I}_{g\mu}$-open.* □

The following theorem gives a characterization of n$\mathcal{I}_{g\mu}$-closed sets in terms of n$\mathcal{I}_{g\mu}$-open sets.

Theorem 2.34. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. Then the following are equivalent.*

(1) A is n$\mathcal{I}_{g\mu}$-closed,

(2) A∪(K−A_n^*) is n$\mathcal{I}_{g\mu}$-closed,

(3) A_n^*−A is n$\mathcal{I}_{g\mu}$-open.

Proof. *(1)⇒(2) Suppose A is n$\mathcal{I}_{g\mu}$-closed. If U is any nμ-open set such that A∪(K−A_n^*)⊆U, then K−U⊆ K−(A∪(K−A_n^*))=K∩(A∪(A_n^*)^c)^c = A_n^*∩A^c=A_n^*−A. Since A is n$\mathcal{I}_{g\mu}$-closed, by Theorem 2.4 (5), it follows that K−U=∅ and so K=U. Therefore A∪(K−A_n^*)⊆U which implies that A∪(K−A_n^*)⊆K and so (A∪(K−A_n^*))$_n^*$⊆K_n^*⊆K=U. Hence A∪(K−A_n^*) is n$\mathcal{I}_{g\mu}$-closed.*

(2)⇒(1) Suppose A∪(K−A_n^*) is n$\mathcal{I}_{g\mu}$-closed. If F is any nμ-closed set such that F⊆A_n^*−A, then F⊆A_n^* and F⊈A which implies that K−A_n^*⊆K−F and A⊆K−F. Therefore A∪(K−A_n^*)⊆A∪(K−F)=K−F and K−F is nμ-open. Since (A∪(K−A_n^*))$_n^*$ ⊆K−F which implies that A_n^*∪(K−A_n^*)$_n^*$⊆K−F and so A_n^*⊆K−F which implies that F⊆K−A_n^*. Since F⊆A_n^*, it follows that F=∅. Hence A is n$\mathcal{I}_{g\mu}$-closed.

(2)⇔(3) Since K−(A_n^*−A)=K∩(A_n^*∩A^c)^c=K∩((A_n^*)^c∪A)=(K∩ (A_n^*)^c)∪(K∩A)=A∪ (K−A_n^*) is n$\mathcal{I}_{g\mu}$-closed. Hence A_n^*−A is n$\mathcal{I}_{g\mu}$-open. □

Theorem 2.35. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every subset of K is n$\mathcal{I}_{g\mu}$-closed if and only if every nμ-open set is n⋆-closed.*

Proof. *Suppose every subset of K is n$\mathcal{I}_{g\mu}$-closed. If U⊆K is nμ-open, then U is n$\mathcal{I}_{g\mu}$-closed and so U_n^*⊆U. Hence U is n⋆-closed.*

Conversely, suppose that every nμ-open set is n⋆-closed. If U is nμ-open set such that A⊆U⊆K, then A_n^*⊆U_n^*⊆U and so A is n$\mathcal{I}_{g\mu}$-closed. □

3 μ-n$\mathcal{I}$-locally closed sets

We introduce the following definition

Definition 3.1. A subset a of of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called an μ -n$\mathcal{I}$-locally closed set(briefly μ-n$\mathcal{I}$-LC) if A = M ∩ N where M is nμ-open and N is n⋆-closed.

Proposition 3.2. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A a subset of K. Then the following hold.*
(1) If A is nμ-open , then A is μ-n$\mathcal{I}$-LC set.
(2) A is n⋆-closed, then A is μ-n$\mathcal{I}$-LC set.
(3) If A is a $\mathcal{L}$-n$\mathcal{I}$-LC-set, then A is an μ-n$\mathcal{I}$-LC set.

Proof. *It is obvious from Definitions 1.11 and 3.1.* □

The converse of the above Proposition 3.2 need not be true as shown in the following examples.

Example 3.3. Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 2.3. Then nμ-open sets are ϕ, K, {4}, {5}, {6}, {4, 5}, {4, 6}, μ-n$\mathcal{I}$-LC sets are power set of K and n⋆-closed sets are ϕ, K, {4}, {5, 6}. It is clear that {5} is μ-n$\mathcal{I}$-LC set but it is not n⋆-closed. Also it is clear that {5, 6} is an μ-n$\mathcal{I}$-LC set but it is not nμ-open.

Example 3.4. Let K = {4, 5, 6} with K/ R= {{6}, {4, 5}, {5, 4}} and X= {4, 5}. Then Nano topology $\mathcal{N}$ = {ϕ, {4, 5}, K} and $\mathcal{I}$={∅}. Then μ-n$\mathcal{I}$-LC sets are ϕ, K, {4}, {5}, {6}, {4, 5} and $\mathcal{L}$-n$\mathcal{I}$-LC-set are ϕ, K, {6}, {4, 5}. It is clear that {4} is μ-n$\mathcal{I}$-LC set but it is not $\mathcal{L}$-n$\mathcal{I}$-LC-set.

Theorem 3.5. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A is an μ -n$\mathcal{I}$-LC-set and B is a n⋆-closed set, then A ∩ B is an μ-n$\mathcal{I}$-LC-set.*

Proof. *Let B be n⋆-closed , then A ∩ B = (O ∩ P) ∩ B = O ∩ (P ∩ B), where P ∩ B is n⋆-closed. Hence A ∩ B is an μ -n$\mathcal{I}$-LC-set.*□

Theorem 3.6. *A subset of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is n⋆-closed if and only if it is*
(1) $\mathcal{L}$-n$\mathcal{I}$-LC-set and n$\mathcal{I}_g$-closed [1].
(2) μ-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{g\mu}$-closed.

Proof. *(2) Necessity is trivial. We prove only sufficiency. Let A be μ -n$\mathcal{I}$-LC-set and n$\mathcal{I}_{g\mu}$-closed set. Since A is μ -n$\mathcal{I}$-LC set, A = O ∩ P , where O is nμ -open and P is n⋆-closed. So we have A = O ∩ P ⊆ O. Since A is n$\mathcal{I}_{g\mu}$-closed, A^*_n⊆ O. Also since A = O ∩ P ⊆ P and P is n⋆-closed, we have A^*_n⊆ P. Consequently, A^*_n⊆ O ∩ P = A and hence A is n⋆-closed.* □

Remark 3.7. (1) The notions of $\mathcal{L}$-n$\mathcal{I}$-LC set and n$\mathcal{I}_g$-closed set are independent[1].
(2) The notions of μ-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{g\mu}$-closed set are independent.

Example 3.8. Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 2.6. Then μ-n$\mathcal{I}$-LC-sets are ϕ, K, {4}, {5}, {6}, {4, 5}, {4, 6}. It is clear that {5} is μ-n$\mathcal{I}$-LC- set but it is not n$\mathcal{I}_{g\mu}$-closed. Also it is clear that {5, 6} is an n$\mathcal{I}_{g\mu}$-closed but it is not μ-n$\mathcal{I}$-LC set.

Definition 3.9. [3] Let A be a subset of a nano topological space (K, $\mathcal{N}$). Then the Nano μ-kernel of the set A, denoted by nμ-ker(A), is the intersection of all nμ-open supersets of A.

Definition 3.10. A subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called n$\wedge_\mu$-set if A = nμ-ker(A).

Definition 3.11. A subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called ζ_μ-n$\mathcal{I}$-closed if A = R ∩ S where R is a n$\wedge_\mu$-set and S is a n⋆-closed.

Lemma 3.12. *(1) Every n$\star$-closed set is ζ_μ-n$\mathcal{I}$-closed but not conversely.*
(2) Every n$\wedge_\mu$-set is ζ_μ-n$\mathcal{I}$-closed but not conversely.

Proof. *(1) Follows from Definitions 1.6 and 3.11.*
(2) Follows from Definitions 3.10 and 3.11. □

Example 3.13. Let K , $\mathcal{N}$ and $\mathcal{I}$ be as in the Example 2.3, n$\star$-closed sets are ϕ, K, $\{4\}$, $\{5, 6\}$, ζ_μ-n$\mathcal{I}$-closed sets are power set of K and n$\wedge_\mu$-sets are ϕ, K, $\{4\}$, $\{5\}$, $\{6\}$, $\{4, 5\}$, $\{4, 6\}$. It is clear that $\{5\}$ is ζ_μ-n$\mathcal{I}$-closed but it is not n$\star$-closed. Also it is clear that $\{5, 6\}$ is ζ_μ-n$\mathcal{I}$-closed but it is not n$\wedge_\mu$-set.

Remark 3.14. The concepts of n$\star$-closed and n$\wedge_\mu$-set are independent.

Example 3.15. Let K , $\mathcal{N}$ and $\mathcal{I}$ be as in the Example 3.4, n$\wedge_\mu$-set are ϕ, K, $\{4\}$, $\{5\}$, $\{4, 5\}$ and n$\star$-closed sets are ϕ, K, $\{6\}$. It is clear that $\{4\}$ is n$\wedge_\mu$-set but it is not n$\star$-closed. Also it is clear that $\{6\}$ is n$\star$-closed set but it is not n$\wedge_\mu$-set.

Lemma 3.16. *For a subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) the following are equivalent.*
(1) A is ζ_μ-n$\mathcal{I}$-closed.
(2) A = O $\cap$ n-cl(A) where O is a n$\wedge_\mu$-set.*
(3) A= nμ-ker(A) $\cap$ n-cl(A).*

Proof. *(1) $\Rightarrow$ (2). Let A be a ζ_μ-n$\mathcal{I}$-closed set. Then A = O $\cap$ P where O is a ζ_μ-n$\mathcal{I}$-closed set and P is a n$\star$-closed. Clearly A $\subseteq$ O $\cap$ n-cl*(A). Since P is a n$\star$-closed, n-cl*(A) $\subseteq$ n-cl*(P) = P and so O $\cap$ n-cl*(A) $\subseteq$ O $\cap$ P = A. Therefore, A = O $\cap$ n-cl*(A).*
(2) $\Rightarrow$ (3). Let A = O $\cap$ n-cl(A), where O is a n$\wedge_\mu$-set. Since O is a n$\wedge_\mu$-set, we have A= nμ-ker(A) $\cap$ n-cl*(A).*
(3) $\Rightarrow$ (1). Let A= nμ-ker(A) $\cap$ n-cl(A). By Definitions 3.10, 3.11 and the notion of n$\star$-closed set, we get A is ζ_μ-n$\mathcal{I}$-closed.* □

Lemma 3.17. *A subset A $\subseteq$ (K, $\mathcal{N}$, $\mathcal{I}$) is n$\mathcal{I}_{g\mu}$-closed if and only if n-cl*(A) $\subseteq$ nμ-ker(A).*

Proof. *Suppose that A $\subseteq$ K is an n$\mathcal{I}_{g\mu}$-closed set. Suppose k $\notin$ nμ-ker(A). Then there exists an nμ-open set U containing A such that k $\notin$ U. Since A is an n$\mathcal{I}_{g\mu}$-closed set, A $\subseteq$ U and U is nμ-open implies that n-cl*(A) $\subseteq$ U and so k $\notin$ n-cl*(A). Therefore n-cl*(A) $\subseteq$ nμ-ker(A).*
Conversely, suppose n-cl(A) $\subseteq$ nμ-ker(A). If A $\subseteq$ U and U is nμ-open, then n-cl*(A) $\subseteq$ nμ-ker(A) $\subseteq$ U . Therefore, A is n$\mathcal{I}_{g\mu}$-closed.* □

Theorem 3.18. *For a subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) the following are equivalent.*
(1) A is n$\star$-closed.
(2) A is n$\mathcal{I}_{g\mu}$-closed and μ-n$\mathcal{I}$-LC.
(3) A is n$\mathcal{I}_{g\mu}$-closed and ζ_μ-n$\mathcal{I}$-closed.

Proof. *(1) $\Rightarrow$ (2) $\Rightarrow$ (3) Obvious.*
(3) $\Rightarrow$ (1). Since a is n$\mathcal{I}_{g\mu}$-closed, by (2), Lemma 3.17, n-cl(A) $\subseteq$ nμ-ker(A). Since A is ζ_μ-n$\mathcal{I}$-closed, by Lemma 3.16, A = nμ-ker(A) $\cap$ n-cl*(A)= n-cl*(A). Hence A is n$\star$-closed.* □

Remark 3.19. The concepts of n$\mathcal{I}_{g\mu}$-closedness and ζ_μ-n$\mathcal{I}$-closedness are independent.

Example 3.20. Let K , $\mathcal{N}$ and $\mathcal{I}$ be as in the Example 3.4, ζ_μ-n$\mathcal{I}$-closed sets are ϕ, K, $\{4\}$, $\{5\}$, $\{6\}$, $\{4, 5\}$ and n$\mathcal{I}_{g\mu}$-closed sets are ϕ, K, $\{6\}$, $\{4, 6\}$, $\{5, 6\}$. It is clear that $\{4\}$ is ζ_μ-n$\mathcal{I}$-closed but it is not n$\mathcal{I}_{g\mu}$-closed. Also it is clear that $\{4, 6\}$ is n$\mathcal{I}_{g\mu}$-closed set but it is not ζ_μ-n$\mathcal{I}$-closed.

4 Decompositions of Nano $\star$-continuity

Definition 4.1. A function f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) is said to be n$\star$-continuous [4] (resp. n$\mathcal{I}_g$-continuous, n$\mathcal{I}_{g\mu}$-continuous, $\mathcal{L}$-n$\mathcal{I}$-LC-continuous, μ-n$\mathcal{I}$-LC-continuous, ζ_μ-n$\mathcal{I}$-continuous) if f^{-1}(A) is n$\star$-closed (resp. n$\mathcal{I}_g$-closed, n$\mathcal{I}_{g\mu}$-closed, $\mathcal{L}$-n$\mathcal{I}$-LC-set, μ-n$\mathcal{I}$-LC-set, ζ_μ-n$\mathcal{I}$-closed) in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$).

Theorem 4.2. *A function f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) is n$\star$-continuous if and only if it is*
(1) $\mathcal{L}$-n$\mathcal{I}$-LC-continuous and n$\mathcal{I}_g$-continuous.
(2) μ-n$\mathcal{I}$-LC-continuous and n$\mathcal{I}_{g\mu}$-continuous.

Proof. *It is an immediate consequence of Theorem 3.6.* □

Theorem 4.3. *A function f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) the following are equivalent.*
(1) f is n$\star$-continuous.
(2)f is n$\mathcal{I}_{g\mu}$–continuous and μ-n$\mathcal{I}$-LC-continuous.
(3) f is n$\mathcal{I}_{g\mu}$-continuous and ζ_μ-n$\mathcal{I}$-continuous.

Proof. *It is an immediate consequence of Theorem 3.18.* □

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Author information

Selvaraj Ganesan, Assistant Professor, PG & Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu.
(Affiliated to Alagappa University, Karaikudi, Tamil Nadu.)
, India.
E-mail: sgsgsgsgsg77@gmail.com

Received: October 22, 2019. Accepted : December 27, 2019