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ON SKEW-RANDIĆ ENERGY OF DIGRAPHS

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Abstract In this paper we compute the skew-Randić energy of the star graph, the $(S_m \wedge P_2)$ graph and the Peterson graph.

1 Introduction

In 2010, Burcu Bozkurt, Dilek Güngör, Gutman and Sinan Çevik [2], have introduced the Randić energy of a graph as follows. Let G be a simple graph and let $v_1, v_2, \ldots, v_n$ be its vertices. For $i = 1, 2, \ldots, n$, let d_i denote the degree of the vertex v_i. Then the Randić matrix of G is defined as $R = (R_{ij})$, where

$$R_{ij} = \begin{cases} 0, & \text{if } i = j, \\ \frac{1}{\sqrt{d_i d_j}}, & \text{if the vertices } v_i \text{ and } v_j \text{ are adjacent,} \\ 0, & \text{if the vertices } v_i \text{ and } v_j \text{ are not adjacent.} \end{cases}$$

The Randić energy of G is defined as the sum of absolute values of the eigenvalues of the Randić matrix.

In the same year Adiga, Balakrishnan and Wasin So [1] have introduced the skew energy of a digraph as follows. Let D be a digraph of order n with vertex set $V(D) = \{v_1, v_2, \ldots, v_n\}$ and arc set $\Gamma(D) \subset V(D) \times V(D)$ where $(v_i, v_i) \notin \Gamma(D)$ for all i and $(v_i, v_j) \in \Gamma(D)$ implies $(v_j, v_i) \notin \Gamma(D)$. The skew-adjacency matrix of D is the $n \times n$ matrix $S(D) = (s_{ij})$ where $s_{ij} = 1$ whenever $(v_i, v_j) \in \Gamma(D)$, $s_{ij} = -1$ whenever $(v_j, v_i) \in \Gamma(D)$ and $s_{ij} = 0$ otherwise. Hence $S(D)$ is a skew symmetric matrix of order n and all its eigenvalues are of the form $i\lambda$ where $i = \sqrt{-1}$ and λ is a real number. The skew energy of G is the sum of the absolute values of eigenvalues of $S(D)$.

Motivated by these works, in 2013, D. D. Somashekara and C. R. Veena [6],[7] have introduced the concept of skew-Randić energy of a digraph as follows. Let D be a digraph of order n with vertex set $V(D) = \{v_1, v_2, \ldots, v_n\}$ and arc set $\Gamma(D) \subset V(D) \times V(D)$ where $(v_i, v_i) \notin \Gamma(D)$ for all i and $(v_i, v_j) \in \Gamma(D)$ implies $(v_j, v_i) \notin \Gamma(D)$. Then the skew-Randić matrix of D is the $n \times n$ matrix $A_{SR} = (r_{ij})$ where

$$r_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}}, & \text{if } (v_i, v_j) \in \Gamma(D), \\ -\frac{1}{\sqrt{d_i d_j}}, & \text{if } (v_j, v_i) \in \Gamma(D), \\ 0, & \text{otherwise.} \end{cases}$$

Then the skew-Randić energy of D was defined as the sum of the absolute values of eigenvalues of A_{SR}. Further, they [6] found the skew-Randić energy of complete bipartite graph, crown graph and $K_2 + \overline{K_{n-2}}$ graph. Recently, Ran Gu, Fie Huang, Xueliang Li [3] have studied the properties of digraphs and in particular the bounds for skew-Randić energy of certain connected graphs.

In this paper we find skew-Randić energy of the star graph, $(S_m \wedge P_2)$ graph and the Peterson graph.

We begin with some basic definitions and notations.

Definition 1.1. [4] A graph G is said to be complete if every pair of its distinct vertices are adjacent. A complete graph on n vertices is denoted by K_n.

Definition 1.2. [4] A bigraph or bipartite graph G is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every line of G joins V_1 with V_2. (V_1, V_2) is a bipartition of G. If G contains every line joining V_1 and V_2, then G is a complete bigraph. If V_1 and V_2 have m and n points, we write $G = K_{m,n}$. A star is a complete bigraph $K_{1,n}$.

Definition 1.3. [5] The conjunction $(S_m \wedge P_2)$ of $S_m = \overline{K}_m + K_1$ and P_2 is the graph having the vertex set $V(S_m) \times V(P_2)$ and edge set $\{(v_i, v_j)(v_k, v_l) | v_i v_k \in E(S_m)\ and\ v_j v_l \in E(P_2)\ and\ 1 \le i, k \le m+1, 1 \le j, l \le 2\}$.

2 Main Results

Theorem 2.1. *Let the vertex set $V(D)$ and arc set $\Gamma(D)$ of S_n star digraph be respectively given by*

$$V(D) = \{v_1, v_2, \ldots, v_n\}\ \textit{and}\ \Gamma(D) = \{(v_1, v_j) \,|\, 2 \le j \le n\}.$$

Then the skew-Randić energy of D is 2.

Proof. The skew-Randić matrix of the star digraph D is given by

$$A_{SR} = \begin{pmatrix} 0 & \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} & \cdots & \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} \\ -\frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ -\frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \\ -\frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

Hence the characteristic polynomial is given by

$$|\lambda I - A_{SR}| = \begin{vmatrix} \lambda & -\frac{1}{\sqrt{n-1}} & -\frac{1}{\sqrt{n-1}} & \cdots & -\frac{1}{\sqrt{n-1}} \\ \frac{1}{\sqrt{n-1}} & \lambda & 0 & \cdots & 0 \\ \frac{1}{\sqrt{n-1}} & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & \lambda \end{vmatrix}$$

$$= \left(\frac{1}{\sqrt{n-1}}\right)^n \begin{vmatrix} \mu & -1 & -1 & \cdots & -1 & -1 \\ 1 & \mu & 0 & \cdots & 0 & 0 \\ 1 & 0 & \mu & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & \mu & 0 \\ 1 & 0 & 0 & \cdots & 0 & \mu \end{vmatrix},$$

where $\mu = \lambda\sqrt{n-1}$. Then $|\lambda I - A_{SR}| = \phi_n(\mu)(\frac{1}{\sqrt{n-1}})^n$

where $\phi_n(\mu)= \begin{vmatrix} \mu & -1 & -1 & \cdots & -1 & -1 \\ 1 & \mu & 0 & \cdots & 0 & 0 \\ 1 & 0 & \mu & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & \mu & 0 \\ 1 & 0 & 0 & \cdots & 0 & \mu \end{vmatrix}.$

Using the properties of the determinants, after some simplifications we obtain

$$\phi_n(\mu) = (\mu^{n-2} + \mu\phi_{n-1}(\mu)).$$

Iterating this, we obtain

$$\phi_n(\mu) = \mu^{n-2}(\mu^2 + n - 1).$$

Therefore

$$|\lambda I - A_{SR}| = \left(\frac{1}{\sqrt{n-1}}\right)^n \left[\left((n-1)\lambda^2 + (n-1)\right)(\lambda\sqrt{(n-1)})^{n-2}\right].$$

Thus the characteristic equation is given by

$$\lambda^{n-2}\left(\lambda^2 + 1\right) = 0.$$

Hence

$$Spec\,(D) = \begin{pmatrix} 0 & i & -i \\ n-2 & 1 & 1 \end{pmatrix}.$$

Hence the skew-Randić energy of D is

$$E_{SR}(D) = 2.$$

□

Theorem 2.2. *Let the vertex set $V(D)$ and arc set $\Gamma(D)$ of $(S_m \wedge P_2)$ digraph be respectively given by*

$$V(D) = \{v_1, v_2, \ldots, v_{2m+2}\} \text{ and}$$
$$\Gamma(D) = \{(v_1, v_j), (v_{m+2}, v_k) \,|\, 2 \le j \le m+1, m+3 \le k \le 2m+2\}.$$

Then the skew-Randić energy of D is 4.

Proof. The skew-Randić matrix of $(S_m \wedge P_2)$ digraph is given by

$$A_{SR} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & \frac{1}{\sqrt{n-1}} & \cdots & \frac{1}{\sqrt{n-1}} \\ 0 & 0 & \cdots & 0 & -\frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & -\frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{n-1}} & \cdots & \frac{1}{\sqrt{n-1}} & 0 & 0 & \cdots & 0 \\ -\frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{2n\times 2n},$$

where $m + 1 = n$. Its characteristic polynomial is given by

$$|\lambda I - A_{SR}| = \begin{vmatrix} \lambda & 0 & \cdots & 0 & 0 & -\frac{1}{\sqrt{n-1}} & \cdots & -\frac{1}{\sqrt{n-1}} \\ 0 & \lambda & \cdots & 0 & \frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda & \frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 \\ 0 & -\frac{1}{\sqrt{n-1}} & \cdots & -\frac{1}{\sqrt{n-1}} & \lambda & 0 & \cdots & 0 \\ \frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{n-1}} & 0 & \cdots & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix}_{2n\times 2n}.$$

Hence the characteristic equation is given by

$$\left(\frac{1}{\sqrt{n-1}}\right)^{2n}\begin{vmatrix}\Lambda & 0 & \cdots & 0 & 0 & -1 & \cdots & -1\\ 0 & \Lambda & \cdots & 0 & 1 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Lambda & 1 & 0 & \cdots & 0\\ 0 & -1 & \cdots & -1 & \Lambda & 0 & \cdots & 0\\ 1 & 0 & \cdots & 0 & 0 & \Lambda & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & \Lambda\end{vmatrix}_{2n\times 2n} = 0,$$

$$where\ \Lambda = \sqrt{n-1}\lambda.$$

Let

$$\phi_{2n}(\Lambda) = \begin{vmatrix}\Lambda & 0 & 0 & \cdots & 0 & 0 & -1 & -1 & \cdots & -1\\ 0 & \Lambda & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & \Lambda & \cdots & 0 & 1 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \Lambda & 1 & 0 & 0 & \cdots & 0\\ 0 & -1 & -1 & \cdots & -1 & \Lambda & 0 & 0 & \cdots & 0\\ 1 & 0 & 0 & \cdots & 0 & 0 & \Lambda & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & \Lambda\end{vmatrix}_{2n\times 2n}.$$

$$= (-1)^{2n+2n}\Lambda\begin{vmatrix}\Lambda & 0 & 0 & \cdots & 0 & 0 & -1 & -1 & \cdots & -1\\ 0 & \Lambda & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & \Lambda & \cdots & 0 & 1 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \Lambda & 1 & 0 & 0 & \cdots & 0\\ 0 & -1 & -1 & \cdots & -1 & \Lambda & 0 & 0 & \cdots & 0\\ 1 & 0 & 0 & \cdots & 0 & 0 & \Lambda & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & \Lambda\end{vmatrix}_{(2n-1)\times(2n-1)}$$

$$+(-1)^{2n+1}\begin{vmatrix}0 & 0 & 0 & \cdots & 0 & 0 & -1 & \cdots & -1 & -1\\ \Lambda & 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0\\ 0 & \Lambda & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \Lambda & -1 & 0 & \cdots & 0 & 0\\ -1 & -1 & -1 & \cdots & -1 & \Lambda & 0 & \cdots & 0 & 0\\ 0 & 0 & 0 & \cdots & 0 & 0 & \Lambda & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \Lambda & 0\end{vmatrix}_{(2n-1)\times(2n-1)}.$$

Let

$$\Psi_{2n-1}(\Lambda)=(-1)^{2n+1}\begin{vmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & -1 & \cdots & -1 & -1 \\ \Lambda & 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 \\ 0 & \Lambda & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \Lambda & -1 & 0 & \cdots & 0 & 0 \\ -1 & -1 & -1 & \cdots & -1 & \Lambda & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \Lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \Lambda & 0 \end{vmatrix}_{(2n-1)\times(2n-1)}$$

Using the properties of the determinants, we obtain, after some simplifications

$$\Psi_{2n-1}(\Lambda)=\Lambda^{n-2}\Theta_n(\Lambda),$$

where $\Theta_n(\Lambda)=\begin{vmatrix} \Lambda & 0 & 0 & \cdots & 1 \\ 0 & \Lambda & 0 & \cdots & 1 \\ 0 & 0 & \Lambda & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & \Lambda \end{vmatrix}_{n\times n}$

Then

$$\phi_{2n}(\Lambda)=\Lambda^{n-2}\Theta_n(\Lambda)+\Lambda\phi_{2n-1}(\Lambda).$$

Now, proceeding as above, we obtain

$$\begin{aligned}\phi_{2n-1}(\Lambda) &= (-1)^{(2n-1)+1}\Psi_{2n-2}(\Lambda)+(-1)^{(2n-1)+(2n-1)}\Lambda\phi_{2n-2}(\Lambda)\\ &= \Lambda^{n-3}\Theta_n(\Lambda)+\Lambda\phi_{2n-2}(\Lambda).\end{aligned}$$

Proceeding like this, we obtain at the $(n-1)^{th}$ step

$$\phi_{2n}(\Lambda)=-(n-1)\Lambda^{n-2}\Theta_n(\Lambda)+\Lambda^{(n-1)}\xi_{n+1}(\Lambda),$$

where $\xi_{n+1}(\Lambda)=\begin{vmatrix} \Lambda & 0 & 0 & \cdots & 0 \\ 0 & \Lambda & 0 & \cdots & 1 \\ 0 & 0 & \Lambda & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -1 & -1 & \cdots & \Lambda \end{vmatrix}_{(n+1)\times(n+1)}$.

$$\begin{aligned}\phi_{2n}(\Lambda) &= (n-1)\Lambda^{n-2}\Theta_n(\Lambda)+\Lambda^{n-1}\Lambda\Theta_n(\Lambda)\\ &= (n-1)\Lambda^{n-2}\Theta_n(\Lambda)+\Lambda^{n}\Theta_n(\Lambda)\\ &= ((n-1)\Lambda^{n-2}+\Lambda^n)\Theta_n(\Lambda).\end{aligned}$$

Using the properties of the determinants, we obtain

$$\Theta_n(\Lambda)=(n-1)\Lambda^{n-2}+\Lambda^n.$$

Therefore

$$\phi_{2n}(\Lambda)=((n-1)\Lambda^{n-2}+\Lambda^n)^2.$$

Hence characteristic equation becomes

$$(\frac{1}{\sqrt{n-1}})^{2n}\phi_{2n}(\Lambda)=0,$$

which is same as

$$(\frac{1}{\sqrt{n-1}})^{2n}((n-1)\Lambda^{n-2}+\Lambda^{n})^{2}=0.$$

This reduces to

$$\lambda^{2n-4}(1+\lambda^{2})^{2}=0.$$

Therefore

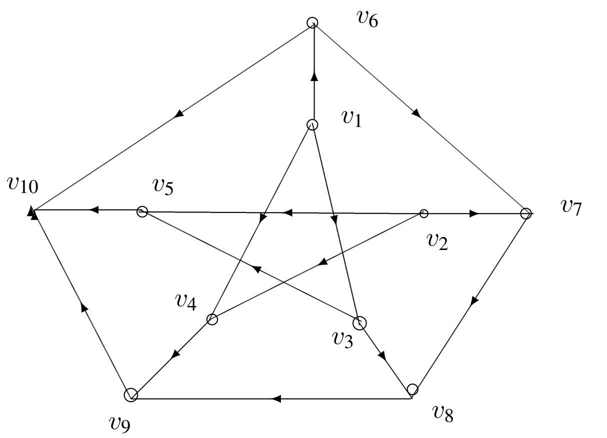
$$Spec\,((S_m \wedge P_2)) = \begin{pmatrix} 0 & i & -i \\ 2n-4 & 2 & 2 \end{pmatrix}.$$

Hence the skew-Randić energy of $(S_m \wedge P_2)$ digraph is

$$E_{SR}((S_m \wedge P_2)) = 4.$$

□

Theorem 2.3. *The skew-Randić energy of the below Peterson digraph is* 5.1654.



Proof. The skew-Randić matrix of the Peterson of digraph is given by

$$A_{SR} = \begin{pmatrix}
0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\
-\frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\
-\frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\
0 & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\
-\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\
0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\
0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} \\
0 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & 0
\end{pmatrix}.$$

Its characteristic polynomial is given by

$$|\lambda I - A_{SR}| = \begin{vmatrix} \lambda & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & \lambda & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & 0 & \lambda & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \lambda & 0 & 0 & 0 & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & 0 & 0 & 0 & \lambda & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} \\ 0 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \lambda \end{vmatrix}.$$

Hence the characteristic equation is given by

$$\begin{vmatrix} \lambda & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & \lambda & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & 0 & \lambda & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \lambda & 0 & 0 & 0 & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & 0 & 0 & 0 & \lambda & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & \lambda & \frac{1}{3} \\ 0 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \lambda \end{vmatrix} = 0.$$

This is same as

$$\lambda^{10} + 1.6667\lambda^8 + 0.9259\lambda^6 + 0.1989\lambda^4 + 0.0152\lambda^2 + 0.003 = 0,$$

on using Matlab software. Again, using the software we found that the eigenvalues of A_{SR} as

$0.8676i, -0.8676i, 0.7328i, -0.7328i, 0.4906i, -0.4906i, 0.1583i, -0.1583i, 0.3333i, -0.3333i$

of multiplicities one. Hence the skew-Randić energy of D is

$$E_{SR}(D) = 5.1654.$$

□

Remark. The skew-Randić energy of a digraph depends on the arc set. In particular, if the arc set of the above Peterson graph is $\Gamma(D)$=$\{(v_i, v_j), (v_3, v_1), (v_4, v_1) \mid 1 \leq i, j \leq 10, i < j\ and\ if\ j = 1, i \neq 3, 4\}$, then its skew-Randić energy is 5.1654.

Conjecture. We conjecture that the skew-Randić energy of the Peterson digraph with any arc set lies between 3 and 6.

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