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Some Characterizations of $\alpha-$Cosymplectic Manifolds Admitting Yamabe Solitons

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Abstract In this paper, we deal with $\alpha-$cosymplectic manifolds. Firstly, we give a characterization for $\alpha-$cosymplectic manifold admitting a Ricci soliton given as to be $\eta-$Einstein and nearly quasi-Einstein. Also, we study yamabe solitons in $\alpha-$cosymplectic manifold and obtain some important classifications about scalar curvature of this manifold. Finally, we find that if an $\alpha-$cosymplectic manifold is conharmonically flat, it is an $\eta-$Einstein.

1 Introduction

The notion of Ricci soliton is a natural generalization of an Einstein manifold (the Ricci tensor S is a constant multiple of the Riemannian metric g). This notion was introduced by Hamilton in 1988 [9]. A Riemannian manifold (M, g) is called a Ricci soliton if

$$(\pounds_V g)(X, Y) + 2S(X, Y) + 2\mu g(X, Y) = 0 \tag{1.1}$$

is satisfied. Here, $\pounds_V g$ denotes the Lie-derivative of the metric tensor g, S is the Ricci tensor of M, μ is a constant and X, Y are the vector fields on M. If $\pounds_V g = 0$ and $\pounds_V g = \rho g$, then potential vector field V is said to be Killing and conformal Killing, respectively, where ρ is a function. Also, when V is zero or Killing in (1.1), then Ricci soliton reduces to Einstein manifold. In addition, a Ricci soliton is called gradient if the potential vector field V is the gradient of a potential function $-f$ (i.e., $V = -\nabla f$) and is called shrinking, steady or expanding depending on $\mu < 0, \mu = 0$ or $\mu > 0$, respectively. There are many studies about Ricci solitons in literature [10], [14], [15], [18], [20] and many others.

A Yamabe soliton is a Riemannian manifold (M, g) if it admits a vector field V such that

$$(\pounds_V g)(X, Y) = (\lambda - r)g(X, Y), \tag{1.2}$$

where λ is a real number and r is the scalar curvature of M [9]. A yamabe soliton which satisfies (1.2) is denoted by (M, g, V, λ). Yamabe solitons correspond to the self-similar solutions of the yamabe flow. A Yamabe soliton is called a gradient if the potential vector field V is the gradient of a potential function β on M and is called shrinking, steady or expanding depending on $\lambda < 0, \lambda = 0$ or $\lambda > 0$, respectively.

The first study of almost cosymplectic structures were introduced by Goldberg and Yano in [8]. The products of almost Kaehler manifolds and the real R line or the circle S^1 are the simplest examples of almost cosymplectic manifolds. Almost cosymplectic manifolds have been studied by many mathematicians in literature ([1], [7], [13], [16] and [17]).

The notion of conharmonic curvature tensor was firstly defined by Ishii in 1957 [12]. Since then, this notion has been studied in some different classes of contact geometry. For example, Çalışkan studied this tensor in Sasakian finsler manifolds [3]. Also, Dwivedi and Kim obtained some necessary and sufficient conditions for $K-$contact manifold to be quasi conharmonically flat, $\xi-$conharmonically flat [6]. Ghosh et al. characterized $N(k)-$contact metric manifolds satisfying certain curvature conditions on the conharmonic curvature tensor [11]. Taleshian et al. proved that $\phi-$conharmonically flat $LP-$Sasakian manifold is an $\eta-$Einstein manifold in [19].

The present paper is organized as follows. In section 2, we give some fundamental definitions and formulas about almost contact metric manifolds. In section 3, we study $\alpha-$cosymplectic manifolds admitting Ricci solitons. In section 4, we analyze $\alpha-$cosymplectic manifolds admitting yamabe solitons and investigate Riemannian manifolds admitting yamabe solitons endowed with concircular vector field. In last section, we give some characterizations for an $\alpha-$cosymplectic manifold with conharmonic curvature tensor.

2 Preliminaries

In this section, we shall recall some basic definitions and formulas of almost contact metric manifolds from [1], [2] and [8].

A $(2n+1)-$dimensional differentiable manifold M is called an almost contact metric manifold if there exists an almost contact metric structure (φ, ξ, η, g) on M and the Riemannian metric g satisfies the following relations:

$$\varphi^2 X = -X + \eta(X)\xi, \ \ \eta(\xi) = 1, \ \ \varphi\xi = 0, \ \ \eta \circ \varphi = 0, \ \ \eta(X) = g(X, \xi) \tag{2.1}$$

and

$$g(\varphi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \ \ g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.2}$$

for any $X, Y \in \Gamma(TM)$, where ξ is a vector field of type $(0, 1)$, (which is so-called the characteristic vector field), $1-$ form η is the $g-$dual of ξ of type $(1, 0)$ and φ is a tensor field of type $(1, 1)$ on M.

On the other hand, in [2], D.E. Blair defined the fundamental $2-$form Φ of M as follows:

$$\Phi(X, Y) = g(X, \varphi Y)$$

for any $X, Y \in \Gamma(TM)$. Furthermore, if the relation

$$\Phi(X, Y) = d\eta(X, Y)$$

holds for all $X, Y \in \Gamma(TM)$, an almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is said to be contact metric manifold such that

$$d\eta(X, Y) = \frac{1}{2}\Big\{X\eta(Y) - Y\eta(X) - \eta([X, Y])\Big\}.$$

The Nijenhuis tensor field of φ is defined by

$$N_\varphi(X, Y) = [\varphi X, \varphi Y] + \varphi^2[X, Y] - \varphi[X, \varphi Y] - \varphi[\varphi X, Y]$$

for all $X, Y \in \Gamma(TM)$. If M is an almost contact metric manifold and the Nijenhuis tensor of φ satisfies

$$N_\varphi + 2d\eta \otimes \xi = 0$$

then, M is called a normal contact metric manifold. A normal contact metric manifold $(M, \varphi, \xi, \eta, g)$ is said to be cosymplectic if the following relations hold:

$$d\eta = 0, \qquad d\Phi = 0.$$

Equivalently,

$$(\nabla_X \varphi)Y = 0, \quad \nabla_X \xi = 0 \tag{2.3}$$

for any $X, Y \in \Gamma(TM)$, where ∇ is the Levi-Civita connection on M. Also, if

$$d\eta = 0, \quad d\Phi = 2\alpha\eta \wedge \Phi$$

are satisfied, then M is called an $\alpha-$cosypmlectic manifold, where α is a real number. Equivalently,

$$(\nabla_X \varphi)Y = \alpha(g(\varphi X, Y)\xi - \eta(Y)\varphi X), \quad (2.4)$$
$$\nabla_X \xi = -\alpha\varphi^2 X. \quad (2.5)$$

If α is equal to zero, then it is easy to see that M is a cosypmlectic manifold. For $\alpha \in \mathbb{R}$, when $\alpha \neq 0$, M is called $\alpha-$Kenmotsu manifold. For an $\alpha-$cosymplectic manifold, we also have

$$R(X,Y)\xi = \alpha^2(\eta(X)Y - \eta(Y)X), \quad (2.6)$$
$$R(X,\xi)Y = \alpha^2(g(X,Y)\xi - \eta(Y)X), \quad (2.7)$$
$$R(X,\xi)\xi = \alpha^2(\eta(X)\xi - X), \quad (2.8)$$
$$S(X,\xi) = -2n\alpha^2\eta(X), \quad (2.9)$$

where S, R denotes the Ricci tensor and Riemann curvature tensor, respectively.

Now, we recal some definitions from [4], [5], [6] and [12] as follows:

The conharmonic curvature tensor of a $(2n+1)-$dimensional $(n \geq 1)$ manifold (M, g) is defined by

$$K(X,Y)Z = R(X,Y)Z - \frac{1}{2n-1}\Big\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\Big\} \quad (2.10)$$

for any $X, Y, Z \in \Gamma(TM)$, where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$.

If K vanishes identically in (2.10), the manifold M is called conharmonically flat. Also, an almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is called $\xi-$conharmonic flat and quasi conharmonically flat, respectively if the followings are satisfied:

$$K(X,Y)\xi = 0,$$

and

$$g(K(X,Y)Z, \varphi W) = 0, \quad (2.11)$$

for any $X, Y, Z, W \in \Gamma(TM)$.

A Riemannian manifold (M, g) is called nearly quasi-Einstein manifold if its Ricci tensor field S satisfies

$$S = ag + bE$$

where a, b are functions and E is a non-vanishing symmetric $(0,2)-$tensor on M. In addition, (M, g) is called an $\eta-$Einstein if there exists two real constants c and d such that the Ricci tensor field S satisfies

$$S = cg + d\eta \otimes \eta.$$

If the constant d is equal to zero, then M becomes Einstein.

On the other hand, a vector field v on a Riemannian manifold (M, g) is called concircular if it satisfies

$$\nabla_X v = fX \quad (2.12)$$

for any $X \in \Gamma(TM)$, where ∇ is the Levi-Civita connection on M and f is a smooth function on M. If f is equal to one in (2.12), then v is called a concurrent vector field.

3 $\alpha-$Cosymplectic Manifolds Admitting Ricci Solitons

In this section, we deal with $\alpha-$cosymplectic manifolds admitting Ricci solitons.

Now, we begin to this section with the following:

Theorem 3.1. *Let M be an $\alpha-$cosymplectic manifold admitting a Ricci soliton. If the potential vector field V is a pointwise collinear with ξ, then M is a nearly quasi-Einstein manifold.*

Proof. Since (M, g, V, μ) is a Ricci soliton whose potential vector field V is a pointwise collinear with ξ, that is, $V = b\xi$, from (2.1) and (2.5), we have

$$\begin{aligned}(\pounds_V g)(X,Y) &= g(\nabla_X V, Y) + g(\nabla_Y V, X)\\ &= g(X(b)\xi + b\nabla_X\xi, Y) + g(Y(b)\xi + b\nabla_Y\xi, X)\\ &= X(b)\eta(Y) + Y(b)\eta(X) + 2b\alpha(g(X,Y) - \eta(X)\eta(Y))\\ &= g(\nabla b, X)\eta(Y) + g(\nabla b, Y)\eta(X)\\ &\quad +2b\alpha(g(X,Y) - \eta(X)\eta(Y))\end{aligned} \tag{3.1}$$

for any $X, Y \in \Gamma(TM)$, where ∇b is the gradient of a smooth fuction b on M.

By combining (1.1) and (3.1), we have

$$\begin{aligned}S(X,Y) &= -(\mu + b\alpha)g(X,Y) + b\alpha\eta(X)\eta(Y)\\ &\quad -\frac{1}{2}\Big\{g(\nabla b, X)\eta(Y) + g(\nabla b, Y)\eta(X)\Big\}.\end{aligned} \tag{3.2}$$

If we denote the the dual $1-$form of ∇b by ϕ, then (3.2) reduces to

$$\begin{aligned}S(X,Y) &= -(\mu + b\alpha)g(X,Y) + b\alpha\eta(X)\eta(Y)\\ &\quad -\frac{1}{2}\Big\{\phi(X)\eta(Y) + \phi(Y)\eta(X)\Big\}.\end{aligned} \tag{3.3}$$

If we take a non-vanishing symmetric $(0,2)-$tensor E in (3.3) such that

$$E(X,Y) = b\alpha\eta(X)\eta(Y) - \frac{1}{2}(\phi(X)\eta(Y) + \phi(Y)\eta(X))$$

then, the equation (3.3) becomes

$$S = -(\mu + b\alpha)g + E.$$

This means that M is a nearly quasi-Einstein manifold. □

As an immediate consequence of Theorem 3.1, we have the following corollary:

Corollary 3.2. *Let M be an $\alpha-$cosymplectic manifold admitting a Ricci soliton whose the potential vector field is the characteristic vector field ξ. Then, M is an $\eta-$Einstein manifold.*

4 $\alpha-$Cosymplectic Manifolds Admitting Yamabe Solitons

The first result of this section is the following:

Proposition 4.1. *Let M be a Riemannian manifold admitting a gradient yamabe soliton as its potential vector field V. Then, V is a concircular vector field on M.*

Proof. Since M is a gradient yamabe soliton, the potential vector field v is the gradient ∇k of a function k on M. Then, from (1.2), we have

$$g(\nabla_X\nabla k, Y) + g(\nabla_Y\nabla k, X) = (\lambda - r)g(X,Y) \tag{4.1}$$

for any $X, Y \in \Gamma(TM)$. On the other hand, for a smooth function k on M, the Hessian H^k of k satisfies

$$H^k(X,Y) = g(\nabla_X\nabla k, Y). \tag{4.2}$$

Since the Hessian H^k of k is a symmetric in X and Y, it follows from (4.1) and (4.2), we derive

$$2g(\nabla_X \nabla k, Y) = (\lambda - r)g(X, Y)$$

which gives

$$\nabla_X \nabla k = \frac{1}{2}(\lambda - r)X.$$

This means that V is a concircular vector field on M. □

Next, we have the following proposition.

Proposition 4.2. *Let M be an $\alpha-$cosymplectic manifold endowed with a concircular vector field v. If M admits a yamabe soliton as its potential vector field v, then the scalar curvature r of M is given by*

$$r = \lambda - 2f,$$

where f is a function on M.

Proof. Since v is a concircular vector field on M, we have

$$\nabla_X v = fX \tag{4.3}$$

for any $X \in \Gamma(TM)$. Using the equality (4.3), one has

$$(\pounds_v g)(X, Y) = 2fg(X, Y). \tag{4.4}$$

Furthermore, from (1.2) and (4.4), we write

$$2fg(X, Y) = (\lambda - r)g(X, Y) \tag{4.5}$$

Substituting $X = Y = \xi$ in (4.5) yields

$$r = \lambda - 2f$$

which proves the propositon. □

As a result of the Proposition 4.2, we have the following:

Remark 4.3. Let M be an $\alpha-$cosymplectic manifold endowed with a concurrent vector field v. If M admits a yamabe soliton as its potential vector field v, then M has constant scalar curvature.

Proposition 4.4. *Let M be an $\alpha-$cosymplectic manifold. If M admits a yamabe soliton as its potential vector field ξ, then M is of constant scalar curvature cosymplectic manifold.*

Proof. For any vector fields $X, Y \in \Gamma(TM)$, it follows from the definition of the Lie-derivative and using (2.5), we have

$$\begin{aligned}(\pounds_\xi g)(X, Y) &= g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) \\ &= 2\alpha(g(X, Y) - \eta(X)\eta(Y)). \end{aligned} \tag{4.6}$$

Since M is a yamabe soliton and from (1.2) and (4.6) we get

$$2\alpha(g(X, Y) - \eta(X)\eta(Y)) = (\lambda - r)g(X, Y). \tag{4.7}$$

If we use $X = Y = \xi$ in equation (4.7), one has

$$r = \lambda$$

which implies that the manifold M has constant scalar curvature. Also, using this fact in (4.6) we get

$$2\alpha g(\varphi X, \varphi Y) = 0.$$

Since $g(\varphi X, \varphi Y) \neq 0$, we have $\alpha = 0$. This means that M is a cosymplectic manifold. Therefore, the proof is completed. □

Now, we shall give the main theorem of this section.

Theorem 4.5. *Let M be an $\alpha-$cosymplectic manifold admitting a yamabe soliton whose non-zero potential vector field V is a pointwise collinear with the structure vector field ξ. Then, M is a cosymplectic manifold if and only if the vector field V is a constant multiple of ξ.*

Proof. Let V be a pointwise collinear with the structure vector field ξ. That is, $V = b\xi$, where b is a smooth function on M. Then, from (1.2), we have

$$g(\nabla_X b\xi, Y) + g(\nabla_Y b\xi, X) = (\lambda - r)g(X, Y) \tag{4.8}$$

for any $X, Y \in \Gamma(TM)$. Also, if we use the equation (2.5) in (4.8), we get

$$X(b)\eta(Y) + Y(b)\eta(X) + 2b\alpha(g(X, Y) - \eta(X)\eta(Y)) = (\lambda - r)g(X, Y). \tag{4.9}$$

On the other hand, let $\{e_1, e_2, ..., e_{n+1} = \xi\}$ be an orthonormal basis of the tangent space T_pM, at each point $p \in M$. For $1 \leq i \leq 2n + 1$, if we take $X = Y = e_i$ in (4.9), one has

$$\xi(b) = \frac{1}{2}(\lambda - r)(2n + 1) - 2nb\alpha. \tag{4.10}$$

By putting $Y = \xi$ in equation (4.9) implies

$$X(b) = (\lambda - r - \xi(b))\eta(X). \tag{4.11}$$

Using $X = \xi$ in equation (4.11) gives

$$\xi(b) = \frac{1}{2}(\lambda - r). \tag{4.12}$$

Therefore, from (4.10) and (4.12), we get

$$\lambda = r + 2b\alpha. \tag{4.13}$$

If we replace (4.13) in (4.11) and use the equation (4.12), one has

$$X(b) = b\alpha\eta(X) \tag{4.14}$$

which yields

$$db = b\alpha\eta. \tag{4.15}$$

Now, if M is a cosymplectic manifold, then from (4.15), we have

$$db = 0$$

which means that b is a constant. Conversely, if the potential vector field V is a constant multiple of ξ, then from (4.14), we write

$$b\alpha\eta(X) = 0 \tag{4.16}$$

Setting $X = \xi$ in (4.16) gives

$$b\alpha = 0.$$

Since V is non-zero vector field, we get $\alpha = 0$. Then M becomes a cosymplectic manifold which completes the proof of the theorem. □

5 On Conharmanically Flat $\alpha-$Cosymplectic Manifolds

In this section, we consider an $\alpha-$cosymplectic manifold which satisfies the condition $K = 0$. Now, we shall give the the following.

Theorem 5.1. *Let $(M, \varphi, \xi, \eta, g)$ be an $\alpha-$cosymplectic manifold of dimension $(2n+1)$. If M is a conharmonically flat, then M is an $\eta-$Einstein manifold.*

Proof. Let us assume that an $\alpha-$ cosymplectic manifold M is conharmonically flat, that is, $K = 0$. If we take the inner product of (2.10) with U, we have

$$\begin{aligned} g(R(X,Y)Z,U) &= \frac{1}{2n-1}\Big\{S(Y,Z)g(X,U) - S(X,Z)g(Y,U) \\ &\quad +g(Y,Z)S(X,U) - g(X,Z)S(Y,U)\Big\} \end{aligned} \tag{5.1}$$

for any $X, Y, Z, U \in \Gamma(TM)$. Also, if we choose $Y = U = \xi$ and use (2.1), (2.7) and (2.9) in (5.1), we get

$$\begin{aligned} \alpha^2(g(X,Z) - \eta(Z)\eta(X)) &= \frac{1}{2n-1}\Big\{-4n\alpha^2\eta(Z)\eta(X) \\ &\quad +2n\alpha^2 g(X,Z) - S(X,Z)\Big\} \end{aligned}$$

which yields

$$S(X,Z) = \alpha^2 g(X,Z) - (2n+1)\alpha^2\eta(X)\eta(Z),$$

which shows that M is an $\eta-$Einstein manifold, which proves the theorem completely. □

Using the above theorem, we can give the following corollary:

Corollary 5.2. *Let $(M, \varphi, \xi, \eta, g)$ be an $\alpha-$cosymplectic manifold of dimension $(2n+1)$. If M is a $\xi-$conharmonic flat, then M is an $\eta-$Einstein manifold.*

The last result of this section is the following.

Theorem 5.3. *Let $(M, \varphi, \xi, \eta, g)$ be an $\alpha-$cosymplectic manifold of dimension $(2n+1)$. If M is a quasi conharmonically flat, then M is an $\eta-$Einstein manifold.*

Proof. Suppose that an $\alpha-$ cosymplectic manifold M is quasi conharmonically flat. If we take the inner product of (2.11) with φW, we get

$$\begin{aligned} g(R(X,Y)Z,\varphi W) &= \frac{1}{2n-1}\Big\{S(Y,Z)g(X,\varphi W) - S(X,Z)g(Y,\varphi W) \\ &\quad +g(Y,Z)S(X,\varphi W) - g(X,Z)S(Y,\varphi W)\Big\} \end{aligned} \tag{5.2}$$

for any $X, Y, Z, W \in \Gamma(TM)$. Moreover, if we set $Y = Z = \xi$ and use (2.1) and (2.8) in (5.2), one has

$$\alpha^2 g(X,\varphi W) = -\frac{1}{2n-1}\Big\{-2n\alpha^2 g(X,\varphi W) + S(X,\varphi W)\Big\}. \tag{5.3}$$

By putting $W = \varphi W$ in equation (5.3) and using (2.1), (2.9) we deduce

$$S(X,W) = \alpha^2 g(X,W) - (2n+1)\alpha^2\eta(X)\eta(W),$$

which implies that M is an $\eta-$Einstein manifold. Thus, we get the requested result. □

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