

## On $r$ -dynamic Coloring of Double Star Graph Families

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**Abstract.** An  $r$ -dynamic proper  $k$ -coloring of a graph  $G$  is a proper  $k$ -coloring of  $G$  such that every vertex in  $V(G)$  has neighbors in at least  $\min\{r, d(v)\}$  different color classes. The  $r$ -dynamic chromatic number of a graph  $G$  is the minimum  $k$  such that  $G$  has an  $r$ -dynamic coloring with  $k$  colors. In this paper we investigate the  $r$ -dynamic chromatic number for the Central graph, Middle graph, Total graph and Line graph of Double star graph.

### 1 Introduction

In this paper, all graphs are assumed to be simple and finite. The  $r$ -dynamic chromatic number, introduced by Montgomery [10] and written as  $\chi_r(G)$ , is the minimum  $k$  such that  $G$  has an  $r$ -dynamic proper  $k$ -coloring. An  $r$ -dynamic coloring of a graph  $G$  is a map  $c$  from  $V(G)$  to the set of colors such that (i) if  $uv \in E(G)$ , then  $c(u) \neq c(v)$ , and (ii) for each vertex  $v \in V(G)$ ,  $|c(N(v))| \geq \min\{r, d(v)\}$ , where  $N(v)$  denotes the set of vertices adjacent to  $v$  and  $d(v)$  its degree. The first condition characterizes proper colorings, the adjacency condition and second condition is double-adjacency condition [13]. The 1-dynamic chromatic number of a graph  $G$  is equal to its chromatic number. The 2-dynamic chromatic number of a graph has been studied under the name dynamic chromatic number in [1, 2, 3, 4, 7].

There are many upper bounds and lower bounds for  $\chi_d(G)$  in terms of graph parameters. For example, For a graph  $G$  with  $\Delta(G) \geq 3$ , Lai et al. [7] proved that  $\chi_d(G) \leq \Delta(G) + 1$ . An upper bound for the dynamic chromatic number of a  $d$ -regular graph  $G$  in terms of  $\chi(G)$  and the independence number of  $G$ ,  $\alpha(G)$ , was introduced in [5]. In fact, it was proved that  $\chi_d(G) \leq \chi(G) + 2\log_2 \alpha(G) + 3$ .

Li et al. proved in [9] that the computational complexity of  $\chi_d(G)$  for a 3-regular graphs is an NP-complete problem. Furthermore, Li and Zhou [8] showed that whether there exists a 3-dynamic coloring, for a claw free graph with the maximum degree 3, is NP-complete.

In this paper, we study the  $r$ -dynamic chromatic number for middle, total, central and line graph of Double star graph. Most known papers concern  $r$ -dynamic coloring only for small values of  $r$ . In this paper, we consider  $r$ -dynamic coloring for all  $r$  between  $\delta$  and  $\Delta$  [14].

### 2 Preliminaries

The middle graph [11] of  $G$ , is defined with the vertex set  $V(G) \cup E(G)$  where two vertices are adjacent iff they are either adjacent edges of  $G$  or one is the vertex and other is an edge incident with it and it is denoted by  $M(G)$ .

The total graph [11] of  $G$ , has vertex set  $V(G) \cup E(G)$ , and edges joining all elements of this vertex set which are adjacent or incident in  $G$

The central graph [12]  $C(G)$  of a graph  $G$  is obtained from  $G$  by adding an extra vertex on each edge of  $G$ , and then joining each pair of vertices of the original graph which were previously non-adjacent.

The line graph [6] of  $G$  denoted by  $L(G)$  is the graph whose vertex set is the edge set of  $G$ . Two vertices of  $L(G)$  are adjacent whenever the corresponding edges of  $G$  are adjacent.

**Theorem 2.1.** For any Double star graph  $K_{1,n,n}$ , the  $r$ -dynamic chromatic number

$$\chi_r(C(K_{1,n,n})) = \begin{cases} n+1, r=1 \\ 2n+1, 2 \leq r \leq \Delta-1 \\ 3n+1, r \geq \Delta \end{cases}$$

*Proof.* First we apply the definition of Central graph on  $K_{1,n,n}$ .

Let the edge  $vv_i, v_iw_i$  be subdivided by the vertices  $e_i(1 \leq i \leq n), e'_i(1 \leq i \leq n)$  in  $K_{1,n,n}$ .

Clearly  $V(C(K_{1,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e'_i : 1 \leq i \leq n\}$ . The vertices  $v_i(1 \leq i \leq n)$  induce a clique of order  $n$  (say  $K_n$ ) and the vertices  $v, u_i(1 \leq i \leq n)$  induce a clique of order  $n+1$  (say  $K_{n+1}$ ) in  $C(K_{1,n,n})$  respectively. Thus we have  $\chi_r(C(K_{1,n,n})) \geq n+1$ .

**Case 1:** For  $r=1$

Consider the color class  $C_1 = \{c_1, c_2, c_3, \dots, c_{(n+1)}\}$

Assign the  $r$ -dynamic coloring to  $C(K_{1,n,n})$  by algorithm 2.1.1

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(C(K_{1,n,n})) = n+1$ .

**Case 2:** For  $r=2 \leq r \leq \Delta-1$

Consider the color class  $C_2 = \{c_1, c_2, c_3, \dots, c_{(2n+1)}\}$

Assign the  $r$ -dynamic coloring to  $C(K_{1,n,n})$  by algorithm 2.1.2

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(C(K_{1,n,n})) = 2n+1$ .

**Case 3:** For  $r \geq \Delta$

Consider the color class  $C_3 = \{c_1, c_2, c_3, \dots, c_{(3n+1)}\}$

Assign the  $r$ -dynamic coloring to  $C(K_{1,n,n})$  by algorithm 2.1.3

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(C(K_{1,n,n})) = 3n+1$ . □

### Algorithm 2.1.1

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $C(K_{1,n,n})$ .

begin

for  $i=1$  to  $n$

{  
 $V_1 = \{e_i\};$   
 $C(e_i) = i;$   
 }

$V_2 = \{v\};$   
 $C(v) = n+1;$

for  $i=1$  to  $n-1$

{  
 $V_3 = \{v_i\};$   
 $C(v_i) = i+1;$   
 }

$C(v_n) = 1;$

for  $i=1$  to  $n$

{  
 $V_4 = \{e'_i\};$   
 $C(e'_i) = n+1;$   
 }

for  $i=1$  to  $n-1$

{  
 $V_5 = \{w_i\};$   
 $C(w_i) = i+1;$   
 }

$C(w_n) = 1;$

$$V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$$

end

**Algorithm 2.1.2**Input: The number " $n$ " of  $K_{1,n,n}$ .Output: Assigning  $r$ -dynamic coloring for the vertices in  $C(K_{1,n,n})$ .

begin

for  $i = 1$  to  $n$ 

{

 $V_1 = \{e_i\};$  $C(e_i) = i;$ 

}

 $V_2 = \{v\};$  $C(v) = n + 1;$ for  $i = 1$  to  $n - 1$ 

{

 $V_3 = \{v_i\};$  $C(v_i) = i + 1;$ 

}

 $C(v_n) = 1;$ for  $i = 1$  to  $n$ 

{

 $V_4 = \{e'_i\};$  $C(e'_i) = n + 1;$ 

}

for  $i = 1$  to  $n$ 

{

 $V_5 = \{w_i\};$  $C(w_i) = n + i + 1;$ 

}

 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 

end

**Algorithm 2.1.3**Input: The number " $n$ " of  $K_{1,n,n}$ .Output: Assigning  $r$ -dynamic coloring for the vertices in  $C(K_{1,n,n})$ .

begin

for  $i = 1$  to  $n$ 

{

 $V_1 = \{e_i\};$  $C(e_i) = i;$ 

}

 $V_2 = \{v\};$  $C(v) = n + 1;$ for  $i = 1$  to  $n - 1$ 

{

 $V_3 = \{v_i\};$  $C(v_i) = i + 1;$ 

}

 $C(v_n) = 1;$ for  $i = 1$  to  $n$ 

{

 $V_4 = \{e'_i\};$  $C(e'_i) = 2n + i + 1;$ 

}

for  $i = 1$  to  $n$ 

{

 $V_5 = \{w_i\};$  $C(w_i) = n + i + 1;$

}  
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ ;  
end

**Theorem 2.2.** For any Double star graph  $K_{1,n,n}$ , the  $r$ -dynamic chromatic number

$$\chi_r(M(K_{1,n,n})) = \begin{cases} n+1, & 1 \leq r \leq n \\ n+2, & r = n+1 \\ n+3, & r \geq \Delta \end{cases}$$

*Proof.* By definition of middle graph, each edge  $vv_i, v_iw_i$  be subdivided by the vertices  $e_i(1 \leq i \leq n)$ ,  $e'_i(1 \leq i \leq n)$  in  $K_{1,n,n}$  and the vertices  $v, e_i$  induce a clique of order  $n+1$  (say  $K_{n+1}$ ) in  $M(K_{1,n,n})$ .

i.e.,  $V(M(K_{1,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e'_i : 1 \leq i \leq n\}$ .

Thus we have,  $\chi_r(M(K_{1,n,n})) \geq n+1$ .

**Case 1:** For  $1 \leq r \leq n$

Consider the color class  $C_1 = \{c_1, c_2, c_3, \dots, c_{(n+1)}\}$

Assign the  $r$ -dynamic coloring to  $M(K_{1,n,n})$  by algorithm 2.2.1

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Thus we have,  $\chi_r(M(K_{1,n,n})) = n+1$  if  $1 \leq r \leq n$ .

**Case 2:** For  $r = n+1$

Consider the color class  $C_2 = \{c_1, c_2, c_3, \dots, c_{(n+1)}, c_{(n+2)}\}$

Assign the  $r$ -dynamic coloring to  $M(K_{1,n,n})$  by algorithm 2.2.2

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled. Hence,  $\chi_r(M(K_{1,n,n})) = n+2$  if  $r = n+1$ .

**Case 3:** For  $r = \Delta$

Consider the color class  $C_3 = \{c_1, c_2, c_3, \dots, c_n, c_{(n+1)}, c_{(n+2)}, c_{(n+3)}\}$

Assign the  $r$ -dynamic coloring to  $M(K_{1,n,n})$  by algorithm 2.2.3

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled. Hence  $\chi_r(M(K_{1,n,n})) = n+3$  if  $r \geq \Delta$ .

□

### Algorithm 2.2.1

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $M(K_{1,n,n})$ .

begin

for  $i = 1$  to  $n$

{  
 $V_1 = \{e_i\}$ ;  
 $C(e_i) = i$ ;  
}

$V_2 = \{v\}$ ;  
 $C(v) = n+1$ ;

for  $i = 1$  to  $n$

{  
 $V_3 = \{v_i\}$ ;  
 $C(v_i) = n+1$ ;  
}

for  $i = 1$  to  $n-1$

{  
 $V_4 = \{e'_i\}$ ;  
 $C(e'_i) = i+1$ ;  
}

```

 $C(e'_n) = 1;$ 
for  $i = 1$  to  $n - 2$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 2;$ 
}
 $C(w_{n-1}) = 1;$ 
 $C(w_n) = 2;$ 
}
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Algorithm 2.2.2**

Input: The number " $n$ " of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $M(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = n + 2;$ 
}
for  $i = 1$  to  $n$ 
{
 $V_4 = \{e'_i\};$ 
 $C(e'_i) = n + 1;$ 
}
for  $i = 1$  to  $n - 1$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 1;$ 
}
 $C(w_n) = 1;$ 
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Algorithm 2.2.3**

Input: The number " $n$ " of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $M(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = n + 2;$ 
}
for  $i = 1$  to  $n$ 

```

```

{
V4 = {e'i};
C(e'i) = n + 3;
}
for i = 1 to n
{
V5 = {wi};
C(wi) = n + 1;
}
V = V1 ∪ V2 ∪ V3 ∪ V4 ∪ V5;
end

```

**Theorem 2.3.** For any Double star graph  $K_{1,n,n}$ , the  $r$ -dynamic chromatic number,

$$\chi_r(T(K_{1,n,n})) = \begin{cases} n + 1, 1 \leq r \leq n \\ r + 1, n + 1 \leq r \leq \Delta - 2 \\ 2n, r = \Delta - 1 \\ 2n + 1, r \geq \Delta \end{cases}$$

*Proof.* By definition of Total graph, each edge  $vv_i, v_iw_i$  be subdivided by the vertices  $e_i (1 \leq i \leq n)$ ,  $e'_i (1 \leq i \leq n)$  in  $K_{1,n,n}$  and the vertices  $v, e_i$  induce a clique of order  $n + 1$  (say  $K_{n+1}$ ) in  $T(K_{1,n,n})$ .

i.e.,  $V(T(K_{1,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e'_i : 1 \leq i \leq n\}$ .

Thus we have  $\chi_r(T(K_{1,n,n})) \geq n + 1$ .

**Case 1:** For  $1 \leq r \leq n$

Consider the color class  $C_1 = \{c_1, c_2, c_3, \dots, c_{(n+1)}\}$

Assign the  $r$ -dynamic coloring to  $T(K_{1,n,n})$  by algorithm 2.3.1

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Thus we have  $\chi_r(T(K_{1,n,n})) = n + 1$  if  $1 \leq r \leq n$ .

**Case 2:** For  $n + 1 \leq r \leq \Delta - 2$

Consider the color class  $C_2 = \{c_1, c_2, c_3, \dots, c_{(2n-1)}\}$

Assign the  $r$ -dynamic coloring to  $T(K_{1,n,n})$  by algorithm 2.3.2

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(T(K_{1,n,n})) = r + 1$  if  $n + 1 \leq r \leq \Delta - 2$ .

**Case 3:** For  $r = \Delta - 1$

Consider the color class  $C_3 = \{c_1, c_2, c_3, \dots, c_{2n}\}$

Assign the  $r$ -dynamic coloring to  $T(K_{1,n,n})$  by algorithm 2.3.3

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(T(K_{1,n,n})) = 2n$  if  $r = \Delta - 1$ .

**Case 4:** For  $r = \Delta$

Consider the color class  $C_4 = \{c_1, c_2, c_3, \dots, c_{2n+1}\}$

Assign the  $r$ -dynamic coloring to  $T(K_{1,n,n})$  if  $r = \Delta$  by algorithm 2.3.4

Thus, an easy check shows that the  $r$ -adjacency condition is fulfilled.

Hence  $\chi_r(T(K_{1,n,n})) = 2n + 1$  if  $r \geq \Delta$ .

□

### Algorithm 2.3.1

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $T(K_{1,n,n})$ .

begin

for  $i = 1$  to  $n$

{

$V_1 = \{e_i\};$

```

 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n - 1$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = i + 1;$ 
}
 $C(v_n) = 1;$ 
for  $i = 1$  to  $n - 2$ 
{
 $V_4 = \{e'_i\};$ 
 $C(e'_i) = i + 2;$ 
}
 $C(e'_{n-1}) = 1;$ 
 $C(e'_n) = 2;$ 
for  $i = 1$  to  $n - 3$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 3;$ 
}
 $C(w_{n-2}) = 1;$ 
 $C(w_{n-1}) = 2;$ 
 $C(w_n) = 3;$ 
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Algorithm 2.3.2**Input: The number " $n$ " of  $K_{1,n,n}$ .Output: Assigning  $r$ -dynamic coloring for the vertices in  $T(K_{1,n,n})$ .

begin

```

for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n - 3$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = r + 1;$ 
}
 $C(v_{n-2}) = n + 2;$ 
 $C(v_{n-1}) = n + 3;$ 
 $C(v_n) = n + 4;$ 
for  $i = 1$  to  $n - 2$ 
{
 $V_4 = \{e'_i\};$ 
 $C(e'_i) = n + i + 2;$ 
}
 $C(e'_{n-1}) = n + 2;$ 
 $C(e'_n) = n + 3;$ 
for  $i = 1$  to  $n - 1$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 1;$ 
}

```

```

}
 $C(w_n) = 1;$ 
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Algorithm 2.3.3**

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $T(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n - 1$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = n + i + 1;$ 
}
 $C(v_n) = n + 2;$ 
for  $i = 1$  to  $n - 2$ 
{
 $V_4 = \{e'_i\};$ 
 $C(e'_i) = n + i + 2;$ 
}
 $C(e'_{n-1}) = n + 2;$ 
 $C(e'_n) = n + 3;$ 
for  $i = 1$  to  $n - 1$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 1;$ 
}
 $C(w_n) = 1;$ 
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Algorithm 2.3.4**

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $T(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
 $V_2 = \{v\};$ 
 $C(v) = n + 1;$ 
for  $i = 1$  to  $n$ 
{
 $V_3 = \{v_i\};$ 
 $C(v_i) = n + i + 1;$ 
}
for  $i = 1$  to  $n - 1$ 
{
 $V_4 = \{e'_i\};$ 
 $C(e'_i) = n + i + 2;$ 
}

```



```

 $C(e'_n) = n + 3;$ 
for  $i = 1$  to  $n - 1$ 
{
 $V_5 = \{w_i\};$ 
 $C(w_i) = i + 1;$ 
}
 $C(w_n) = 1;$ 
 $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5;$ 
end

```

**Theorem 2.4.** For any Double star graph  $K_{1,n,n}$ , the  $r$ -dynamic chromatic number

$$\chi_r(L(K_{1,n,n})) = \begin{cases} n, & 1 \leq r \leq n - 1 \\ n + 1, & r \geq \Delta \end{cases}$$

*Proof.* First we apply the definition of Line graph on  $K_{1,n,n}$

By the definition of line graph, each edge of  $K_{1,n,n}$  taken to be as vertex in  $L(K_{1,n,n})$ . The vertices  $e_1, e_2, \dots, e_n$  induce a clique of order  $n$  in  $L(K_{1,n,n})$ .  
i.e.,  $V(L(K_{1,n,n})) = E(K_{1,n,n}) = \{e_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$ .

Thus we have  $\chi_r(L(K_{1,n,n})) \geq n$ .

**Case 1:** For  $(1 \leq i \leq \Delta - 1)$

Consider the vertex set  $V(L(K_{1,n,n}))$  and color class  $C_1 = \{c_1, c_2, \dots, c_n\}$

Assign  $r$  dynamic coloring to  $L(K_{1,n,n})$  by algorithm 2.4.1

Thus, an easy check shows that the  $r$ - adjacency condition is fulfilled.

Hence  $\chi_r(L(K_{1,n,n})) = n$ .

**Case 2:** For  $(r \geq \Delta)$

Consider the vertex set  $V(L(K_{1,n,n}))$  and color class  $C_2 = \{c_1, c_2, \dots, c_n, c_{(n+1)}\}$

Assign  $r$  dynamic coloring to  $L(K_{1,n,n})$  by algorithm 2.4.2

Thus, an easy check shows that the  $r$ - adjacency condition is fulfilled.

Hence  $\chi_r(L(K_{1,n,n})) = n + 1$ . □

#### Algorithm 2.4.1

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $L(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 
 $C(e_i) = i;$ 
}
for  $i = 1$  to  $n - 1$ 
{
 $V_2 = \{w_i\};$ 
 $C(w_i) = i + 1;$ 
}
 $C(w_n) = 1;$ 
 $V = V_1 \cup V_2;$ 
end

```

#### Algorithm 2.4.2

Input: The number "n" of  $K_{1,n,n}$ .

Output: Assigning  $r$ -dynamic coloring for the vertices in  $L(K_{1,n,n})$ .

```

begin
for  $i = 1$  to  $n$ 
{
 $V_1 = \{e_i\};$ 

```

```

 $C(e_i) = i;$ 
}
for  $i = 1$  to  $n$ 
{
 $V_2 = \{w_i\};$ 
 $C(w_i) = n + 1;$ 
}
 $V = V_1 \cup V_2;$ 
end

```

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