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FURTHER RESULTS ON 4-DIFFERENCE CORDIAL GRAPHS

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Abstract In this paper we investigate the 4-difference cordial labeling behaviour of jelly fish, jewel graph, comb, subdivision of star, subdivision of bistar, book with triangle pages.

1 Introduction

Graphs in this paper are finite, simple and undirected. k-difference cordial labeling of graphs have been introduced in [3] and they investigate the 3-difference cordial labeling behaviour of path, cycle, star, bistar and complete graph. In this paper, we investigate the 4-difference cordial labeling behaviour of jelly fish, jewel graph, comb, subdivision of star, subdivision of bistar, book with triangle pages. $[x]$ denote the greatest integer $\leq x$. Terms are not defined here follow from Harary[2] and Gallian[1].

2 4-difference cordial graphs

Definition 2.1. Let G be a (p, q) graph and $f : V(G) \to \{1, 2, \dots, k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $x \in \{1, 2, \dots, k\}$, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

3 Preliminaries

Definition 3.1. If $e = uv$ is an edge of G and w is a vertex not in G then e is said to be *subdivided* when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.

Definition 3.2. The *jelly fish graph* JF_n is a graph with $V(JF_n) = \{u, v, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and edge set $E(JF_n) = \{uu_i : 1 \leq i \leq n\} \cup \{vv_j : 1 \leq j \leq n-2\} \cup \{u_{n-1}u_n, vu_n, vu_{n-1}\}$.

Definition 3.3. The *jewel graph* J_n is the graph with the vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and the edge set
$E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Definition 3.4. The *shadow graph* $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say G_1 and G_2. Join each vertex u_1 in G_1 to the the neighbours of corresponding vertex u_2 in G_2.

4 Main results

Theorem 4.1. The jelly fish graph JF_n is 4-difference cordial for all values of n.

Proof. Take the vertex set and edge set as in Definition 3.2. Then JF_n is of order $2n$ and size $2n + 1$.
Assign the labels 2,3,1 and 4 respectively to the vertices u,v,u_{n-1} and u_n.

Case 1. n is odd.
Now we consider the pendent vertices $u_1, u_2, \dots, u_{n-2}$. Assign the label 1 to the $\frac{n-1}{2}$ vertices $u_1, u_2, \dots, u_{\frac{n-1}{2}}$, then assign the label 4 to the next $\frac{n-3}{2}$ vertices $u_{\frac{n+1}{2}}, u_{\frac{n+3}{2}}, \dots, u_{n-2}$. We now move to the pendent vertices $v_1, v_2, \dots, v_{n-2}$. Assign the label 2 to the $\frac{n-3}{2}$ vertices $v_1, v_2, \dots, v_{\frac{n-3}{2}}$ and finally assign the label 3 to the next $\frac{n-1}{2}$ vertices $v_{\frac{n-1}{2}}, v_{\frac{n+1}{2}}, \dots, v_{n-2}$.

Case 2. n is even.
Assign the label 1 to the $\frac{n-2}{2}$ vertices $u_1, u_2, \dots, u_{\frac{n-2}{2}}$, then assign the label 4 to the next $\frac{n-2}{2}$ vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, \dots, u_{n-2}$. Next assign the label 2 to the $\frac{n-2}{2}$ vertices $v_1, v_2, \dots, v_{\frac{n-2}{2}}$ and finally assign the label 3 to the next $\frac{n-2}{2}$ vertices $v_{\frac{n}{2}}, v_{\frac{n+2}{2}}, \dots, v_{n-2}$.

Thus this vertex labeling f is 4-difference cordial labeling of jelly fish follows from Tabel 1

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n is odd	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$n+1$	n
n is even	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$n+1$	n

Table 1.

□

Theorem 4.2. The jewel J_n is 4-difference cordial for all n.

Proof. Take the vertex set and edge set as in Definition 3.3. Therefore J_n is of order $n + 4$ and size $2n + 5$.
Assign the labels 2, 3, 1, 4 respectively to the vertices u, v, x, y.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4t$, $t \in N$.
Now we consider the vertices $u_1, u_2, \dots, u_n$. Assign the label 1 to the t vertices $u_1, u_2, \dots, u_t$, then assign the label 2 to the next t vertices $u_{t+1}, u_{t+2}, \dots, u_{2t}$, next assign the label 3 to the t vertices $u_{2t+1}, u_{2t+2}, \dots, u_{3t}$ and finally assign the label 4 to the remaining t vertices $u_{3t+1}, u_{3t+2}, \dots, u_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4t + 1$, $t \in Z^+$.
Assign the label 1 to the $t + 1$ vertices $u_1, u_2, \dots, u_{t+1}$, then assign the label 2 to the next t vertices $u_{t+2}, u_{t+3}, \dots, u_{2t+1}$, next assign the label 3 to the t vertices $u_{2t+2}, u_{2t+3}, \dots, u_{3t+1}$ and assign the label 4 to the remaining non-labelled t vertices $u_{3t+2}, u_{3t+3}, \dots, u_{4t+1}$.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4t + 2$, $t \in Z^+$.
Assign the label 1 to the $t + 1$ vertices $u_1, u_2, \dots, u_{t+1}$, then assign the label 2 to the next t vertices $u_{t+2}, u_{t+3}, \dots, u_{2t+1}$ next assign the label 3 to the t vertices $u_{2t+2}, u_{2t+3}, \dots, u_{3t+1}$ and assign the label 4 to the remaining non-labelled $t + 1$ vertices $u_{3t+2}, u_{3t+3}, \dots, u_{4t+2}$.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4t + 3$, $t \in Z^+$.
Assign the label 1 to the $t + 1$ vertices $u_1, u_2, \dots, u_{t+1}$, then assign the label 2 to the next $t + 1$ vertices $u_{t+2}, u_{t+3}, \dots, u_{2t+2}$ next assign the label 3 to the t vertices $u_{2t+3}, u_{2t+4}, \dots, u_{3t+2}$ and assign the label 4 to the remaining non-labelled $t + 1$ vertices $u_{3t+3}, u_{3t+4}, \dots, u_{4t+3}$.

This vertex labeling f is 4-difference cordial labeling follows from Tabel 2

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$m \equiv 0 \pmod 4$	$\frac{n+4}{4}$	$\frac{n+4}{4}$	$\frac{n+4}{4}$	$\frac{n+4}{4}$	$n+3$	$n+2$
$m \equiv 1 \pmod 4$	$\frac{n+7}{4}$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$n+3$	$n+2$
$m \equiv 2 \pmod 4$	$\frac{n+6}{4}$	$\frac{n+6}{4}$	$\frac{n+2}{4}$	$\frac{n+2}{4}$	$n+3$	$n+2$
$m \equiv 3 \pmod 4$	$\frac{n+5}{4}$	$\frac{n+5}{4}$	$\frac{n+5}{4}$	$\frac{n+1}{4}$	$n+3$	$n+2$

Table 2.

□

Theorem 4.3. The complete bipartite graph $K_{2,n}$ is 4-difference cordial for all values of n.

Proof. Let $V(K_{2,n}) = \{u_i, v_j : 1 \le i \le 2, 1 \le j \le n\}$ and
$E(K_{2,n}) = \{u_1v_j, u_2v_j : 1 \le i \le n\}$.
Assign the labels 2, 3 to the vertices u_1, u_2 respectively.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4t$, $t \in N$.
Now we consider the vertices $v_1, v_2, \ldots, v_n$. Assign the label 1 to the first t vertices $v_1, v_2, \ldots, v_t$, then assign the label 2 to the next t vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$, next assign the label 3 to the t vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{3t}$ and finally assign the label 4 to the t vertices $v_{3t+1}, v_{3t+2}, \ldots, v_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4t+1$, $t \in Z^+$.
Assign the label 1 to the first $t+1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next t vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+1}$, next assign the label 3 to the t vertices $v_{2t+2}, v_{2t+3}, \ldots, v_{3t+1}$ and finally assign the label 4 to the t vertices $v_{3t+2}, v_{3t+3}, \ldots, v_{4t+1}$.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4t+2$, $t \in Z^+$.
Assign the label 1 to the first $t+1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next t vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+1}$, next assign the label 3 to the t vertices $v_{2t+2}, v_{2t+3}, \ldots, v_{3t+1}$ and finally assign the label 4 to the $t+1$ vertices $v_{3t+2}, v_{3t+3}, \ldots, v_{4t+2}$.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4t+3$, $t \in Z^+$.
Assign the label 1 to the first $t+1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next $t+1$ vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+2}$, next assign the label 3 to the t vertices $v_{2t+3}, v_{2t+4}, \ldots, v_{3t+2}$ and finally assign the label 4 to the $t+1$ vertices $v_{3t+3}, v_{3t+4}, \ldots, v_{4t+3}$.

The Tabel 3, establish that this vertex labeling f is 4-difference cordial labeling

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{n}{4}$	$\frac{n+4}{4}$	$\frac{n+4}{4}$	$\frac{n}{4}$	n	n
$n \equiv 1 \pmod 4$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$\frac{n-1}{4}$	n	n
$n \equiv 2 \pmod 4$	$\frac{n+2}{4}$	$\frac{n+2}{4}$	$\frac{n+2}{4}$	$\frac{n+2}{4}$	n	n
$n \equiv 3 \pmod 4$	$\frac{n+1}{4}$	$\frac{n+5}{4}$	$\frac{n+1}{4}$	$\frac{n+1}{4}$	n	n

Table 3.

□

Theorem 4.4. The Book with triangle pages, $K_2 + mK_1$ is 4-difference cordial for all values of m.

Proof. Let $V(K_2 + mK_1) = \{u, v, v_j : 1 \le j \le m\}$ and
$E(K_2 + mK_1) = \{uv, uv_j, vv_j : 1 \le j \le m\}$.
Note that the order and size of $K_2 + mK_1$ are $m + 2$ and $2m + 1$ respectively. Assign the labels 2, 3 respectively to the vertices u, v.

Case 1. $m \equiv 0 \pmod 4$.
Let $m = 4t$, $t \in N$.
Now we consider the vertices $v_1, v_2, \ldots, v_m$. Assign the label 1 to the t vertices $v_1, v_2, \ldots, v_t$, then assign the label 2 to the next t vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$, next assign the label 3 to the t vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{3t}$ and finally assign the label 4 to the remaining t vertices $v_{3t+1}, v_{3t+2}, \ldots, v_{4t}$.

Case 2. $m \equiv 1 \pmod 4$.
Let $m = 4t + 1$, $t \in Z^+$.
Assign the label 1 to the $t+1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next t vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+1}$, next assign the label 3 to the t vertices $v_{2t+2}, v_{2t+3}, \ldots, v_{3t+1}$and assign the label 4 to the remaining non-labelled t vertices $v_{3t+2}, v_{3t+3}, \ldots, v_{4t+1}$.

Case 3. $m \equiv 2 \pmod 4$.
Let $m = 4t + 2$, $t \in Z^+$.
Assign the label 1 to the $t+1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next t vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+1}$ next assign the label 3 to the t vertices $v_{2t+2}, v_{2t+3}, \ldots, v_{3t+1}$ and assign the label 4 to the remaining non-labelled $t + 1$ vertices $v_{3t+2}, v_{3t+3}, \ldots, v_{4t+2}$.

Case 4. $m \equiv 3 \pmod 4$.
Let $m = 4t + 3$, $t \in Z^+$.
Assign the label 1 to the $t + 1$ vertices $v_1, v_2, \ldots, v_{t+1}$, then assign the label 2 to the next $t + 1$ vertices $v_{t+2}, v_{t+3}, \ldots, v_{2t+2}$ next assign the label 3 to the t vertices $v_{2t+3}, v_{2t+4}, \ldots, v_{3t+2}$ and assign the label 4 to the remaining non-labelled $t + 1$ vertices $v_{3t+3}, v_{3t+4}, \ldots, v_{4t+3}$.

Thus this vertex labeling f is 4-difference cordial labeling follows from Tabel 4

Nature of m	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$m \equiv 0 \pmod 4$	$\frac{m}{4}$	$\frac{m+4}{4}$	$\frac{m+4}{4}$	$\frac{m}{4}$	m	$m+1$
$m \equiv 1 \pmod 4$	$\frac{m+3}{4}$	$\frac{m+3}{4}$	$\frac{m+3}{4}$	$\frac{m-1}{4}$	m	$m+1$
$m \equiv 2 \pmod 4$	$\frac{m+2}{4}$	$\frac{m+2}{4}$	$\frac{m+2}{4}$	$\frac{m+2}{4}$	m	$m+1$
$m \equiv 3 \pmod 4$	$\frac{m+1}{4}$	$\frac{m+5}{4}$	$\frac{m+1}{4}$	$\frac{m+1}{4}$	m	$m+1$

Table 4.

□

Theorem 4.5. The comb $P_n \odot K_1$ is 4-difference cordial for all values of n.

Proof. Let $u_1 u_2 \ldots u_n$ be the path P_n.
Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \le i \le n\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_i v_i : 1 \le i \le n\}$.
Therefore $P_n \odot K_1$ is of order $2n$ and size $2n - 1$.

Case 1. n is odd.
Now we consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 1 to the $\frac{n+1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$, then assign the label 3 to the next $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. We now move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 2 to the $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$ and finally assign the label 4 to the next $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$.

Case 2. n is even.
Assign the label 1 to the $\frac{n}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n}{2}}$, then assign the label 3 to the next $\frac{n}{2}$ vertices

$u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n$. Next assign the label 2 to the $\frac{n}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$ and finally assign the label 4 to the next $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$.

This vertex labeling f is 4-difference cordial labeling follows from Tabel 5

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n is odd	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$n-1$	n
n is even	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$	$n-1$	n

Table 5.

□

Theorem 4.6. $S(K_{1,n})$ is 4-difference cordial for all n.

Proof. Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \leq i \leq n\}$ and
$E(S(K_{1,n})) = \{uv_i, v_iu_i : 1 \leq i \leq n\}$.

Assign the label 4 to the vertex u.

Case 1. $n \equiv 0 \pmod 2$.
Now we consider the pendent vertices $u_1, u_2, \ldots, u_n$. Assign the label 2 to $\frac{n}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n}{2}}$, then assign the label 3 to the next $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n$. We now move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 1 to $\frac{n}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$ and assign the label 4 to the next $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$.

Case 2. $n \equiv 1 \pmod 2$.
Assign the label 2 to the $\frac{n+1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$, then assign the label 3 to the next $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. Assign the label 1 to the $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$ and assign the label 4 to the next $\frac{n-1}{3}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$.

The Table 6, given below establish that this vertex labeling f is 4-difference cordial labeling of the graph $S(K_{1,n})$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(3)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n+2}{2}$	n	n
$n \equiv 1 \pmod 2$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	n	n

Table 6.

□

Theorem 4.7. $S(B_{n,n})$ is 4-difference cordial for all values of n.

Proof. Let $V(S(B_{n,n})) = \{u, v, w, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and
$E(S(B_{n,n})) = \{uw, vw, ux_i, x_iu_i, vy_i, y_iv_i : 1 \leq i \leq n\}$

Assign the labels 2,3,1 respectively to the vertices u,v,w.

Assign the label 4 to the n vertices $u_1, u_2, \ldots, u_n$, then assign the label 1 to the n vertices $x_1, x_2, \ldots, x_n$, next assign the label 2 to the n vertices $v_1, v_2, \ldots, v_n$ and assign the label 3 to the n vertices $y_1, y_2, \ldots, y_n$.
Clearly $v_f(1) = n+1$, $v_f(2) = n+1$, $v_f(3) = n+1$, $v_f(4) = n$, $e_f(0) = 2n+1$ and $e_f(1) = 2n+1$.

□

Theorem 4.8. $D_2(B_{n,n})$ is 4-difference cordial for all n.

Proof. Let $V(D_2(B_{n,n})) = \{u,v,x,y\} \cup \{u_i, x_i, v_i, y_i : 1 \le i \le n\}$ and $E(D_2(B_{n,n})) = \{uv, xy, uy, xv\} \cup \{uu_i, ux_i, xu_i, xx_i, vv_i, vy_i, yv_i, yy_i : 1 \le i \le n\}$. Then, $D_2(B_{n,n})$ is of order $4n+4$ and size $8n+4$.
Assign the labels 1,3,4,2 respectively to the vertices u,v,x,y.

Assign the label 2 to the n vertices $u_1, u_2, \dots, u_n$, then assign the label 3 to the n vertices $x_1, x_2, \dots, x_n$. Assign the label 1 to the n vertices $v_1, v_2, \dots, v_n$, then assign the label 4 to the n vertices $y_1, y_2, \dots, y_n$.
Obviously $v_f(1) = n+1$, $v_f(2) = n+1$, $v_f(3) = n+1$, $v_f(4) = n+1$, $e_f(0) = 4n+2$ and $e_f(1) = 4n+2$.

□

Theorem 4.9. The square of the path, P_n^2 is 4-difference cordial for all values of n..

Proof. Let $u_1 u_2 \dots u_n$ be the path P_n. Let $V(P_n^2) = V(P_n)$ and
$E(P_n^2) = E(P_n) \cup \{u_i u_{i+2} : 1 \le i \le n-2\}$.
Therefore P_n^2 is of order n and size $2n-3$.

Case 1. $n \equiv 0 \pmod 4$
Let $n = 4t$, $t \in N$.
Assign the labels $1,2,4,3$ respectively to the vertices u_1, u_2, u_3, u_4. Next assign the labels $1,2,4,3$ to the vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this until reach the vertex u_{4t}. It is easy to verify that the last four vertices $u_{4t-3}, u_{4t-2}, u_{4t-1}, u_{4t}$ received the labels $1,2,4,3$.

Case 2. $n \equiv 1 \pmod 4$
Let $n = 4t+1$, $t \in N$.
As in Case 1 assign the label to the vertices u_i $(1 \le i \le 4t)$. Finally assign the label 1 to the vertex u_{4t+1}.

Case 3. $n \equiv 2 \pmod 4$
Let $n = 4t+2$, $t \in N$.
Label the vertices u_i $(1 \le i \le 4t+1)$ as in Case 2. Next assign the label 2 to the vertex u_{4t+2}.

Case 4. $n \equiv 3 \pmod 4$
Let $n = 4t+3$, $t \in Z^+$.
In this case assign the label for the vertices u_i $(1 \le i \le 4t+2)$ as in Case 3. We now assign the label 4 to the vertex u_{4t+3}.

This vertex labeling f is a 4-difference cordial labeling of P_n^2 follows from Tabel 7

Order of P_n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(3)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$	$n-2$	$n-1$
$n \equiv 1 \pmod 4$	$\frac{n+3}{4}$	$\frac{n-1}{4}$	$\frac{n-1}{4}$	$\frac{n-1}{4}$	$n-1$	$n-2$
$n \equiv 2 \pmod 4$	$\frac{n+2}{4}$	$\frac{n+2}{4}$	$\frac{n-2}{4}$	$\frac{n-2}{4}$	$n-2$	$n-1$
$n \equiv 3 \pmod 4$	$\frac{n+1}{4}$	$\frac{n+1}{4}$	$\frac{n-3}{4}$	$\frac{n+1}{4}$	$n-1$	$n-2$

Table 7.

□

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