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The projective character tables of the maximal subgroups of the Mathieu groups M_{23} and M_{24}

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Abstract It is well known that all the irreducible projective characters of a finite group G can be obtained from the ordinary irreducible characters of a so-called representation group $R = M(G).G$ of G, where $M(G)$ denotes the Schur multiplier of G. Using this theory, a routine written in the computational algebra system GAP is presented to compute the irreducible projective characters IrrProj(G, α_i) with associated factor sets α_i for all of the maximal subgroups of the sporadic simple Mathieu groups M_{23} and M_{24}. In fact, this routine can be applied to any finite group G provided the ordinary irreducible characters of a representation group R of G can be found.

1 Introduction

The GAP routine developed by the current author in [13] computes all the sets of irreducible projective characters IrrProj(G, α_i), i = $1, 2, \ldots, m$, of a finite group G with factor sets α_i from a so-called representation group $R \cong M(G).G$ of G, where $M(G)$ denotes the Schur multiplier of the group G and m the number of cohomology classes $[\alpha_i]$ in $M(G)$. The said GAP routine is a result of the work in [14], [15], [16], [17] and [18] by the current author. The current paper is part of a series of papers to compute all the irreducible projective characters IrrProj(G, α_i) of each maximal subgroup of the sporadic simple Mathieu groups M_{11}, M_{12}, M_{22}, M_{23}, M_{24} and their automorphism groups. The ones for M_{11}, M_{12}, M_{22} and their automorphism groups were computed in [12] and [13]. In this paper, all the sets IrrProj(G, α_i) of irreducible projective characters for each maximal subgroup of M_{23} and M_{24} are computed using the methods in [12] and [13], except for the ones whose sets IrrProj(G, α_i) already appeared in the ATLAS [3] or GAP library [6]. It is noteworthy to mention that as the group G increases in size, the calculations involving a representation group (Schur cover) of G in GAP might become unfeasible, and therefore it can becomes extremely difficult to compute the sets IrrProj(G, α_i) of G. Since the orders of the maximal subgroups of M_{23} and M_{24} are relatively large compare to the ones of the other Mathieu groups, we have to make use first of a MAGMA routine (see [1]) to convert the finitely presented Schur covers of these groups into permutations groups and then use the aforementioned GAP routine to compute the sets IrrProj(G, α_i) for the maximal subgroups of M_{23} and M_{24}. Note that M_{23} and M_{24} have trivial Schur multipliers. In addition, readers who are interested can read up on the relevance of the projective representations of the centralizers $C_{M_{24}}(g)$ of elements g in M_{24} in the Generalised Mathieu Moonshine [5].

In Section 2, a brief theoretical background is given on how to obtain all the irreducible projective presentations of a finite group G from the ordinary irreducible representations of a representation group R of G. A brief discussion on the GAP routine, which was used to compute the irreducible projective characters of the maximal subgroups of M_{23} and M_{24}, will follows in Section 3. In Section 4, all the information concerning the sets IrrProj(G, α_i) for each maximal subgroup G of M_{23} and M_{24} are tabulated. Computations are done in GAP [6] and MAGMA [2] and notations in both GAP and the ATLAS are followed.

2 Preliminary Results on Projective Characters

In this section, definitions of concepts pertaining to our study are given which will lead up to a theoretical construction of finding all the irreducible projective characters of a finite group G from the ordinary irreducible characters of a central extension C of G. In this regard, the outline given in [11] is followed closely. For a more detailed treatment on ordinary and projective character theory readers are referred to [7], [8], [9], [10] and [19].

Definition 2.1. A projective representation of a group G of degree n over the complex numbers $\mathbb{C}$ is a map $P{:}G \to GL(n, \mathbb{C})$, such that

(i) $P(1) = I_n$, and

(ii) $P(x)P(y) = \alpha(x, y)P(xy)$ for $x, y \in G$ and $\alpha(x, y) \in \mathbb{C}^*$.

The map $\alpha{:}G \times G \to \mathbb{C}^*$ is called a factor set (or 2-cocycle) α of G and satisfies the relation $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ because of the associativity of multiplication in G and $GL(n, \mathbb{C})$. Then P is called a projective representation with factor set α and ξ, defined as $\xi(g) = Trace(P(g))$ for all $g \in G$, is called a projective character of G with factor set α. An irreducible projective representation P of a group G is essentially defined in a similar way then an ordinary irreducible representation of G.

Definition 2.2. Two projective representations P_1 and P_2 of G of degree n with factor sets α_1 and α_2 respectively are said to be projectively equivalent if there exist a mapping $\phi : G \to \mathbb{C}^*$ and a matrix $T \in GL(n, \mathbb{C})$ such that $P_1(x)$=$\phi(g)T^{-1}P_2(g)T$, $\forall x \in G$.

For such P_1 and P_2 in Definition 2.2 it follows that $\alpha_2(x, y)$ = $\phi(x)\phi(y)(\phi(xy))^{-1}\alpha_1(x, y)$, $\forall x, y \in G$ and it defines an equivalence relation where the equivalence class of the factor set α_1 is denoted by $[\alpha_1]$. The set of all equivalence classes of factor sets of G forms a finite abelian group and is called the Schur multiplier $M(G)$ (also known as the second cohomology group $H^2(G, \mathbb{C}^*)$ of G). Also, in each cohomology class $[\alpha]$ of $M(G)$ there is a so-called special factor set α (see discussion below or [7]) such that the projective characters IrrProj(G, α) associated with it are constant on the classes of G, in other words they are class functions. If the order of the class $[\alpha]$ in $M(G)$ is k, then the special factor set α takes values in powers of a kth-root of unity.

Now a theoretical account of how to obtain the irreducible projective representations of a group G with factor set α from the ordinary irreducible representations of a central extension $C = A.G$ of G, will follows.

Definition 2.3. A group $C = A.G$ is a central extension for G if there exists a homomorphism π from C onto G such that $A = ker(\pi) \leq Z(C) \cap C^{'}$. In addition, if $A \cong M(G)$, then we call the central extension C a representation group R of G.

Let $C = A.G$ be a central extension of the group G with $A = ker(\pi)$. Let $X = \{x_g | g \in G\}$ be a set of coset representatives of A in C, such that $\pi(x_g) = g$ (one-to-one correspondence of elements of X with the elements of G). Therefore, $C = \bigcup_{g \in G} Ax_g$. Then, for all $g, h \in G$, let $a(g, h)$ be the unique element in A such that $x_g x_h = a(g, h)x_{gh}$. Since the product operation to combine two elements in C and G is associative, then it follows that $a(g, h)a(gh, k) = a(g, hk)a(h, k)$ for all $g, h, k \in G$. Now, let λ be a linear character of the abelian group A and put $\alpha(g, h) = \lambda(a(g, h))$ for all $g, h \in G$, then it follows from the relation in the previous sentence that α is a factor set of G (the so-called special factor set mentioned above). Now, let T be an ordinary irreducible representation of C of degree n and let $P(g) = T(x_g)$ for all $g \in G$, then P is an irreducible projective representation of G with factor set α, i.e., $P(g)P(h) = \lambda(a(g, h))P(gh)$ for all $g, h \in G$. Hence we can formulate the following definition:

Definition 2.4. A projective representation P of G constructed from an ordinary irreducible representation T of C in the above manner is said to be linearized by the ordinary representation T (or lifted to C). Furthermore, P is irreducible if and only if T is irreducible.

Each irreducible projective representation of G with corresponding factor set α can be linearized by an ordinary irreducible representation of a representation group R of G. So the problem of constructing all irreducible projective characters of a finite group G reduces to that of finding the ordinary irreducible characters of a representation group R of G.

Definition 2.5. A covering group D for G will be a quotient $D \cong R/B$ of a representation group $R = M(G).G$ of G by a subgroup B of $M(G)$. If $M(G)/B$ has order n we refer to the covering group as a n-fold cover of G.

The projective characters of G associated with each equivalence class $[\alpha]$ of factors sets in $M(G)$ are found in the representation group R whereas in a n-fold cover D of G only the projective characters coming from the n equivalence classes covered by D will be represented [7].

Definition 2.6. An element $x \in G$ is said to be α-regular if $\alpha(x,g) = \alpha(g,x)$ for all $g \in C_G(x)$. Furthermore, it is well known that $g \in G$ is α-regular if and only if $\xi(g) \neq 0$ for some $\xi \in \mathrm{IrrProj}(G,\alpha)$ or equivalently that g is α-irregular if and only if $\xi(g) = 0$ for all $\xi \in \mathrm{IrrProj}(G,\alpha)$.

Now, if $x \in G$ is α-regular, then we called the conjugacy class $[x]$ of G which contains x an α-regular class. The number $|\mathrm{IrrProj}(G,\alpha)|$ of irreducible projective characters with factor set α equals the number of α-regular classes of a group G. Projective characters also satisfy the usual orthogonality relations and have analogues to ordinary characters.

3 A GAP routine to compute IrrProj(G, α_i)

The GAP routine which was developed in [13] has its origin in Proposition 3.1 below. Since our groups under consideration are relatively large, the said GAP routine will be slightly adjusted in the sense that we will convert the finitely presented Schur cover (representation group) of G first to a permutation group by using appropriate commands in GAP or use the MAGMA routine in [1] before the GAP routine is applied.

Proposition 3.1. *Let $R = M(G).G$ be a representation group of a finite group G, where $M(G)$ denotes the Schur multiplier of G. Then the number of irreducible characters $\chi_j \in Irr(R)$ of R which lies over a linear character λ of $M(G)$ is less or equal to $|Irr(G)|$.*

Proof. (see [14] , [15] or [20]). □

The quantity $\sum_{\chi\in \mathrm{Irr}(R)} \frac{<\chi\downarrow_{M(G)},\lambda>}{\chi(1)}$ (see proof of Proposition 3.1) determines the number of irreducible characters $\chi_j \in \mathrm{Irr}(R)$ of R which lies over a linear character $\lambda \in \mathrm{Irr}(M(G))$ (in other words $\lambda \in \mathrm{Irr}(M(G))$ is an irreducible constituent of $\chi_j \downarrow_{M(G)}$ which implies that $< \chi_j \downarrow_{M(G)}, \lambda > \neq 0$). Using this fact (see the line of the GAP routine starting with "n"), all the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, ..., |\mathrm{Irr}(M(G)|$, of G can be computed. Furthermore, it is shown in the proof of Proposition 3.1 that the quantity $\sum_{\chi\in \mathrm{Irr}(R)} \frac{<\chi\downarrow_{M(G)},\lambda>}{\chi(1)} \leq |\mathrm{Irr}(G)|$ and the inequality becomes strict if there is a non-identity element $x \in M(G) \setminus \ker(\lambda)$ which is a commutator in R. Hence the number $|\mathrm{IrrProj}(G, \alpha_i)|$ of irreducible projective characters of G with factor set α_i is always less or equal to the number |Irr(G)| of ordinary irreducible characters of G. The last line of the GAP routine ("N") gives $|\mathrm{Irr}(M(G))|$ blocks coming from a representation group R (denoted as "Source(f)" in the below routine), where each block will contains one of the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, ..., |\mathrm{Irr}(M(G)|$. Whereas the output "Display(ct)" will display each individual irreducible projective character table of G with factor set α_i in GAP.

If a finite group G has a relatively small order then the GAP routine below can computes easily all the distinct sets $\mathrm{IrrProj}(G, \alpha_i)$ of G from a suitable representation group R (Schur cover) of G as it was in the case of the groups in [13]. But if the group G becomes too large then GAP does not has sufficient enough functions to compute the Schur cover of G and its ordinary irreducible characters. Then additional techniques in computing these sets IrrProj(G,α_i) are required. This was the case with most of the maximal subgroups of M_{22} and Aut(M_{22}) in [12] and with all of the maximal subgroups of M_{24} which were dealt with in the current paper. So, to overcome this constraint, we compute the representation group R of "Perm:= G" using the GAP command "S:=SchurCover(Perm)" and then use the GAP commands "iso:=IsomorphismPermGroup(S)" and "x:=Image(iso)" to convert R into a permutation group "x". Then to find the normal subgroup "z" of the group "x" such that "x/z ≅ G", the command "Nor:=Filtered(NormalSubgroups(x),h1−>Size(h1)=Size(x)/Size(Perm))" is used. Furthermore, the command "f:=NaturalHomomorphismByNormalSubgroup(x,z)" is employed and then we can follow the rest of the GAP routine below from the input line "I1:=ImagesSource(f)" to compute the sets IrrProj(G,α_i) of G successfully. The MAGMA routine in [1] was used to convert the Schur covers of most of the maximal subgroups of M_{23} and M_{24} under consideration in this paper into a permutation group before applying the below GAP routine.

```
gap> Perm:= G;; (Permutation group with generators found in [22] or can be generated in GAP.)
gap> f:= EpimorphismSchurCover(Perm);;
gap> z:= Kernel(f);;
gap> x:= Source(f);;
gap> I1:=ImagesSource(f);; (Quotient group I1 ≅ G)
gap> t:=Irr(I1);;
gap> 2t:=Irr(x);;
gap> F:=FusionConjugacyClassesOp(f);
gap> map:=ProjectionMap(F);
gap> N:=[];
gap> for i in [1..Size(Irr(z))] do
> n:=Filtered(Irr(x),chi−>not IsZero(ScalarProduct(RestrictedClassFunction(chi,z),Irr(z)[i])));
> s:=List(n,x−>x{map});
> Add(N,s);
>Cen:=SizesCentralizers(CharacterTable(I1));
>Cl:=OrdersClassRepresentatives(CharacterTable(I1));
> ct:=function()local ct ;ct:=rec();
>ct.SizesCentralizers:=Cen;;
>ct.OrdersClassRepresentatives:=Cl;;
> ct.Irr:=N[i];;
>ct.UnderlyingCharacteristic:=0;ct.Id:="G";
> ConvertToLibraryCharacterTableNC(ct);
>return ct;end;ct:=ct();
>SetInfoLevel(InfoCharacterTable,2);
>Display(ct);
>od;
gap>N;
```

4 The sets IrrProj(G,α_i) for the maximal subgroups of M_{23} and M_{24}

In this section, the sets IrrProj(G,α_i) (see Tables 2 to 6) are only computed for the maximal subgroups $2^4{:}A_7$, $2^4{:}(3\times A_5){:}2$ of M_{23} and the ones $2^4{:}A_8$, $2^6{:}(3.S_6)$ and $2^6{:}(L_3(2)\times S_3)$ of M_{24}. The GAP routine discussed in Section 3 is used for this purpose. The irreducible projective characters for the rest of the maximal subgroups of M_{23} and M_{24} (see Table 1) can be found amongst the ATLAS, GAP library, [12] or [13]. Note that the ordinary irreducible characters Irr(G) of the group G appear always in the first block of Tables 2 to 6. The information about the structure of the Schur multiplier $M(G)$ and the number |IrrProj(G,α_i)| of irreducible projective characters of a maximal subgroup G of M_{23} and M_{24} associated with each cohomology class $[\alpha_i]$ in $M(G)$ is listed in Table 1.

Most of the groups for which the sets IrrProj(G, α_i) were computed in this section have Schur multipliers of order 2 except for the maximal subgroup $2^4{:}A_7$ of M_{23}. The Schur multiplier $M(G)$ of the group $2^4{:}A_7$ is cyclic of order six. Let α_2 be a generator for the Schur multiplier $M(G) \cong 6$ of $2^4{:}A_7$. Then the group $2^4{:}A_7$ will have 5 sets IrrProj$(2^4{:}A_7, \alpha_i)$ with non-trivial factor sets α_2, $\alpha_3 = \alpha_2^2$, $\alpha_4 = \alpha_2^3$, $\alpha_5 = \alpha_2^4$ and $\alpha_6 = \alpha_2^5$ of order 6, 3, 2, 3 and 6, respectively. The trivial factor set $\alpha_1 = \alpha_2^6 = 1$ is associated with the ordinary irreducible characters Irr$(2^4{:}A_7)$ of $2^4{:}A_7$. Since $\alpha_2^5 = \alpha_2^{-1} = \overline{\alpha_2}$, the entries of the set IrrProj$(2^4{:}A_7, \alpha_2)$ of $2^4{:}A_7$ in the fourth block of Table 3 are just the complex conjugates of the entries of the set IrrProj$(2^4{:}A_7, \alpha_2^5)$. Therefore, only one set IrrProj$(2^4{:}A_7, \alpha_2)$ of $2^4{:}A_7$ with factor set of order six is found in Table 3 since IrrProj$(2^4{:}A_7, \alpha_2^5)$ can be easily deduced from it. Similarly, the sets IrrProj$(2^4{:}A_7, \alpha_2^2)$ and IrrProj$(2^4{:}A_7, \alpha_2^4)$ with factor sets of order 3 are just complex conjugates of each and hence we will only find one set IrrProj$(2^4{:}A_7, \alpha_2^2)$ of irreducible projective characters of order 3 in the third block of Table 3. The set IrrProj$(2^4{:}A_7, \alpha_2^3)$ with factor set of order 2 is found in the the second block of Table 3. Hence only three disctinct sets IrrProj$(2^4{:}A_7, \alpha_i)$ with non-trivial factor sets occur for the group $2^4{:}A_7$.

It is interesting to note that the maximal subgroup $M_1 = 2^4{:}A_8$ of M_{24} has a Schur multiplier of order 2 and therefore a representation group $2.(2^4{:}A_8) \cong 2^{5\cdot}A_8$ exists which is isomorphic to a non-split extension of the shape $G_1 = 2^{5\cdot}A_8$. The current author in [16] computed the Fischer-Clifford matrices and associated ordinary character table of a non-split extension $G_2 = 2^{5\cdot}A_8$ of the same shape as G_1 but they are not isomorphic to each other although G_1 and G_2 have the same number of conjugacy classes. In fact the group G_2 is a 2-fold cover group for the maximal subgroup $M_2 = 2^{4\cdot}A_8$ of the smallest sporadic Conway simple group Co_3. And interesting enough the two groups M_1 and M_2 share the same irreducible projective character table with factor set of order two. The irreducible projective character table of M_1 with factor set of order two was also computed in [1] and it coincides with the one in Table 4. In conclusion, it was mentioned in [21] that the sets IrrProj(G, α_i) of G are only defined universally up to sign and it is possible that one can obtain different signs if the sets IrrProj(G, α_i) are re-calculated using a different representation group $R = M(G).G$. But these signs are calculated consistently with a "special factor set " and so the inner product and conjugacy results of [7] apply to the sets IrrProj(G, α_i).

Table 1. $M(G)$ **and** $|$**IrrProj**$(G, \alpha_i)|$ **of maximal subgroups** G **of** M_{23} **and** M_{24}

Maximal subgroups of M_{23}	$\lvert G\rvert$	$[M_{23}{:}G]$	$M(G)$	$\lvert\text{IrrProj}(G, \alpha_i)\rvert$
M_{22}	$443520 = 2^7.3^2.5.7.11$	23	12	[12,11,8,11,10,7]
$L_3(4){:}2_2$	$40320 = 2^7.3^2.5.7$	253	4	[14,13,11,11]
$2^4{:}A_7$	$40320 = 2^7.3^2.5.7$	253	6	[15,12,13,13,10,10]
A_8	$20160 = 2^6.3^2.5.7$	506	2	[14,9]
M_{11}	$7920 = 2^4.3^2.5.11$	1288	1	[10]
$2^4{:}(3\times A_5){:}2$	$5760 = 2^7.3^2.5$	1771	2	[17,12]
$23{:}11$	$253 = 11.23$	40320	1	[13]
Maximal subgroups of M_{24}	$\lvert G\rvert$	$[M_{24}{:}G]$	$M(G)$	$\lvert\text{IrrProj}(G, \alpha_i)\rvert$
M_{23}	$10200960 = 2^7.3^2.5.7.11.23$	24	1	[17]
$M_{22}{:}2$	$887040 = 2^8.3^2.5.7.11$	276	2	[21,16]
$2^4{:}A_8$	$322560 = 2^{10}.3^2.5.7$	759	2	[25,14]
$M_{12}{:}2$	$190080 = 2^7.3^3.5.11$	1288	2	[21,13]
$2^6{:}(3.S_6)$	$138240 = 2^{10}.3^3.5$	1771	2	[33,18]
$L_3(4){:}S_3$	$120960 = 2^7.3^3.5.7$	2024	1	[20]
$2^6{:}(L_3(2)\times S_3)$	$64512 = 2^{10}.3^2.7$	3795	2	[33,24]
$L_2(23)$	$6072 = 2^3.3.11.23$	40320	2	[14,13]
$L_2(7)$	$168 = 2^3.3.7$	1457280	2	[6,5]

Table 2. IrrProj(G,α_i) for 2^4:$(3 \times A_5)$:2, $\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1	2	3	4	2	4	2	6	3	3	6	6	4	8	15	5	15
$\lvert C_G(g)\rvert$	5760	384	180	16	48	32	96	12	18	36	12	6	8	8	15	15	15
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	1	1
χ_3	2	2	-1	0	0	2	2	-1	2	-1	-1	0	0	0	-1	2	-1
χ_4	4	4	4	-2	-2	0	0	0	1	1	1	1	0	0	-1	-1	-1
χ_5	4	4	4	2	2	0	0	0	1	1	1	-1	0	0	-1	-1	-1
χ_6	5	5	5	-1	-1	1	1	1	-1	-1	-1	-1	1	1	0	0	0
χ_7	5	5	5	1	1	1	1	1	-1	-1	-1	1	-1	-1	0	0	0
χ_8	6	6	6	0	0	-2	-2	-2	0	0	0	0	0	0	1	1	1
χ_9	6	6	-3	0	0	-2	-2	1	0	0	0	0	0	0	C	1	$\overline{\mathrm{C}}$
χ_{10}	6	6	-3	0	0	-2	-2	1	0	0	0	0	0	0	$\overline{\mathrm{C}}$	1	C
χ_{11}	8	8	-4	0	0	0	0	0	2	-1	-1	0	0	0	1	-2	1
χ_{12}	10	10	-5	0	0	2	2	-1	-2	1	1	0	0	0	0	0	0
χ_{13}	15	-1	0	1	-3	-1	3	0	0	3	-1	0	-1	1	0	0	0
χ_{14}	15	-1	0	-1	3	-1	3	0	0	3	-1	0	1	-1	0	0	0
χ_{15}	30	-2	0	0	0	-2	6	0	0	-3	1	0	0	0	0	0	0
χ_{16}	45	-3	0	1	-3	1	-3	0	0	0	0	0	1	-1	0	0	0
χ_{17}	45	-3	0	-1	3	1	-3	0	0	0	0	0	-1	1	0	0	0
χ_1	4	4	4	0	0	0	0	0	2	2	2	0	0	0	-1	-1	-1
χ_2	4	4	-2	0	0	0	0	0	2	-1	-1	0	0	0	-$\overline{\mathrm{C}}$	-1	-C
χ_3	4	4	-2	0	0	0	0	0	2	-1	-1	0	0	0	-C	-1	-$\overline{\mathrm{C}}$
χ_4	4	4	4	0	0	0	0	0	-1	-1	-1	A	0	0	-1	-1	-1
χ_5	4	4	4	0	0	0	0	0	-1	-1	-1	-A	0	0	-1	-1	-1
χ_6	6	6	6	0	0	0	0	0	0	0	0	0	B	B	1	1	1
χ_7	6	6	6	0	0	0	0	0	0	0	0	0	-B	-B	1	1	1
χ_8	8	8	-4	0	0	0	0	0	-2	1	1	0	0	0	1	-2	1
χ_9	12	12	-6	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1
χ_{10}	30	-2	0	0	0	0	0	0	0	3	-1	0	B	-B	0	0	0
χ_{11}	30	-2	0	0	0	0	0	0	0	3	-1	0	-B	B	0	0	0
χ_{12}	60	-4	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0

A = $-\sqrt{3}$i, B = $-\sqrt{2}$i, C = $\frac{-1+\sqrt{15}\mathrm{i}}{2}$

Table 3. IrrProj(G,α_i) for $2^4{:}A_7$, $\alpha_1 = \alpha_4^2 = \alpha_3^3 = \alpha_2^6 = 1$

$[g]_G$	1a	2a	5a	7a	14a	14b	7b	2b	4a	3a	6a	3b	6b	8a	4b
$\lvert C_G(g)\rvert$	40320	2688	5	14	14	14	14	96	32	36	12	36	12	8	8
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	6	6	1	-1	-1	-1	-1	2	2	3	-1	0	0	0	0
χ_3	10	10	0	B	B	$\overline{B}$	$\overline{B}$	-2	-2	1	1	1	1	0	0
χ_4	10	10	0	$\overline{B}$	$\overline{B}$	B	B	-2	-2	1	1	1	1	0	0
χ_5	14	14	-1	0	0	0	0	2	2	-1	-1	2	2	0	0
χ_6	14	14	-1	0	0	0	0	2	2	2	2	-1	-1	0	0
χ_7	15	15	0	1	1	1	1	-1	-1	3	-1	0	0	-1	-1
χ_8	15	-1	0	1	-1	-1	1	3	-1	0	0	3	-1	-1	1
χ_9	21	21	1	0	0	0	0	1	1	-3	1	0	0	-1	-1
χ_{10}	35	35	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1
χ_{11}	45	-3	0	B	-B	$-\overline{B}$	$\overline{B}$	-3	1	0	0	0	0	-1	1
χ_{12}	45	-3	0	$\overline{B}$	$-\overline{B}$	-B	B	-3	1	0	0	0	0	-1	1
χ_{13}	90	-6	0	-1	1	1	-1	6	-2	0	0	0	0	0	0
χ_{14}	105	-7	0	0	0	0	0	-3	1	0	0	3	-1	1	-1
χ_{15}	120	-8	0	1	-1	-1	1	0	0	0	0	-3	1	0	0
χ_1	4	4	-1	$-\overline{B}$	$-\overline{B}$	-B	-B	0	0	2	0	1	1	0	0
χ_2	4	4	-1	-B	-B	$-\overline{B}$	$-\overline{B}$	0	0	2	0	1	1	0	0
χ_3	14	14	-1	0	0	0	0	0	0	-2	0	-1	-1	H	H
χ_4	14	14	-1	0	0	0	0	0	0	-2	0	-1	-1	-H	-H
χ_5	20	20	0	-1	-1	-1	-1	0	0	-2	0	2	2	0	0
χ_6	20	20	0	-1	-1	-1	-1	0	0	4	0	-1	-1	0	0
χ_7	36	36	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_8	60	-4	0	-B	B	$\overline{B}$	$-\overline{B}$	0	0	0	0	3	-1	0	0
χ_9	60	-4	0	$-\overline{B}$	$\overline{B}$	B	-B	0	0	0	0	3	-1	0	0
χ_{10}	90	-6	0	-1	1	1	-1	0	0	0	0	0	0	H	-H
χ_{11}	90	-6	0	-1	1	1	-1	0	0	0	0	0	0	-H	H
χ_{12}	120	-8	0	1	-1	-1	1	0	0	0	0	-3	1	0	0
χ_1	6	6	A	-1	-1	$-\overline{A}$	$-\overline{A}$	E	E	0	$\overline{E}$	0	0	0	0
χ_2	15	15	0	1	1	$\overline{A}$	$\overline{A}$	F	F	0	0	0	0	1	1
χ_3	15	15	0	1	1	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	0	$\overline{E}$	0	0	-1	-1
χ_4	15	-1	0	1	-1	$-\overline{A}$	$\overline{A}$	F	$-\overline{A}$	0	0	0	E	-1	1
χ_5	21	21	A	0	0	0	0	-F	-F	0	0	0	0	1	1
χ_6	21	21	A	0	0	0	0	$\overline{A}$	$\overline{A}$	0	$-\overline{E}$	0	0	-1	-1
χ_7	24	24	-A	B	B	C	C	0	0	0	0	0	0	0	0
χ_8	24	24	-A	$\overline{B}$	$\overline{B}$	D	D	0	0	0	0	0	0	0	0
χ_9	45	-3	0	B	-B	-C	C	-F	$\overline{A}$	0	0	0	0	-1	1
χ_{10}	45	-3	0	$\overline{B}$	$-\overline{B}$	-D	D	-F	$\overline{A}$	0	0	0	0	-1	1
χ_{11}	90	-6	0	-1	1	$\overline{A}$	$-\overline{A}$	G	-E	0	0	0	0	0	0
χ_{12}	105	-7	0	0	0	0	0	-F	$\overline{A}$	0	0	0	E	1	-1
χ_{13}	120	-8	0	1	-1	$-\overline{A}$	$\overline{A}$	0	0	0	0	0	-E	0	0
χ_1	6	6	A	-1	-1	$-\overline{A}$	$-\overline{A}$	0	0	0	0	0	0	H	H
χ_2	6	6	A	-1	-1	$-\overline{A}$	$-\overline{A}$	0	0	0	0	0	0	-H	-H
χ_3	24	24	-A	B	B	C	C	0	0	0	0	0	0	0	0
χ_4	24	24	-A	$\overline{B}$	$\overline{B}$	D	D	0	0	0	0	0	0	0	0
χ_5	36	36	A	1	1	$\overline{A}$	$\overline{A}$	0	0	0	0	0	0	0	0
χ_6	60	-4	0	-B	B	C	-C	0	0	0	0	0	E	0	0
χ_7	60	-4	0	$-\overline{B}$	$\overline{B}$	D	-D	0	0	0	0	0	E	0	0
χ_8	90	-6	0	-1	1	$\overline{A}$	$-\overline{A}$	0	0	0	0	0	0	H	-H
χ_9	90	-6	0	-1	1	$\overline{A}$	$-\overline{A}$	0	0	0	0	0	0	-H	H
χ_{10}	120	-8	0	1	-1	$-\overline{A}$	$\overline{A}$	0	0	0	0	0	-E	0	0

A = $\frac{-1+\sqrt{3}i}{2}$, B = $\frac{-1-\sqrt{7}i}{2}$, C = $E(21)^5 + E(21)^{17} + E(21)^{20}$,
D = $E(21)^2$+$E(21)^8$+$E(21)^{11}$, E = -1-$\sqrt{3}$i, F = $\frac{-3-3\sqrt{3}i}{2}$,
G = -3-3$\sqrt{3}$i, H = -$\sqrt{2}$

Table 4. IrrProj(G,α_i) for $2^4{:}A_8$, $\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1a	2a	7a	14a	14b	7b	5a	3a	15a	15b	2b	4a	8a	4b	4c	2c	2d	6a	3b	6b	12a	6c	4d	4e	4f
$\lvert C_G(g)\rvert$	322560	21504	14	14	14	14	15	180	15	15	384	128	16	16	384	512	1536	24	72	12	12	12	64	32	64
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	7	7	0	0	0	0	2	4	-1	-1	3	3	1	1	-1	-1	-1	1	1	-1	-1	0	-1	-1	-1
χ_3	14	14	0	0	0	0	-1	-1	-1	-1	2	2	0	0	6	6	6	2	2	0	0	-1	2	2	2
χ_4	15	-1	1	-1	-1	1	0	0	0	0	3	-1	-1	1	-1	-1	7	-1	3	1	-1	0	3	-1	-1
χ_5	20	20	-1	-1	-1	-1	0	5	0	0	4	4	0	0	4	4	4	-1	-1	1	1	1	0	0	0
χ_6	21	21	0	0	0	0	1	6	1	1	1	1	-1	-1	-3	-3	-3	0	0	0	0	-2	1	1	1
χ_7	21	21	0	0	0	0	1	-3	B	$\overline{B}$	1	1	-1	-1	-3	-3	-3	0	0	0	0	1	1	1	1
χ_8	21	21	0	0	0	0	1	-3	$\overline{B}$	B	1	1	-1	-1	-3	-3	-3	0	0	0	0	1	1	1	1
χ_9	28	28	0	0	0	0	-2	1	1	1	4	4	0	0	-4	-4	-4	1	1	-1	-1	1	0	0	0
χ_{10}	35	35	0	0	0	0	0	5	0	0	-5	-5	-1	-1	3	3	3	2	2	0	0	1	-1	-1	-1
χ_{11}	45	45	A	A	$\overline{A}$	$\overline{A}$	0	0	0	0	-3	-3	1	1	-3	-3	-3	0	0	0	0	0	1	1	1
χ_{12}	45	45	$\overline{A}$	$\overline{A}$	A	A	0	0	0	0	-3	-3	1	1	-3	-3	-3	0	0	0	0	0	1	1	1
χ_{13}	45	-3	$\overline{A}$	-$\overline{A}$	-A	A	0	0	0	0	-3	1	-1	1	-3	5	-3	0	0	0	0	0	1	1	-3
χ_{14}	45	-3	A	-A	-$\overline{A}$	$\overline{A}$	0	0	0	0	-3	1	-1	1	-3	5	-3	0	0	0	0	0	1	1	-3
χ_{15}	56	56	0	0	0	0	1	-4	1	1	0	0	0	0	8	8	8	-1	-1	-1	-1	0	0	0	0
χ_{16}	64	64	1	1	1	1	-1	4	-1	-1	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0
χ_{17}	70	70	0	0	0	0	0	-5	0	0	2	2	0	0	-2	-2	-2	1	1	1	1	-1	-2	-2	-2
χ_{18}	90	-6	-1	1	1	-1	0	0	0	0	6	-2	0	0	-6	2	18	0	0	0	0	0	2	-2	2
χ_{19}	105	-7	0	0	0	0	0	0	0	0	-3	1	1	-1	1	-7	17	-1	3	-1	1	0	1	1	-3
χ_{20}	105	-7	0	0	0	0	0	0	0	0	-3	1	1	-1	-7	9	1	-1	3	1	-1	0	-3	1	1
χ_{21}	105	-7	0	0	0	0	0	0	0	0	9	-3	-1	1	1	1	-7	-1	3	-1	1	0	-3	1	1
χ_{22}	120	-8	1	-1	-1	1	0	0	0	0	0	0	0	0	-8	8	8	1	-3	-1	1	0	0	0	0
χ_{23}	210	-14	0	0	0	0	0	0	0	0	6	-2	0	0	2	-6	10	1	-3	1	-1	0	-2	2	-2
χ_{24}	315	-21	0	0	0	0	0	0	0	0	-9	3	-1	1	3	-5	3	0	0	0	0	0	-1	-1	3
χ_{25}	315	-21	0	0	0	0	0	0	0	0	3	-1	1	-1	3	3	-21	0	0	0	0	0	3	-1	-1
χ_1	8	8	-1	-1	-1	-1	-2	-4	1	-1	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
χ_2	24	24	-$\overline{A}$	-$\overline{A}$	-A	-A	-1	-6	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	24	24	-A	-A	-$\overline{A}$	-$\overline{A}$	-1	-6	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	48	48	1	1	1	1	-2	6	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_5	56	56	0	0	0	0	1	-4	1	-1	0	0	0	0	0	0	0	-1	-1	C	C	0	0	0	0
χ_6	56	56	0	0	0	0	1	-4	1	-1	0	0	0	0	0	0	0	-1	-1	-C	-C	0	0	0	0
χ_7	56	56	0	0	0	0	1	2	$\overline{B}$	-B	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
χ_8	56	56	0	0	0	0	1	2	B	-$\overline{B}$	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
χ_9	64	64	-1	-1	-1	-1	-1	4	-1	1	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0
χ_{10}	120	-8	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	-2	6	0	0	0	0	0	0
χ_{11}	120	-8	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	-3	C	-C	0	0	0	0
χ_{12}	120	-8	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	-3	-C	C	0	0	0	0
χ_{13}	360	-24	-A	A	$\overline{A}$	-$\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	360	-24	-$\overline{A}$	$\overline{A}$	A	-A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A = $\frac{-1-\sqrt{7}i}{2}$, B = $\frac{-1+\sqrt{15}i}{2}$, C = -$\sqrt{3}$i

Table 5. IrrProj(G,α_i) for $2^6{:}(3.S_6)$, $\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1a	2a	2b	3a	2c	4a	4b	2d	4c	4d	2e	6a	6b	3b	12a	6c	6d	3c	4e	4f	2f	4g	4h	8a	4i	4j	12b	15a	15b	10a	5a	12c	6e
$\lvert C_G(g)\rvert$	138240	3072	7680	1080	384	64	384	768	128	128	256	24	24	72	12	12	24	72	128	96	384	32	32	16	96	32	12	15	15	20	60	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1
χ_3	5	5	5	5	1	1	1	1	1	1	1	1	-1	-1	1	1	2	2	-3	-3	-3	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_4	5	5	5	5	3	3	3	1	1	1	1	1	2	2	0	0	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	-1	-1
χ_5	5	5	5	5	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	2	2	3	3	3	1	1	1	-1	-1	-1	0	0	0	0	0	0
χ_6	5	5	5	5	-3	-3	-3	1	1	1	1	1	2	2	0	0	-1	-1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	1	1
χ_7	6	6	6	-3	0	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	D	$\overline{D}$	1	1	0	0
χ_8	6	6	6	-3	0	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	$\overline{D}$	D	1	1	0	0
χ_9	9	9	9	9	-3	-3	-3	1	1	1	1	1	0	0	0	0	0	0	-3	-3	-3	1	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_{10}	9	9	9	9	3	3	3	1	1	1	1	1	0	0	0	0	0	0	3	3	3	-1	-1	-1	1	1	1	-1	-1	-1	-1	0	0
χ_{11}	10	10	10	10	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	2	2	2	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_{12}	10	10	10	10	2	2	2	-2	-2	-2	-2	-2	1	1	-1	-1	1	1	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	1	1
χ_{13}	12	12	12	-6	0	0	0	4	4	4	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	0	0
χ_{14}	16	16	16	16	0	0	0	0	0	0	0	0	-2	-2	0	0	-2	-2	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_{15}	18	18	18	-9	0	0	0	2	2	2	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	1	1	-2	-2	0	0
χ_{16}	18	2	-6	0	-4	0	4	6	2	-2	-2	0	-1	3	1	-1	0	0	0	0	0	2	-2	0	0	0	0	0	0	-1	3	0	0
χ_{17}	18	2	-6	0	4	0	-4	6	2	-2	-2	0	-1	3	-1	1	0	0	0	0	0	-2	2	0	0	0	0	0	0	-1	3	0	0
χ_{18}	30	30	30	-15	0	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	1	0	0	0	0	0	0
χ_{19}	45	-3	5	0	-3	1	-3	9	-3	1	1	0	0	0	0	0	-1	3	1	1	-7	-1	-1	1	3	-1	0	0	0	0	0	1	-1
χ_{20}	45	-3	5	0	3	-1	3	9	-3	1	1	0	0	0	0	0	-1	3	-1	-1	7	1	1	-1	3	-1	0	0	0	0	0	-1	1
χ_{21}	45	-3	5	0	-3	1	-3	-3	1	-3	5	0	0	0	0	0	-1	3	-3	1	5	1	1	-1	-3	1	0	0	0	0	0	1	-1
χ_{22}	45	-3	5	0	3	-1	3	-3	1	-3	5	0	0	0	0	0	-1	3	3	-1	-5	-1	-1	1	-3	1	0	0	0	0	0	-1	1
χ_{23}	72	8	-24	0	-8	0	8	0	0	0	0	0	-1	3	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0
χ_{24}	72	8	-24	0	8	0	-8	0	0	0	0	0	-1	3	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0
χ_{25}	90	-6	10	0	-6	2	-6	6	-2	-2	6	0	0	0	0	0	1	-3	-2	2	-2	0	0	0	0	0	0	0	0	0	0	-1	1
χ_{26}	90	-6	10	0	6	-2	6	6	-2	-2	6	0	0	0	0	0	1	-3	2	-2	2	0	0	0	0	0	0	0	0	0	0	1	-1
χ_{27}	90	10	-30	0	-4	0	4	6	2	-2	-2	0	1	-3	1	-1	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{28}	90	10	-30	0	4	0	-4	6	2	-2	-2	0	1	-3	-1	1	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{29}	108	12	-36	0	0	0	0	-12	-4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	0	0
χ_{30}	135	-9	15	0	-3	1	-3	3	-1	3	-5	0	0	0	0	0	0	0	1	-3	9	-1	-1	1	-3	1	0	0	0	0	0	0	0
χ_{31}	135	-9	15	0	3	-1	3	3	-1	3	-5	0	0	0	0	0	0	0	-1	3	-9	1	1	-1	-3	1	0	0	0	0	0	0	0
χ_{32}	135	-9	15	0	-3	1	-3	-9	3	-1	-1	0	0	0	0	0	0	0	5	-3	-3	1	1	-1	3	-1	0	0	0	0	0	0	0
χ_{33}	135	-9	15	0	3	-1	3	-9	3	-1	-1	0	0	0	0	0	0	0	-5	3	3	-1	-1	1	3	-1	0	0	0	0	0	0	0
χ_1	4	4	4	4	0	0	0	0	0	0	0	0	-2	-2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	E	E
χ_2	4	4	4	4	0	0	0	0	0	0	0	0	-2	-2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-E	-E
χ_3	4	4	4	4	0	0	0	0	0	0	0	0	1	1	A	A	2	2	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0
χ_4	4	4	4	4	0	0	0	0	0	0	0	0	1	1	-A	-A	2	2	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0
χ_5	12	12	12	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	-1	-1	2	2	0	0
χ_6	12	12	12	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-C	-1	-1	2	2	0	0
χ_7	16	16	16	16	0	0	0	0	0	0	0	0	-2	-2	0	0	2	2	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_8	20	20	20	20	0	0	0	0	0	0	0	0	2	2	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	24	24	24	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{D}$	-D	-1	-1	0	0
χ_{10}	24	24	24	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-D	$-\overline{D}$	-1	-1	0	0
χ_{11}	72	8	-24	0	0	0	0	0	0	0	0	0	2	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0
χ_{12}	72	8	-24	0	0	0	0	0	0	0	0	0	-1	3	A	-A	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0
χ_{13}	72	8	-24	0	0	0	0	0	0	0	0	0	-1	3	-A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0
χ_{14}	108	12	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	B	0	0	0	0	0	-1	3	0	0
χ_{15}	108	12	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-B	0	0	0	0	0	-1	3	0	0
χ_{16}	180	-12	20	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	180	-12	20	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	E	-E
χ_{18}	180	-12	20	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-E	E

A = $-\sqrt{3}$i, B = $-2\sqrt{2}$i, C = $-\sqrt{6}$i, D = $\frac{-1-\sqrt{15}i}{2}$, E=$\sqrt{3}$

Table 6. IrrProj(G,α_i) for $2^6{:}(L_3(2) \times S_3)$, $\alpha_1 = \alpha_2^2 = 1$

$[g]_G$	1a	2a	2b	3a	4a	2c	4b	4c	2d	4d	6a	4e	4f	2e	2f	3b	6b	3c	6c	12a	6d	12b	4g	4h	4i	4j	8a	14a	7a	21a	14b	7b	21b
$\lvert C_G(g)\rvert$	64512	3072	1536	504	384	2688	32	128	128	64	24	128	128	256	768	36	12	72	24	12	12	12	32	96	32	32	16	14	42	21	14	42	21
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	1
χ_3	2	2	2	-1	0	0	0	0	0	0	-1	2	2	2	2	-1	-1	2	2	0	0	-1	2	2	0	0	0	0	2	-1	0	2	-1
χ_4	3	3	3	3	-3	-3	1	1	1	1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	-1	-1	-1	M	-M	-M	$\overline{M}$	-$\overline{M}$	-$\overline{M}$
χ_5	3	3	3	3	-3	-3	1	1	1	1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	-1	-1	-1	$\overline{M}$	-$\overline{M}$	-$\overline{M}$	M	-M	-M
χ_6	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1	1	-$\overline{M}$	-$\overline{M}$	-$\overline{M}$	-M	-M	-M
χ_7	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1	1	-M	-M	-M	-$\overline{M}$	-$\overline{M}$	-$\overline{M}$
χ_8	6	6	6	6	-6	-6	-2	-2	-2	-2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	-1	-1
χ_9	6	6	6	6	6	6	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{10}	6	6	6	-3	0	0	0	0	0	0	1	-2	-2	-2	-2	0	0	0	0	0	0	-1	2	2	0	0	0	0	O	M	0	$\overline{O}$	$\overline{M}$
χ_{11}	6	6	6	-3	0	0	0	0	0	0	1	-2	-2	-2	-2	0	0	0	0	0	0	-1	2	2	0	0	0	0	$\overline{O}$	$\overline{M}$	0	O	M
χ_{12}	7	7	7	7	-7	-7	1	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	1	1	0	0	0	0	0	0
χ_{13}	7	7	7	7	7	7	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_{14}	8	8	8	8	-8	-8	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	0	0	0	0	0	0	-1	1	1	-1	1	1
χ_{15}	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1	1
χ_{16}	12	12	12	-6	0	0	0	0	0	0	-2	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	-2	1
χ_{17}	14	14	14	-7	0	0	0	0	0	0	1	-2	-2	-2	-2	-1	-1	2	2	0	0	1	-2	-2	0	0	0	0	0	0	0	0	0
χ_{18}	16	16	16	-8	0	0	0	0	0	0	0	0	0	0	0	1	1	-2	-2	0	0	0	0	0	0	0	0	0	2	-1	0	2	-1
χ_{19}	21	5	-3	0	-1	7	-1	3	3	-1	0	-3	1	1	9	0	0	3	-1	-1	1	0	-1	3	1	1	-1	0	0	0	0	0	0
χ_{20}	21	5	-3	0	1	-7	1	-3	-3	1	0	-3	1	1	9	0	0	3	-1	1	-1	0	-1	3	-1	-1	1	0	0	0	0	0	0
χ_{21}	21	5	-3	0	-1	7	-1	-1	-1	3	0	1	-3	5	-3	0	0	3	-1	-1	1	0	1	-3	-1	-1	1	0	0	0	0	0	0
χ_{22}	21	5	-3	0	1	-7	1	1	1	-3	0	1	-3	5	-3	0	0	3	-1	1	-1	0	1	-3	1	1	-1	0	0	0	0	0	0
χ_{23}	42	10	-6	0	-2	14	-2	2	2	2	0	-2	-2	6	6	0	0	-3	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	42	10	-6	0	2	-14	2	-2	-2	-2	0	-2	-2	6	6	0	0	-3	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	42	-6	2	0	0	0	0	4	-4	0	0	2	-2	-2	6	3	-1	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0
χ_{26}	42	-6	2	0	0	0	0	-4	4	0	0	2	-2	-2	6	3	-1	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0
χ_{27}	63	15	-9	0	-3	21	1	1	1	-3	0	-1	3	-5	3	0	0	0	0	0	0	0	1	-3	-1	-1	1	0	0	0	0	0	0
χ_{28}	63	15	-9	0	3	-21	-1	-1	-1	3	0	-1	3	-5	3	0	0	0	0	0	0	0	1	-3	1	1	-1	0	0	0	0	0	0
χ_{29}	63	15	-9	0	-3	21	1	-3	-3	1	0	3	-1	-1	-9	0	0	0	0	0	0	0	-1	3	1	1	-1	0	0	0	0	0	0
χ_{30}	63	15	-9	0	3	-21	-1	3	3	-1	0	3	-1	-1	-9	0	0	0	0	0	0	0	-1	3	-1	-1	1	0	0	0	0	0	0
χ_{31}	84	-12	4	0	0	0	0	0	0	0	0	4	-4	-4	12	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	126	-18	6	0	0	0	0	4	-4	0	0	-2	2	2	-6	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0
χ_{33}	126	-18	6	0	0	0	0	-4	4	0	0	-2	2	2	-6	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0
χ_1	4	4	4	4	A	A	0	0	0	0	0	0	0	0	0	1	1	1	1	G	G	0	0	0	0	0	0	N	-M	-M	-$\overline{N}$	-$\overline{M}$	-$\overline{M}$
χ_2	4	4	4	4	A	A	0	0	0	0	0	0	0	0	0	1	1	1	1	G	G	0	0	0	0	0	0	-$\overline{N}$	-$\overline{M}$	-$\overline{M}$	N	-M	-M
χ_3	4	4	4	4	-A	-A	0	0	0	0	0	0	0	0	0	1	1	1	1	-G	-G	0	0	0	0	0	0	$\overline{N}$	-$\overline{M}$	-$\overline{M}$	-N	-M	-M
χ_4	4	4	4	4	-A	-A	0	0	0	0	0	0	0	0	0	1	1	1	1	-G	-G	0	0	0	0	0	0	-N	-M	-M	$\overline{N}$	-$\overline{M}$	-$\overline{M}$
χ_5	6	6	6	6	B	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	H	H	H	K	K	K	-G	1	1	-G	1	1
χ_6	6	6	6	6	B	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-H	-H	-H	-K	-K	-K	-G	1	1	-G	1	1
χ_7	6	6	6	6	-B	-B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	H	H	H	-K	-K	-K	G	1	1	G	1	1
χ_8	6	6	6	6	-B	-B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-H	-H	-H	K	K	K	G	1	1	G	1	1
χ_9	8	8	8	8	C	C	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	G	G	0	0	0	0	0	0	G	-1	-1	G	-1	-1
χ_{10}	8	8	8	8	-C	-C	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-G	-G	0	0	0	0	0	0	-G	-1	-1	-G	-1	-1
χ_{11}	8	8	8	-4	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	0	0	0	0	0	0	0	0	0	O	M	0	$\overline{O}$	$\overline{M}$
χ_{12}	8	8	8	-4	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	0	0	0	0	0	0	0	0	0	$\overline{O}$	$\overline{M}$	0	O	M
χ_{13}	12	12	12	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-H	I	I	0	0	0	0	2	-1	0	2	-1
χ_{14}	12	12	12	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	H	-I	-I	0	0	0	0	2	-1	0	2	-1
χ_{15}	16	16	16	-8	0	0	0	0	0	0	0	0	0	0	0	1	1	-2	-2	0	0	0	0	0	0	0	0	0	-2	1	0	-2	1
χ_{16}	42	10	-6	0	D	E	0	0	0	0	0	0	0	0	0	0	0	-3	1	G	-G	0	H	J	K	K	-K	0	0	0	0	0	0
χ_{17}	42	10	-6	0	D	E	0	0	0	0	0	0	0	0	0	0	0	-3	1	G	-G	0	-H	-J	-K	-K	K	0	0	0	0	0	0
χ_{18}	42	10	-6	0	-D	-E	0	0	0	0	0	0	0	0	0	0	0	-3	1	-G	G	0	H	J	-K	-K	K	0	0	0	0	0	0
χ_{19}	42	10	-6	0	-D	-E	0	0	0	0	0	0	0	0	0	0	0	-3	1	-G	G	0	-H	-J	K	K	-K	0	0	0	0	0	0
χ_{20}	84	20	-12	0	-A	F	0	0	0	0	0	0	0	0	0	0	0	3	-1	-G	G	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}	84	20	-12	0	A	-F	0	0	0	0	0	0	0	0	0	0	0	3	-1	G	-G	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	84	-12	4	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	L	-L	0	0	0	0	0	0	0
χ_{23}	84	-12	4	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	-L	L	0	0	0	0	0	0	0
χ_{24}	168	-24	8	0	0	0	0	0	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A = -4i, B = 6i, C = 8i, D = 2i, E = -14i, F = -28i, G = i, H = $\sqrt{2}$, I = $2\sqrt{2}$, J = $-3\sqrt{2}$, K = $-\sqrt{2}$i, L = $-2\sqrt{2}$i, M = $\frac{1-\sqrt{7}i}{2}$,
N = $E(28)^{11}+E(28)^{15}+E(28)^{23}$, O= $-1+\sqrt{7}i$

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