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Numerical solution of linear and nonlinear elliptic partial differential equations using lifting scheme

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Abstract Wavelet theory is a recently developed promising tool in science and engineering research; particularly wavelets are successfully used in fast algorithms for easy implementation. In this paper, we present wavelet-based numerical solutions of elliptic boundary value problems encountered in mathematical physics, using lifting scheme. Here we used orthogonal and biorthogonal wavelets to demonstrate the effectiveness and efficiency of the proposed technique. The proposed scheme speeds up convergence in lesser computational time. Numerical examples are given to prove the robustness of proposed technique.

1 Introduction

The full-approximation scheme (FAS) is a well known computational technique for accelerating convergence to the steady state for flow problems and has proved to be very successful for solving elliptic equations. The ill-conditioned systems are arising in the solution of nonlinear system of equations. The suitable remedy is FAS for such systems and is largely applicable in increasing the efficiency of the iterative methods to solve nonlinear system of equations. In the history of numerical analysis, the development of efficient error minimization techniques for the system of equations has been a significant research. Recently, it is renowned that FAS iterative solvers are extremely efficient for nonlinear equations, introduced by Brandt [1]. For a detailed treatment of FAS is presented in Briggs et al. [2]. An introduction of FAS is available in Hackbusch and Trottenberg [3], Wesseling [4] and Trottenberg et al. [5]. However, for the elliptic differential equations with highly oscillatory coefficients, the standard FAS is not efficient. But, wavelets, particularly, Daubechies wavelets have great potential in this regard.

Wavelet analysis has its significance due to successful applications in signal and image processing during the 1980s. The smooth orthonormal basis obtained by the translation and dilation of a single function in a hierarchical fashion proved very useful to develop compression algorithms for signals and images upto a chosen threshold of relevant amplitudes. While the existence of the Haar type of wavelet functions has been known for a long time, the study of wavelets acquired the present growth after the mathematical analysis of wavelets by Stromberg [6], Grossmann and Morlet [7], and Meyer [8]. The multiresolution approximation of Mallat [9] and Meyer [10] led to Daubechies [11] orthogonal family of wavelets. Recently wavelets have been applied in a wide range of engineering disciplines; particularly, wavelets are successfully used in signal analysis, time-frequency analysis and fast algorithms for easy implementation. Wavelet based numerical methods are used for solving the system of equations with better convergence in less computational cost. Some of the earlier works on wavelet based methods can be found in Dahmen et al. [12].

A collection of the discrete wavelet transforms (DWT) and the wavelet based full-approximation scheme (WFAS) were introduced recently in [13-16]. Shiralashetti et al. [17] had proposed the

new wavelet based full-approximation scheme for the numerical solution of non-linear elliptic partial differential equations. Similarly, the biorthogal wavelet method is applied for the solution of elliptic partial differential equations [18]. The method can be either used as an iterative solver or as a preconditioning technique, offering in many cases a better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, the new work had built i. e orthogonal/biorthogonal discrete wavelet transforms using lifting scheme [19]. Wavelet based lifting technique was introduced by Sweldens [20], which permits some improvements on the properties of existing wavelet transforms. The technique has many numerical benefits such as a reduced number of operations, which are fundamental in the context of the iterative solvers. Shiralashetti et al. [21] applied wavelet lifting technique for the solution of non-linear partial differential equations. In addition to this, the present paper illustrates that the application of the lifting technique to the science and engineering problems.
The paper is divided as follows: Preliminaries of wavelets are given in section 2. Lifting technique is presented in section 3. Section 4 highlights the method of solution. Numerical solutions of the test problems are presented in section 5. Finally, concluding remarks of the paper are discussed in section 6.

2 Preliminaries of wavelets

Orthogonal and biorthogonal wavelet coefficients based on the orthogonality and smoothness conditions that must be satisfied by scaling and wavelet functions. These conditions impose restrictions on the value of filter coefficients through dilation equations. Fortunately we have two distinct functions called scaling functions and wavelet functions with coefficients $\{h_k\}$ and $\{g_k\}$ that define the refinement relation. These coefficients decide shape of the scaling and wavelet functions and act as signal filters, in which the application where we can use the particular wavelet. The huge amount of literature is available on filter design for specific application but it is isolated to a regular reader since practically all methods use frequency domain as well as complex analysis concepts to arrive at the filter.

2.1 Wavelet filters

The most important classes of filters are those of finite impulse response (FIR). The main characteristics of these filters are the convenient time-localization properties. These filters are initiated from wavelets with compact support and are such that,

$$h_n = 0 \quad \text{for} \quad n < 0 \quad \text{and} \quad n > L$$

in which L is the length of the filter.
The minimum requirements for these compact FIR filters are:

(i) The length of the scaling filter h_n must be even.

(ii) $\sum_n h_n = \sqrt{2}$

(iii) $\sum_n (h_n \cdot h_{n-2k}) = \delta(k)$,

in which $\delta(k)$ is the Kronecker delta, such that, it is equal to 1 for $k = 0$ or 0 for $k = 1$.

2.2 Haar wavelet filter coefficients

We know that low pass filter coefficients $h = [h_0,\ h_1]^T = \left[\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right]^T$ and high pass filter coefficients $g = [g_0,\ g_1]^T = \left[\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right]^T$ play an important role in the decomposition. Thus it is natural to wonder that, it possible to model the decomposition in terms of linear transformations. Moreover, since the digital signals and images are composed of discrete data, then we need a discrete analog of the decomposition algorithm so that we can process the signal and image data.

2.3 Daubechies wavelet filter coefficients

Daubechies introduced scaling functions that satisfy the above requirements and distinguished by having the shortest possible support. The scaling function ϕ has support$[0,\ L-1]$, while the corresponding wavelet ψ has support in the interval $[1-L/2,\ L/2]$. We have filter coefficients [22], $h = [h_0,\ h_1,\ h_2,\ h_3]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$ are low pass filter coefficients and $g = [g_0,\ g_1,\ g_2,\ g_3]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}},\ -\frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$ are the high pass filter coefficients.

2.4 Biorthogonal (CDF (2, 2)) wavelet filter coefficients

In many filtering applications, we need filter coefficients having symmetry to get a better accuracy. None of the orthogonal wavelet systems except Haar are having symmetrical coefficients. But Haar is too insufficient for many applications in science and engineering. Biorthogonal wavelet system can be constructed to have this feature. This is the motivation for designing such wavelet systems. The following are the biorthogonal (CDF (2, 2)) wavelet filter coefficients [23, 24] are,
low pass filters: $h = [h_0,\ h_1,\ h_2] = \left[\frac{1}{2\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{1}{2\sqrt{2}}\right]$ and

$$\tilde{h} = \left[\tilde{h}_0,\ \tilde{h}_1,\ \tilde{h}_2,\ \tilde{h}_3,\ \tilde{h}_4\right] = \left[\frac{-\sqrt{2}}{8},\ \frac{\sqrt{2}}{4},\ \frac{3\sqrt{2}}{4},\ \frac{\sqrt{2}}{4},\ \frac{-\sqrt{2}}{8}\right].$$

Similarly, high pass filters: $g_k = (-1)^k \tilde{h}_{4-k}$ and $\tilde{g}_k = (-1)^{k+1} h_{2-k}$.

3 Lifting Technique

The wavelet transform customs averages and differences, brings us to the definition of the lifting procedure. The operations, average and difference, can be observed as distinct cases of more general operations. If two data are almost equal the difference is, of course small and it is therefore obvious to think of the first data as a prediction of the second one. It is a good prediction, if the difference is small. We also calculated the average of the two data. This can be viewed in two cases. Either as an operation, this preserves some properties of the original data, or as an extraction of essential properties of the data. The concluding viewpoint is based on the fact that the pair-wise average values containing the overall structure of the data, but with only half the number of data. The lifting procedure has three steps i.e. split, prediction and update.
Split: The given data are split into the even and odd entries. It is important to observe that we do this only to explain the functionality in the algorithm.
Prediction: The given value at the data 2n, we predict that the value at data 2n+1 is the same. Then we replace the value at 2n+1 with the correction to the prediction, which is the difference. Generally, the idea is to have a prediction procedure P then compute

$$d = \text{odd} - \text{P}(\text{even}).$$

In the data *d*, each entry is one odd data minus some prediction on an even data.
Update: Given an even entry we have predicted that the next odd entry has same value, and stored the difference. Then we update even entry. In general we decide an updating procedure, and then compute

$$s = \text{even} + \text{U}(d).$$

The algorithm described here is called one step lifting. It requires the choice of a prediction P and an update U. The discrete wavelet transformed is obtained by combining a number of lifting steps.
Now we look in to lifting technique in general. First let us see how we can invert the lifting procedure. It is observed by just reverse in the arrows and changing the signs. Thus the direct transform

$$d_j = \text{odd}_j - \text{P}(\text{even}_j)$$

$$s_j = \text{even}_j + \text{U}(d_j)$$

is inverted by the steps

$$\text{even}_j = s_j - \text{U}(d_j)$$
$$\text{odd}_j = d_j + \text{P}(\text{even}_j)$$

The last step, where the sequences even_j and odd_j are merged to the form of sequence s and is given to explain in the algorithm.
It is observed that the generalization is crucial in applications. There are many important transforms, where the above steps do not occur in pairs. Furthermore, in last we add a new type of operation which is called normalization or sometimes rescaling [19]. The detailed algorithm using different wavelets is given in the next section.

4 Method of solution

Consider the elliptic partial differential equation of the form,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,\, y) \tag{4.1}$$

where $F(x,\, y)$ is a non homogeneous term in x and y subjected to Dirichlet boundary conditions. By applying the finite difference scheme to Eq. (4.1), which gives the system of algebraic equations,

$$Au = F_{i\,j}, \quad 1 \le i,\, j \le N \tag{4.2}$$

where $N = 2^J$, N is the number of grid points and J is the level of resolution.
By solving Eq. (4.2), we get an approximate solution u. An approximate solution containing some error, thus required solution equals to sum of approximate solution and some error.
There are many methods to minimize such error to get the accurate solution. Some of them are FAS, WFAS and BWFAS etc. Now, we are using the advanced technique based on different wavelets called as lifting scheme. Recently, lifting schemes are very useful in the signal analysis and image processing in the science and engineering field. But nowadays, extends to approximations in the numerical analysis. Here, we are discussing the algorithm [19] of the lifting schemes as follows.

4.1 Lifting scheme via Haar wavelet (HWLS)

In [25], Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [19, 20]. The representation of Haar wavelet via lifting form presented as;
Decomposition:
Consider an approximate solution $S = u$ like as signal, and then apply the HWLS decomposition (finer to coarser) procedure as,

$$\begin{aligned} d_1 &= S(2j) - S(2j-1), \\ s_1 &= S(2j-1) + \tfrac{1}{2} d_1, \\ S_1 &= \sqrt{2}\, s_1 \quad \text{and} \\ D &= \tfrac{1}{\sqrt{2}} d_1 \end{aligned} \tag{4.3}$$

Reconstruction:
Now apply the HWLS reconstruction (coarser to finer) procedure as,

$$\begin{aligned} d_1 &= \sqrt{2}\, D, \\ s_1 &= \tfrac{1}{\sqrt{2}} S_1, \\ S(2j-1) &= s_1 - \tfrac{1}{2} d_1 \quad \text{and} \\ S(2j) &= d_1 + S(2j-1) \end{aligned} \tag{4.4}$$

which is the required solution of the given equation.

4.2 Lifting scheme via Daubechies wavelet (DWLS)

As discussed in the previous section 4.1, we follow the same procedure but using different wavelet i.e., Daubechies 4^{th} order (Db4) wavelet coefficients. The DWLS procedure is as follows;

Decomposition:

$$\begin{aligned} s_1 &= S(2j-1) + \sqrt{3}\,S(2j), \\ d_1 &= S(2j) - \tfrac{\sqrt{3}}{4}s_1 - \tfrac{\sqrt{3}-2}{4}s_1(j-1), \\ s_2 &= s_1 - d_1(j+1), \\ S_1 &= \tfrac{\sqrt{3}-1}{\sqrt{2}}s_2 \quad \text{and} \\ D &= \tfrac{\sqrt{3}+1}{\sqrt{2}}d_1 \end{aligned} \tag{4.5}$$

Reconstruction:
Now we apply the DWLS reconstruction (coarser to finer) procedure as,

$$\begin{aligned} d_1 &= \tfrac{\sqrt{2}}{\sqrt{3}+1}D, \\ s_2 &= \tfrac{\sqrt{2}}{\sqrt{3}-1}\,S_1, \\ s_1 &= s_2 + d_1(j+1), \\ S(2j) &= d_1 + \tfrac{\sqrt{3}}{4}s_1 + \tfrac{\sqrt{3}-2}{4}s_1(j-1) \quad \text{and} \\ S(2j-1) &= s_1 - \sqrt{3}\,S(2j) \end{aligned} \tag{4.6}$$

which is the required solution of the given equation.

4.3 Lifting scheme via biorthogonal wavelet (BWLS)

As discussed in the previous sections 4.1 and 4.2, here also we follow the same procedure but we used another wavelet i.e., biorthogonal wavelet (CDF (2, 2)) coefficients. The BWLS procedure is as follows;

Decomposition:

$$\begin{aligned} d_1 &= S(2j) - \tfrac{1}{2}\left[S(2j-1) + S(2j+2)\right], \\ s_1 &= S(2j-1) + \tfrac{1}{4}\left[d_1(j-1) + d_1\right], \\ D &= \tfrac{1}{\sqrt{2}}d_1, \\ S_1 &= \sqrt{2}\,s_1 \end{aligned} \tag{4.7}$$

Reconstruction:
Now we apply the BWLS reconstruction (coarser to finer) procedure as,

$$\begin{aligned} s_1 &= \tfrac{1}{\sqrt{2}}S_1, \\ d_1 &= \sqrt{2}\,D, \\ S(2j-1) &= s_1 - \tfrac{1}{4}\left[d_1(j-1) + d_1\right] \\ S(2j) &= d_1 + \tfrac{1}{2}\left[S(2j-1) + S(2j+2)\right], \end{aligned} \tag{4.8}$$

which is the required solution of the given equation.
The coefficients $s_1(j)$ and $d_1(j)$ are the average and detailed coefficients respectively of the approximate solution u. The new approaches are illustrated through some of the numerical problems and the results are shown in next section.

5 Numerical examples

Here, we present some of the test problems to demonstrate the validity and applicability of the HWLS, DWLS and BWLS.

Test Problem 5.1: First, we consider the linear elliptic partial differential equation (Poisson equation),

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2\pi^2 \left(\sin(\pi x) + \sin(\pi y)\right) \tag{5.1}$$

subject to Dirichlet's boundary conditions. The exact solution for this problem is given by $u(x, y) = \sin(\pi x)\sin(\pi y)$. The wavelet based numerical solutions of Eqn. (5.1) are obtained as per the procedure explained in section 4 and are presented in comparison with exact solution in figure 1.
The maximum error $E_{\text{max}} = \max |u_e - u_a|$, where $u_e \& u_a$ are exact and approximate solutions respectively. The error analysis and CPU time versus grid points is given in table 1.

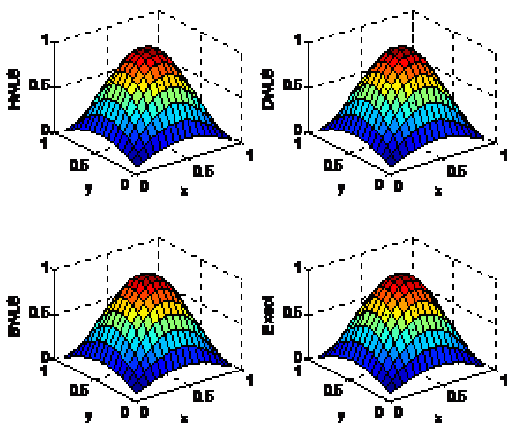

Figure 1. Comparison of numerical solutions with exact solution for N=16 X 16 of Test Problem 5.1.

Table 1. The maximum error with CPU time (in seconds) versus grid points of Test Problem 5.1.

N	Method	E_{max}	Setup time	Running time	Total time
8X8	HWLS	9.9080e-03	7.1466e-04	1.3209e-03	2.0355e-03
	WFAS	9.9080e-03	2.6126e-02	8.8605e-04	2.7012e-02
	DWLS	9.9080e-03	6.0348e-04	6.1302e-03	6.7337e-03
	BWFAS	9.9080e-03	3.7374e-02	8.7202e-04	3.8246e-02
	BWLS	9.9080e-03	5.0632e-04	1.3305e-03	1.8368e-03
16X16	HWLS	2.8265e-03	6.9447e-04	1.2805e-03	1.9750e-03
	WFAS	2.8265e-03	2.6091e-01	2.0345e-03	2.6295e-01
	DWLS	2.8265e-03	5.9766e-04	6.0915e-03	6.6892e-03
	BWFAS	2.8265e-03	2.6091e-01	2.0345e-03	2.6295e-01
	BWLS	2.8265e-03	4.8202e-04	2.6753e-03	3.1573e-03
32X32	HWLS	7.5388e-04	6.7155e-04	1.2716e-03	1.9432e-03
	WFAS	7.5388e-04	2.6349e-01	4.3437e-03	2.6784e-01
	DWLS	7.5388e-04	5.8500e-04	6.0871e-03	6.6721e-03
	BWFAS	7.5388e-04	3.6823e+00	3.1822e-03	3.6855e+00
	BWLS	7.5388e-04	4.8545e-04	2.7050e-03	3.1905e-03

Test Problem 5.2: Now, we consider the nonlinear elliptic partial differential equation,

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + ue^u = 2(x - x^2) + 2(y - y^2) + (x - x^2)(y - y^2)e^{(x-x^2)(y-y^2)} \tag{5.2}$$

with respect to Dirichlet's boundary conditions. The exact solution of the problem is given by$u(x, y) = (x - x^2)(y - y^2)$. As in the previous example, the numerical results of Eqn. (5.2) are presented in figure 2. The error analysis and CPU time versus grid points is given in table 2.

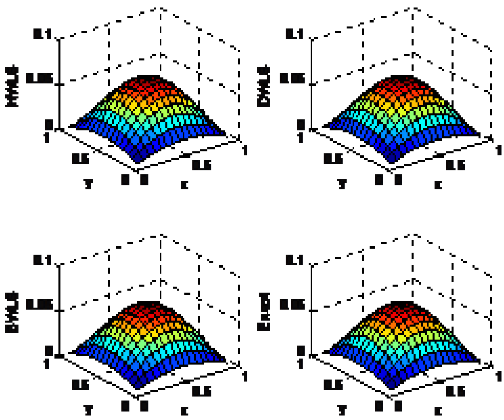

Figure 2. Comparison of numerical solutions with exact solution for N=16 X 16 of Test Problem 5.2.

Table 2. The maximum error with CPU time (in seconds) versus grid points of Test Problem 5.2.

N	Method	$E_{\max}$	Setup time	Running time	Total time
8X8	HWLS	1.6302e-05	5.2718e-04	8.1831e-04	1.3455e-03
	WFAS	1.5771e-05	2.0915e-02	2.4905e-04	2.1164e-02
	DWLS	1.6302e-05	4.0300e-04	8.2197e-03	8.6227e-03
	BWFAS	1.5934e-05	2.1791e-02	2.3160e-04	2.2023e-02
	BWLS	1.6302e-05	7.1329e-04	2.6818e-03	3.3950e-03
16X16	HWLS	6.3642e-06	5.2445e-04	8.2413e-04	1.3486e-03
	WFAS	6.2978e-06	1.6368e-02	2.5692e-04	1.6625e-02
	DWLS	6.3642e-06	4.0026e-04	7.9751e-03	8.3754e-03
	BWFAS	6.3402e-06	1.7256e-02	2.3229e-04	1.7488e-02
	BWLS	6.3642e-06	7.2389e-04	2.6876e-03	3.4115e-03
32X32	HWLS	3.6802e-06	5.4702e-04	8.3610e-04	1.3831e-03
	WFAS	3.6702e-06	1.6552e-02	2.5418e-04	1.6807e-02
	DWLS	3.6802e-06	3.9924e-04	8.1592e-03	8.5584e-03
	BWFAS	3.6768e-06	1.9340e-02	2.4016e-04	1.9580e-02
	BWLS	3.6802e-06	7.2389e-04	2.7122e-03	3.4361e-03

Test Problem 5.3: Finally, we consider the nonlinear elliptic partial differential equation,

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + ue^u = \left(\left(9\pi^2 + e^{(\mathrm{x}^2-\mathrm{x}^3)\sin(3\pi \mathrm{y})}\right)(\mathrm{x}^2 - \mathrm{x}^3) + 6x - 2\right)\sin(3\pi \mathrm{y}) \tag{5.3}$$

subjected to Dirichlet's boundary conditions. The exact solution of the problem is given by $u(x, y) = (x^2 - x^3)\sin(3\pi y)$. As in the previous examples, the numerical solutions of Eqn. (5.3) are presented in figure 3. The error analysis and CPU time versus grid points is given in table 3.

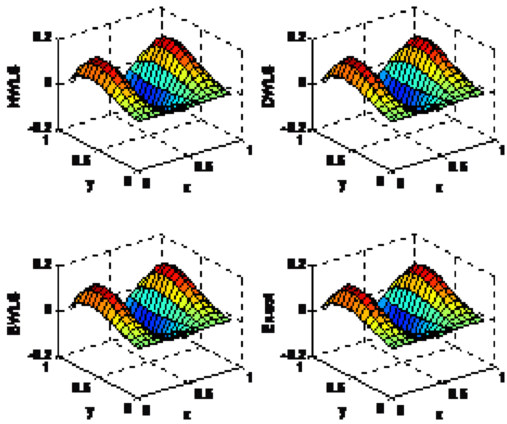

Figure 3. Comparison of numerical solutions with exact solution for N=16 X 16 of Test Problem 5.3.

Table 3. The maximum error with CPU time (in seconds) versus grid points of Test Problem 5.3.

N	Method	$E_{\max}$	Setup time	Running time	Total time
8X8	HWLS	1.0530e-02	5.4634e-04	8.5116e-04	1.3975e-03
	WFAS	1.0530e-02	1.7440e-02	1.2076e-04	1.7561e-02
	DWLS	1.0530e-02	4.3276e-04	8.0364e-03	8.4691e-03
	BWFAS	1.0530e-02	1.8024e-02	5.3916e-04	1.8563e-02
	BWLS	1.0530e-02	7.2184e-04	2.6705e-03	3.3923e-03
16X16	HWLS	3.2596e-03	5.4566e-04	8.5834e-04	1.4040e-03
	WFAS	3.2596e-03	1.7372e-02	1.2110e-04	1.7493e-02
	DWLS	3.2596e-03	4.3208e-04	7.8448e-03	8.2769e-03
	BWFAS	3.2596e-03	1.8069e-02	5.3984e-04	1.8609e-02
	BWLS	3.2596e-03	7.8547e-04	2.9154e-03	3.7009e-03
32X32	HWLS	8.6292e-04	5.6687e-04	8.7202e-04	1.4389e-03
	WFAS	8.6292e-04	1.7537e-02	1.2247e-04	1.7660e-02
	DWLS	8.6292e-04	4.3379e-04	7.7685e-03	8.2023e-03
	BWFAS	8.6292e-04	1.8244e-02	5.4394e-04	1.8788e-02
	BWLS	8.6292e-04	7.1466e-04	2.7334e-03	3.4481e-03

6 Conclusions

In this paper, we developed an efficient wavelet based lifting technique for the numerical solution of elliptic problems. From the figures and tables, the proposed schemes are very convenient and effective. However the CPU time of the proposed scheme shows the super convergence than the existing ones (Wavelet based FAS). Hence the scheme has wide range of applications in science and engineering field.

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