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SOME CURVES ON THREE-DIMENSIONAL ALMOST KENMOTSU MANIFOLDS

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Abstract. The object of the present paper is to study some curves on three-dimensional almost Kenmotsu manifolds with the $(\kappa,\mu)'$-nullity distribution. In this paper, we study biharmonic almost contact curves and slant curves on three-dimensional almost Kenmotsu manifolds with the $(\kappa,\mu)'$-nullity distribution. We also study ϕ-symmetric curves on almost Kenmotsu manifolds and a nontrivial example of Legendre curve is given.

1 INTRODUCTION

The notion of k-nullity distribution was first introduced by Gray [9] and Tanno [22] in the study of Riemannian manifold (M,g), which is defined for any $p\in M$ as follows

$$N_p(k)=\{Z\in T_pM : R(X,Y)Z=k[g(Y,Z)X-g(X,Z)Y]\}, \tag{1.1}$$

where X,Y and Z denote arbitrary vectors in T_pM and $k\in R$. Blair et al. [5] introduced a generalized notion of the k-nullity distribution named the (k,μ)-nullity distribution on a contact metric manifold $(M^{2n+1},\phi,\xi,\eta,g)$, which is defined for any $p\in M^{2n+1}$ as follows

$$\begin{aligned}N_p(k,\mu)=\{Z\in T_pM : R(X,Y)Z=&k[g(Y,Z)X-g(X,Z)Y]\\&+\mu[g(Y,Z)hX-g(X,Z)hY]\},\end{aligned} \tag{1.2}$$

where $h=\frac{1}{2}L_\xi\phi$ and L denotes the Lie differentiation and $(k,\mu)\in R^2$. Later, Dileo and Pastore [7] introduced another generalized notion of the k-nullity distribution which is named $(k,\mu)'$-nullity distribution on an almost Kenmotsu manifold $(M^{2n+1},\phi,\xi,\eta,g)$ and is defined for any $p\in M^{2n+1}$ as follows

$$\begin{aligned}N_p(k,\mu)'=\{Z\in T_pM : R(X,Y)Z=&k[g(Y,Z)X-g(X,Z)Y]\\&+\mu[g(Y,Z)h'X-g(X,Z)h'Y]\},\end{aligned} \tag{1.3}$$

where $h'=h\circ\phi$ and $(k,\mu)\in R^2$. If both k and μ in relation (1.2) are smooth function on M^{2n+1}, then such a nullity distribution is called a generalized (k,μ)-nullity distribution.

In [1], M. Atceken studied warped product semi-slant submanifolds and proved non-existence of such submanifolds in Kenmotsu manifolds. The geometry of warped product pointwise semi-slant submanifolds of locally product Riemannian manifolds were studied in [27]. The warped product semi-slant submanifolds of Kenmotsu manifolds and bi-slant submanifolds of Kaehler manifolds were studied in ([6], [23], [24], [25]). Also the Classification of totally umbilical slant submanifolds of a Kenmotsu manifold were studied in [26].

After the work of Baikoussis and Blair [2], the study of curves on contact manifolds has become a popular topic. They have studied Legendre curves on contact three-manifolds. Initially, Legendre curves were studied only on contact manifolds. Later, the Legendre curves on almost contact manifolds have been also studied on almost contact manifolds. Legendre curves have been given the name almost contact curves. For detailed we refer the references given in [19]. Legendre curves have been studied on manifolds with Lorentzian metric side by side of Riemannian metric. In [3], the authors have studied Legendre curves on Lorentzian Sasakian manifolds.

Trans-Sasakian manifolds form an important class of almost contact manifolds. It generalizes a large number of contact and almost contact manifolds. Recently, in [8] Lorentzian trans-Sasakian manifolds were studied. In [11] a large class of almost contact manifolds were studied admitting different types of curves. The present authors have studied some curves on trans-Sasakian manifolds admitting semi-symmetric metric connections [20]. The present author also studied some curves on three-dimensional Kenmotsu space form [13].

The present paper is organized as follows:
We give the required preliminaries and some basic results of almost Kenmotsu manifolds in Section 2. Section 3, contains some well known results on almost Kenmotsu manifolds with generalized $(\kappa,\mu)'$-nullity distribution. In Section 4, we study slant curves and C-parallel slant curves on three-dimensional almost Kenmotsu manifolds with the $(\kappa,\mu)'$-nullity distribution. Section 5, is devoted to study biharmonic Legendre curves on almost Kenmotsu manifolds with the $(\kappa,\mu)'$-nullity distribution. In Section 6, we study locally ϕ-symmetric Legendre curves on almost Kenmotsu manifolds with the $(\kappa,\mu)'$-nullity distribution. Finally we construct an example of three-dimensional almost Kenmotsu manifold.

2 PRELIMINARIES

Let M be a $(2n+1)$-dimensional connected differentiable manifold together with an almost contact metric structure (ϕ,ξ,η,g), where ϕ is a $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and g is a compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.1}$$

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X,\xi), \tag{2.2}$$

and

$$g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(X,\phi Y) = -g(\phi X,Y). \tag{2.3}$$

An almost contact structure endowed with a compatible Riemannian metric is said to be an almost contact metric structure. Moreover, a manifold endowed with an almost contact metric structure is said to be an almost contact metric manifold. The fundamental 2-form Φ on an almost contact metric manifold M^{2n+1} is defined by $\Phi(X,Y) = g(X,\phi Y)$ for any vector fields X,Y on M^{2n+1}. An almost Kenmotsu manifold is defined as an almost contact metric manifold such that $d\eta = 0$ and $d\Phi = 2\eta\wedge\Phi$. An almost contact metric manifold is normal if $[\phi,\phi](X,Y) + 2d\eta(X,Y)\xi = 0$. We see that the normality of an almost Kenmotsu manifold is expressed by [7]

$$(\nabla_X\phi)Y = g(\phi X + hX, Y)\xi - \eta(Y)(\phi X + hX), \tag{2.4}$$

for any vector fields X,Y on M^{2n+1}.

Let us denote the distribution orthogonal to ξ by D and denoted by $D = ker(\eta) = im(\phi)$. In an almost Kenmotsu manifold, since η is closed, D is an integrable distribution. We consider two tensor fields $l = R(.,\xi)\xi$ and $h = \frac{1}{2}L_\xi\phi$ on an almost Kenmotsu manifold $(M^{2n+1},\phi,\xi,\eta,g)$, where R is the Riemannian curvature tensor of g and L is the Lie differentiation. From Dileo and Pastore [7] and Kim and Pak [12], we know that the two $(1,1)$ type tensor fields l and h are symmetric and satisfy

$$h\xi = 0, \quad l\xi = 0, \quad trh = 0, \quad tr(h\phi) = 0, \quad h\phi + \phi h = 0, \tag{2.5}$$

$$\nabla_X\xi = -\phi^2X - \phi hX, \tag{2.6}$$

$$\phi h\phi - l = 2(h^2 - \phi^2), \tag{2.7}$$

$$tr(l) = S(\xi,\xi) = g(\phi\xi,\xi) = -2n - trh^2, \tag{2.8}$$

$$R(X,Y)\xi = \eta(X)(Y - \phi hY) - \eta(Y)(X - \phi hX) + (\nabla_Y\phi h)X - (\nabla_X\phi h)Y, \tag{2.9}$$

for any vector fields X and Y, where S, Q, ∇ denote the Ricci curvature tensor, the Ricci operator with respect to the metric g, the Levi-Civita connection of g, respectively. The (1,1)-type symmetric tensor field $h' = h \circ \phi$ is anticommuting with ϕ and $h'\xi = 0$. Also it is clear that

$$h = 0 \Leftrightarrow h' = 0, \qquad {h'}^2 = (k+1)\phi^2 (\Leftrightarrow h^2 = (k+1)\phi^2). \tag{2.10}$$

3 ALMOST KENMOTSU MANIFOLD WITH ξ BELONGING TO THE $(k, \mu)'$-NULLITY DISTRIBUTION

In this section we study almost Kenmotsu manifolds with ξ belonging to the $(k, \mu)'$-nullity distribution.

Let $X \in D$ be the eigen vector of h' corresponding to the eigen value λ. Then $h'X = \lambda X$ implies $h'^2 X = \lambda^2 X$. From (2.10) we get $\lambda^2 X = (k+1)\phi^2 X$. Hence $\lambda^2 X = -(k+1)X$ which implies $\lambda^2 = -(k+1)$, a constant. Therefore $k \leq -1$ and $\lambda = \pm\sqrt{-k-1}$. We denote by $[\lambda]'$ and $[-\lambda]'$ the corresponding eigen space related to the non-zero eigen value λ and $-\lambda$ of h', respectively.

Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold such that $h' \neq 0$ and ξ belongs to the $(k, \mu)'$-nullity distribution. Then for any $X_\lambda, Y_\lambda, Z_\lambda \in [\lambda]'$ and $X_{-\lambda}, Y_{-\lambda}, Z_{-\lambda} \in [-\lambda]'$, the Riemannian tensor satisfies (*Proposition 4.2 of [7]*)

$$R(X_\lambda, Y_\lambda)Z_{-\lambda} = 0, \tag{3.1}$$

$$R(X_{-\lambda}, Y_{-\lambda})Z_\lambda = 0, \tag{3.2}$$

$$R(X_\lambda, Y_{-\lambda})Z_\lambda = (k+2)g(X_\lambda, Z_\lambda)Y_{-\lambda}, \tag{3.3}$$

$$R(X_\lambda, Y_{-\lambda})Z_{-\lambda} = -(k+2)g(Y_{-\lambda}, Z_{-\lambda})X_\lambda, \tag{3.4}$$

$$R(X_\lambda, Y_\lambda)Z_\lambda = (k-2\lambda)[g(Y_\lambda, Z_\lambda)X_\lambda - g(X_\lambda, Z_\lambda)Y_\lambda], \tag{3.5}$$

$$R(X_{-\lambda}, Y_{-\lambda})Z_{-\lambda} = (k+2\lambda)[g(Y_{-\lambda}, Z_{-\lambda})X_{-\lambda} - g(X_{-\lambda}, Z_{-\lambda})Y_{-\lambda}], \tag{3.6}$$

where $\lambda^2 = -(k+1)$.

Let M be a 3-dimensional Riemannian manifold. Let $\gamma : I \to M$, I being an interval, be a curve in M which is parameterized by the arc length, and let $\nabla_{\dot\gamma}$ denotes the covariant differentiation along γ with respect to the Levi-Civita connection on M. It is said that γ is a Frenet curve if one of the following three cases holds:

(a) γ is of osculating order 1, if, $\nabla_T T = 0$ (geodesic), $T = \dot\gamma$. Here, . denotes differentiation with respect to the arc length.

(b) γ is of osculating order 2, if,there exist two orthonormal vector fields $T(= \dot\gamma)$, N and a non-negative function κ (curvature) along γ such that $\nabla_T T = \kappa N$, $\nabla_T N = -\kappa T$.

(c) γ is of osculating order 3, if, there exist three orthonormal vectors $T = (\dot\gamma), N, B$ and two non-negative function κ(curvature) and τ(torsion) along γ such that

$$\nabla_T T = \kappa N, \tag{3.7}$$

$$\nabla_T N = -\kappa T + \tau B, \tag{3.8}$$

$$\nabla_T B = -\tau N, \tag{3.9}$$

where $T = \dot{\gamma}$ and $\{T, N, B\}$ is the Frenet frame κ and τ are the curvature and torsion of the curve. With respect to Levi-Civita connection, a Frenet curve of osculating order 3 is called a geodesic if $\kappa = 0$. It is called a circle if κ is a positive constant and $\tau = 0$. The curve is called a helix in M if κ and τ both are positive constants and the curve is called a generalized helix if $\frac{\kappa}{\tau} =$ constant.

A Frenet curve γ in an almost contact metric manifold is said to be a Legendre curve or almost contact curve if it is an integral curve of the contact distribution $D = \ker\eta$. Formally, it is said that a Frenet curve γ in an almost contact metric manifold is a Legendre curve if and only if $\eta(\dot{\gamma}) = 0$ and $g(\dot{\gamma}, \dot{\gamma}) = 1$. For more details we refer [19].

4 SLANT CURVE ON THREE-DIMENSIONAL ALMOST KENMOTSU MANIFOLDS WITH THE $(\kappa, \mu)'$-NULLITY DISTRIBUTION

Definition 4.1. A unit speed curve γ in an almost contact metric manifold $M(\phi, \xi, \eta, g)$ is said to be slant if its tangent vector field makes constant angle θ with ξ i.e., $\eta(\dot{\gamma}) = \cos\theta$ is constant along γ.

By definition, slant curves with constant angle $\frac{\pi}{2}$ are called Legendre curves or almost contact curves.

Consider a slant curve γ on an almost Kenmotsu manifold. We get by definition

$$g(T, \xi) = \cos\theta, \tag{4.1}$$

where θ is a constant.

Differentiating both sides with respect to T we get from (4.1)

$$g(\nabla_T T, \xi) + g(T, \nabla_T \xi) = 0. \tag{4.2}$$

If γ is a Legendre slant curve the from (2.1), (2.2), (2.6) and (3.7) we obtain

$$\kappa\eta(N) + 1 - \cos^2\theta - g(\phi hT, T) = 0, \tag{4.3}$$

where $\{T, N, B\}$ is a Frenet frame with $T = \dot{\gamma}$.
From above we get

$$\kappa\eta(N) = \cos^2\theta - 1 + g(\phi hT, T). \tag{4.4}$$

In particular, let $\theta = \frac{\pi}{2}$, then $\kappa \neq 0$, provided $g(\phi hT, T) \neq 0$.
Thus we obtain the following:

Theorem 4.2. *A Legendre slant curve on a three-dimensional almost Kenmotsu manifold with the $(k, \mu)'$-nullity distribution is never geodesic, provided $g(\phi hT, T) \neq 0$.*

5 BIHARMONIC LEGENDRE CURVE ON ALMOST KENMOTSU MANIFOLDS WITH THE $(k, \mu)'$-NULLITY DISTRIBUTION

Definition 5.1. A unit speed smooth curve γ on an almost Kenmotsu manifold with the $(k, \mu)'$-nullity distribution is called a Legendre curve [3] if it satisfies $\eta(\dot{\gamma}) = 0$.

Definition 5.2. A Legendre curve γ on three dimensional almost Kenmotsu manifold with the $(k, \mu)'$-nullity distribution will be called biharmonic [19] if it satisfies

$$\nabla_T^3 T + R(\nabla_T T, T)T = 0, \tag{5.1}$$

where $T = \dot{\gamma}$.

Let γ be a biharmonic Legendre curve on almost Kenmotsu manifold with the $(k,\mu)'$-nullity distribution. Let T be the unit tangent vector field of the Legendre curve. Where $T=\gamma'$. Let $\{T,N,B\}$ as a Frenet frame on a tabular neighbourhood of the image of γ, with T, $N=-\phi T$ and $B=\xi$ is consider to fixed an orientation.

Then the equation (5.1) reduces to the following

$$\nabla_T^3 T+\kappa R(N,T)T=0, \tag{5.2}$$

where $T=\dot{\gamma}$.

For Legendre curve $\eta(T)=0$, $\eta(N)=0$, because we have considered the Frenet frame $T,N=-\phi T,B=\xi$. Using these facts in (3.5) we get, after simplification

$$R(N,T)T=(k-2\lambda)N. \tag{5.3}$$

By Serret-Frenet formula we get

$$\nabla_T^3 T=-3\kappa\kappa' T+(\kappa''-\kappa^3-\kappa\tau^2)N+(2\tau\kappa'+\kappa\tau')\xi. \tag{5.4}$$

From (5.2), (5.3) and (5.4), it follows that

$$-3\kappa\kappa' T+\{\kappa''-\kappa^3-\kappa\tau^2+\kappa(k-2\lambda)\}N+(2\tau\kappa'+\kappa\tau')\xi=0. \tag{5.5}$$

Consider the curve is not geodesic. Then from the first component we get

$$\kappa=\text{constant}.$$

From third component we get

$$\tau=\text{constant}.$$

Hence the curve helix. From second component we get

$$\kappa^2+\tau^2=(k-2\lambda)=(\lambda+1)^2,$$

since $\lambda^2=(-k-1)$.

Hence we are in position to state the following:

Theorem 5.3. *The biharmonic Legendre curve on three-dimensional almost Kenmotsu manifold with the $(k,\mu)'$-nullity distribution is a helix which satisfies*

$$\kappa^2+\tau^2=(\lambda+1)^2,$$

where λ is the eigen value such that $\lambda^2=-(k+1)$ and $k\leq -1$.

6 LOCALLY ϕ-SYMMETRIC LEGENDRE CURVE IN AN ALMOST KENMOTSU MANIFOLD WITH $(k,\mu)'$-NULLITY DISTRIBUTION

The notion of locally ϕ-symmetric manifolds was introduced by T. Takahasi [21]. Since every smooth curve is one-dimensional differentiable manifold we may apply the concept of local ϕ-symmetry on a smooth curve. In the following we introduce the definition of locally ϕ-symmetric Legendre curves.

Definition 6.1. A Legendre curve γ on a three dimensional almost Kenmotsu manifold will be called locally ϕ-symmetry if it satisfies

$$\phi^2(\nabla_T R)(\nabla_T T,T)T=0, \tag{6.1}$$

where $T=\dot{\gamma}$.

By definition of covariant derivative of R we get

$$\begin{aligned}(\nabla_T R)(\nabla_T T, T)T =& \nabla_T R(\nabla_T T, T)T - R(\nabla_T^2 T, T)T \\ &- R(\nabla_T T, \nabla_T T)T - R(\nabla_T T, T)\nabla_T T.\end{aligned} \tag{6.2}$$

Let us consider a Legendre curve γ. Now, proceeding in the same way as in the previous section we get

$$\nabla_T R(\nabla_T T, T)T = -\kappa[d(k-2\lambda)N + (k-2\lambda)(-\kappa T + \tau\xi)], \tag{6.3}$$

$$R(\nabla_T^2 T, T)T = (k-2\lambda)[-\kappa\tau\xi - \kappa' N], \tag{6.4}$$

and

$$R(\nabla_T T, T)\nabla_T T = -\kappa^2(k-2\lambda)T. \tag{6.5}$$

Hence from (6.2) we get

$$\begin{aligned}(\nabla_T R)(\nabla_T T, T)T =& -\kappa[d(k-2\lambda)N + (k-2\lambda)(-\kappa T + \tau\xi)] \\ &- (k-2\lambda)[-\kappa\tau\xi - \kappa' N] + \kappa^2(k-2\lambda)T.\end{aligned} \tag{6.6}$$

Applying ϕ^2 in the both sides of the above equation and using (2.1) we obtain

$$\phi^2(\nabla_T R)(\nabla_T T, T)T = [\kappa d(k-2\lambda) - (k-2\lambda)\kappa']N - 2\kappa^2(k-2\lambda)T. \tag{6.7}$$

By virtue of Definition 6.1 and the above equation we can state the following:

Theorem 6.2. *A necessary and sufficient condition for an almost Legendre curve on three-dimensional almost Kenmotsu manifold with the $(k,\mu)'$-nullity distribution to be locally ϕ-symmetric is $\kappa = 0$, that is the curve is a geodesic.*

7 EXAMPLE OF A LEGENDRE CURVE IN A THREE-DIMENSIONAL ALMOST KENMOTSU MANIFOLD

Let us consider a 3-dimensional manifold $M = \{(x,y,z) \in R^3 : z \neq 0\}$, where (x,y,z) are the standard coordinates in R^3.

Let e_1, e_2, e_3 be three linearly independent vector fields in R^3 which satisfies $[e_1, e_2] = (1+\lambda)e_3$, $[e_1, e_3] = -(1-\lambda)e_3$, $[e_2, e_3] = 2e_1$, where λ is a real number.

Let g be a Riemannian metric defined by

$$g(e_1, e_2) = g(e_2, e_3) = g(e_1, e_3) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -1.$$

Let η be a 1-form defined by $\eta(Z) = g(Z, e_1)$, for any $Z \in TM$ and ϕ be the tensor field of type $(1,1)$ defined by $\phi e_1 = 0$, $\phi e_2 = e_3$, $\phi e_3 = -e_2$. Then by applying linearity of ϕ and g, we have

$$\eta(e_1) = 1, \quad \phi^2 Z = -Z + \eta(Z)e_1,$$

$$g(\phi Z, \phi U) = g(Z, U) - \eta(Z)\eta(U),$$

for any $Z, U \in TM$.
Moreover $he_1 = 0$, $\quad he_2 = \lambda e_2$ $\quad$ and $\quad he_3 = \lambda e_3$

By using Koszul's formula for Levi-Civita connection ∇ with the metric tensor g, we obtain

$$\begin{array}{lll} \nabla_{e_1} e_3 = 0, & \nabla_{e_2} e_3 = (1+\lambda)e_1, & \nabla_{e_3} e_3 = 0, \\ \nabla_{e_1} e_2 = 0, & \nabla_{e_2} e_2 = 0, & \nabla_{e_3} e_2 = -(1-\lambda)e_1, \\ \nabla_{e_1} e_1 = 0, & \nabla_{e_2} e_1 = -(1+\lambda)e_3, & \nabla_{e_3} e_1 = (1-\lambda)e_2. \end{array}$$

From the above relations, we see that the manifold satisfies $h\xi = 0, \quad h\phi + \phi h = 0, \quad \nabla_X \xi = -\phi X - \phi h X$, for any vector X in TM and $\xi = e_1$.
Hence the manifold $M(\phi, \xi, \eta, g)$ is a contact metric manifold.

With the help of the above relations it can be verified that

$$\begin{array}{lll} R(e_1,e_2)e_3 = 0, & R(e_2,e_3)e_3 = -(1-\lambda^2)e_2, & R(e_1,e_3)e_3 = (1-\lambda^2)e_1, \\ R(e_1,e_2)e_2 = (1-\lambda^2)e_1, & R(e_2,e_3)e_2 = (1-\lambda^2)e_3, & R(e_1,e_3)e_2 = 0, \\ R(e_1,e_2)e_1 = -(1-\lambda^2)e_2, & R(e_2,e_3)e_1 = 0, & R(e_1,e_3)e_1 = -(1-\lambda^2)e_3. \end{array}$$

In view of the expressions of the curvature tensors we conclude that the characteristic vector field ξ belongs to $(\kappa, \mu)'$-nullity distribution with the relation $\lambda = \frac{\kappa}{2}$.

Now we give an example of unit speed non geodesic Legendre curves on the manifold.

Consider a curve $\gamma : I \longrightarrow M$ defined by $\gamma(s) = (1, \sqrt{\frac{2}{3}}s, \sqrt{\frac{1}{3}}s)$. Hence $\dot{\gamma_1} = 0$, $\dot{\gamma_2} = \sqrt{\frac{2}{3}}$, and $\dot{\gamma_3} = \sqrt{\frac{1}{3}}$.
Now

$$\eta(\dot{\gamma}) = g(\dot{\gamma}, e_1) = g(\dot{\gamma_1}e_1 + \dot{\gamma_2}e_2 + \dot{\gamma_3}e_3, e_1) = 0.$$

$$\begin{aligned} g(\dot{\gamma}, \dot{\gamma}) =& g(\dot{\gamma_1}e_1 + \dot{\gamma_2}e_2 + \dot{\gamma_3}e_3, \dot{\gamma_1}e_1 + \dot{\gamma_2}e_2 + \dot{\gamma_3}e_3) \\ &= \dot{\gamma_1^2} + \dot{\gamma_2^2} + \dot{\gamma_3^2} \\ &= \dot{\gamma_2^2} + \dot{\gamma_3^2} \\ &= 1. \end{aligned}$$

Hence, the curve is a Legendre curve. For this curve $\nabla_{\dot{\gamma}}\dot{\gamma} = \frac{2\sqrt{2}}{3}\lambda e_1$. Hence the curve is not a geodesic. It is geodesic if the eigen value is zero.

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