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Anisotropic cosmological model involving null radiation flow and magnetic field

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Abstract The spatially homogeneous and totally anisotropic Bianchi type-II cosmological models has been investigated with null radiation flow and magnetic field for perfect fluid in the framework of general relativity. With the help of special law of variation for Hubble's parameter proposed by Berman (Nuovo Cimento 74B:182, 1983) cosmological model is obtained in this theory. We use the power law relation between average Hubble parameter H and average scale factor a to find the solution. The assumption of constant deceleration parameter leads to two models of universe, i.e. power law model and exponential model. Some physical and kinematical properties of the model are also discussed.

1 Introduction

Having studied the physics of stars and stellar remnants, one might be tempted to proceed directly to the next level in astrophysical structure, as galaxies. But the formation of galaxies is so closely linked up with cosmological considerations that it is necessary to grapple with the aspects of cosmology to understand galaxies in their totality. The matter in the visible universe is concentrated in galaxies, of which our own is typical, its stars populates a pancake-shaped region of diameter 30 kpc and about 1 kpc thick. It contains about 10^{11} stars whose average mass is comparable with that of our own sun. The galaxies form clusters and super clusters extending over tens to hundreds of mega parsec. One large cluster Virgo containing about 2000 galaxies, is about 15 Mpc from the earth, it forms the core of the local super cluster that includes our own galaxy and the local group of some 24 galaxies. Structure is visible up to the largest distances studied, super clusters and clusters form threads and sheets that are separated by huge voids where galaxies are very rare. These voids can by 100 Mpc across. The evolution of the universe in its early stages near the big bang singularity, the only thing we can say about the nature of matter is that it would be in a highly dense state having very exotic and unusual behavior.

Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the universe . The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structure that we see in the universe today. Some Bianchi type cosmologies, for example, are natural hosts of large scale magnetic fields and therefore, their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family is the Bianchi type-I space time that contains the flat FRW universe as a special case. Several authors studied the cosmological models with constant and time dependent displacement field [1]-[15]. The matter content universe is not expected to attain thermal equilibrium at the time of its evolution, it is evident that there would be heat flow in the universe. The dominant component of matter is supposed to be the incoherent radiation which could be some pure null radiation flowing along a particular direction. Patel and Dadhich [16] have discussed cylindrically symmetric models with the Kasnerian time evolution containing heat flow and null radiation flow with the perfect fluid. The null radiation flux has been investigated by Vaidya and Patel [17]-[18] and Singh [19].

In this paper we discuss Bianchi type-II cosmological model with null radiation flow and magnetic field in general relativity.

2 Metric and Field Equations

The Bianchi type-II metric is given by

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2, \tag{0.1}$$

where the metric potentials A, B and C are functions of cosmic time t.
The Einstein field equations for nonempty space-time is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij}. \tag{0.2}$$

The energy-momentum tensor T_{ij} for perfect fluid with null radiation flow and magnetic field which is along the direction of the null radiation flow is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} + Xw_i w_j + \frac{1}{4\pi}\left(-g^{\alpha\beta}F_{i\alpha}F_{j\beta} + \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right), \tag{0.3}$$

where ρ is the energy density, p the pressure of the fluid and X is the null radiation density. The u^i describe the unit time-like four-velocity vector and the unit space-like vector w^i denotes the direction of the null radiation flow which can be taken along any one of the three directions $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial z}$. Without loss of generality let us choose y-direction as the direction of the null radiation flow along which the magnetic field is assumed to be present. So that

$$w^i = \left(0, \frac{1}{\sqrt{A^2z^2 + B^2}}, 0, 0\right). \tag{0.4}$$

The electromagnetic field tensor F_{ij} has only one non-zero component F_{31} because the magnetic field is assumed to be along the y-direction. Subsequently Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \quad , \quad F^{ij}_{;j} = 0, \tag{0.5}$$

lead to

$$F^{31} = K, \tag{0.6}$$

where K is a constant. Now assuming commoving coordinate system, the field Eqs. (2) for the metric (1) with the help of (3) and (6) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3}{4}\frac{A^2}{B^2C^2} = 8\pi p + 2A^2C^2K^2 - \frac{(A^2z^2 + B^2)A^2C^2K^2}{B^2}, \tag{0.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{1}{4}\frac{A^2}{B^2C^2} = 8\pi(p + X) - A^2C^2K^2, \tag{0.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{A^2}{B^2C^2} = 8\pi p + \frac{(A^2z^2 + B^2)A^2C^2K^2}{B^2}, \tag{0.9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{4}\frac{A^2}{B^2C^2} = -8\pi\rho - \frac{(A^2z^2 + B^2)A^2C^2K^2}{B^2}. \tag{0.10}$$

where an overhead dot denotes derivative with respect to cosmic time t.
Now we define some physical parameters before solving the field equations. The average scale factor a and the volume scale factor V are define as

$$V = a^3 = ABC. \tag{0.11}$$

The generalized mean Hubble parameter H is given in the form

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{0.12}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z axis respectively. Using Eqs. (11) and (12), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a}. \tag{0.13}$$

The mean anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{0.14}$$

The shear scalar σ^2 is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right). \tag{0.15}$$

The absolute temperature T for barotropic fluid $p = \gamma\rho$ $(0 \leq \gamma \leq 1)$ is defined as

$$T = T_0 \rho^{\frac{\gamma}{1+\gamma}}, \tag{0.16}$$

where T_0 is a constant. The entropy density s for barotropic fluid $p = \gamma\rho$ $(0 \leq \gamma \leq 1)$ is defined as

$$s = \frac{(1+\gamma)}{T_0}\rho^{\frac{1}{1+\gamma}}. \tag{0.17}$$

The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{0.18}$$

It is mentioned here that q was supposed to be positive initially but recent observations from the supernova experiments suggest that it is negative. Thus the behavior of the universe models depends upon the sign of q. The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. We also use a well-known relation [24] between the average Hubble parameter H and average scale factor a given as

$$H = \chi a^{-n}, \tag{0.19}$$

where $\chi > 0$ and $n \geq 0$.This is an important relation because it gives the constant value of the deceleration parameter. From Eqs. (13) and (19), we get

$$\dot{a} = \chi a^{-n+1}, \tag{0.20}$$

Using this value, we find that deceleration parameter is constant, i.e. $q = n - 1$. Integrating Eq. (20), it follows that

$$a = (n\chi t + k_1)^{\frac{1}{n}},\ n \neq 0 \tag{0.21}$$

$$a = k_2\ exp(\chi t),\ n = 0 \tag{0.22}$$

where k_1 and k_2 are constants of integration. Thus we obtain two values of the average scale factor that correspond to two different models of the universe.

3 Cosmological model of the Universe when n≠0

Here we discuss the model of universe when n≠0 i.e., $a = (n\chi t + k_1)^{\frac{1}{n}}$. The field equations $(7) - (10)$ constitute a system of four independent equations with six unknown parameters A, B, C, X, ρ and p and therefore some additional constraint equations relating these parameters are required to obtain explicit solutions of the system of the equations. Assuming

$$A = C^m, \tag{0.23}$$

$$B = C^l, \tag{0.24}$$

where m and l is the proportionality constant. This condition is explained with reference to [25]. Solving Eq. (11) with the help of Eqs. (21), (23) and (24) we get

$$C = (n\chi t + k_1)^{\frac{3}{n(m+l+1)}} . \tag{0.25}$$

From Eqs. (23) and (25) we get

$$A = (n\chi t + k_1)^{\frac{3m}{n(m+l+1)}} . \tag{0.26}$$

From Eqs. (24) and (25) we get

$$B = (n\chi t + k_1)^{\frac{3l}{n(m+l+1)}} . \tag{0.27}$$

The directional Hubble parameters H_i become

$$H_1 + H_2 = (m+l)H_3, \tag{0.28}$$

$$H_3 = \frac{3\chi}{(m+l+1)(n\chi t + k_1)}. \tag{0.29}$$

The mean generalized Hubble parameter becomes

$$H = \frac{\chi}{n\chi t + k_1}. \tag{0.30}$$

while the volume scale factor turns out to be

$$V = a^3 = (n\chi t + k_1)^{\frac{3}{n}} . \tag{0.31}$$

The expansion scalar Θ and shear scalar σ^2 take the form

$$\Theta = \frac{3\chi}{n\chi t + k_1}, \tag{0.32}$$

$$\sigma^2 = \frac{3\chi^2}{2(n\chi t + k_1)^2} \left\{ \frac{3(m^2 + l^2 + 1)}{(m+l+1)^2} - 1 \right\}. \tag{0.33}$$

The mean anisotropy parameter $\triangle$ become

$$\triangle = \frac{3(m^2 + l^2 + 1)}{(m+l+1)^2} - 1. \tag{0.34}$$

Subtracting two times Eq. (8) after adding Eqs. (7), (9) and using Eqs. (23), (24) and (25) we get null radiation density

$$\begin{aligned} X = \frac{1}{16\pi} \Bigg[& \frac{3\chi^2}{(m+l+1)(n\chi t + k_1)^2} \Big\{ n(2l - m - 1) \\ & + \frac{3(m^2 - 2l^2 - ml + 2m + 1)}{(m+l+1)} \Big\} + (n\chi t + k_1)^{\frac{6(m-l-1)}{n(m+l+1)}} \Bigg] \\ & - \frac{K^2}{4\pi} (n\chi t + k_1)^{\frac{6(m+1)}{n(m+l+1)}} . \end{aligned} \tag{0.35}$$

Subtracting Eq. (7) from Eq. (10) and using Eqs. (23), (24), (25) and assume a barotropic equation of state $p = \gamma\rho$ $(0 \le \gamma \le 1)$ we get energy density

$$\begin{aligned} \rho = \frac{1}{8\pi(1+\gamma)} & \left[\frac{-3\chi^2}{(m+l+1)(n\chi t + k_1)^2} \left\{ n(l+1) + \frac{3(m + ml - l^2 - 1)}{(m+l+1)} \right\} \right. \\ & \left. - \frac{1}{2} (n\chi t + k_1)^{\frac{6(m-l-1)}{n(m+l+1)}} \right] + \frac{K^2}{4\pi(1+\gamma)} (n\chi t + k_1)^{\frac{6(m+1)}{n(m+l+1)}} . \end{aligned} \tag{0.36}$$

Using Eq. (36) in $p = \gamma\rho$ $(0 \leq \gamma \leq 1)$ we get pressure of the barotropic fluid

$$p = \frac{\gamma}{8\pi(1+\gamma)}\left[\frac{-3\chi^2}{(m+l+1)(n\chi t+k_1)^2}\left\{n(l+1)+\frac{3(m+ml-l^2-1)}{(m+l+1)}\right\}\right.$$
$$\left.-\frac{1}{2}(n\chi t+k_1)^{\frac{6(m-l-1)}{n(m+l+1)}}\right]+\frac{K^2}{4\pi(1+\gamma)}(n\chi t+k_1)^{\frac{6(m+1)}{n(m+l+1)}}. \qquad (0.37)$$

Using Eq. (36) in Eq. (16) we get absolute temperature

$$T = T_0\left[\frac{1}{8\pi(1+\gamma)}\left\{\frac{-3\chi^2}{(m+l+1)(n\chi t+k_1)^2}\left\{n(l+1)+\frac{3(m+ml-l^2-1)}{(m+l+1)}\right\}\right.\right.$$
$$\left.\left.-\frac{1}{2}(n\chi t+k_1)^{\frac{6(m-l-1)}{n(m+l+1)}}\right\}+\frac{K^2}{4\pi(1+\gamma)}(n\chi t+k_1)^{\frac{6(m+1)}{n(m+l+1)}}\right]^{\frac{\gamma}{1+\gamma}}. \qquad (0.38)$$

Using Eq. (36) in Eq. (17) we get entropy density

$$s = \frac{(1+\gamma)}{T_0}\left[\frac{1}{8\pi(1+\gamma)}\left\{\frac{-3\chi^2}{(m+l+1)(n\chi t+k_1)^2}\left\{n(l+1)+\frac{3(m+ml-l^2-1)}{(m+l+1)}\right\}\right.\right.$$
$$\left.\left.-\frac{1}{2}(n\chi t+k_1)^{\frac{6(m-l-1)}{n(m+l+1)}}\right\}+\frac{K^2}{4\pi(1+\gamma)}(n\chi t+k_1)^{\frac{6(m+1)}{n(m+l+1)}}\right]^{\frac{1}{1+\gamma}}. \qquad (0.39)$$

4 Cosmological model of the Universe when $n = 0$

Here we discuss the model of universe when $n = 0$ the average scale factor for this model of the universe is $a = k_2\, exp\,(\chi t)$. Solving Eq. (11) with the help of Eqs. (22), (23) and (24) we get

$$C = k_2^{\left(\frac{3}{m+l+1}\right)} exp\left(\frac{3\chi t}{m+l+1}\right). \qquad (0.40)$$

From Eqs. (23) and (40) we get

$$A = k_2^{\left(\frac{3m}{m+l+1}\right)} exp\left(\frac{3\chi mt}{m+l+1}\right), \qquad (0.41)$$

From Eqs. (24) and (40) we get

$$B = k_2^{\left(\frac{3l}{m+l+1}\right)} exp\left(\frac{3\chi st}{m+l+1}\right), \qquad (0.42)$$

The directional Hubble parameters H_i become

$$H_1 + H_2 = (m+l)H_3, \qquad (0.43)$$

$$H_3 = \frac{3\chi}{m+l+1}. \qquad (0.44)$$

The mean generalized Hubble parameter become

$$H = \chi. \qquad (0.45)$$

The volume scale factor become

$$V = a^3 = k_2^3 exp(3\chi t). \qquad (0.46)$$

The expansion scalar Θ and shear scalar σ^2 take the form

$$\Theta = 3\chi, \qquad (0.47)$$

$$\sigma^2 = \frac{3\chi^2}{2}\left\{\frac{3(m^2+l^2+1)}{(m+l+1)^2} - 1\right\}. \tag{0.48}$$

The mean anisotropy parameter Δ become

$$\triangle = \frac{3(m^2+l^2+1)}{(m+l+1)^2} - 1. \tag{0.49}$$

Subtracting two times Eq. (8) after adding Eqs. (7), (9) and using Eqs. (40), (41) and (42) we get null radiation density

$$X = \frac{1}{16\pi}\left[\frac{9\chi^2}{(m+l+1)^2}\left\{(m+1)^2 - 2(l+1)^2 - l(m-1) + 4\right\}\right.$$
$$\left. + k_2^{\frac{6(m-l-1)}{(m+l+1)}} exp\left\{\frac{6\chi(m-l-1)t}{(m+l+1)}\right\}\right] - \frac{K^2 k_2^{\frac{6(m+1)}{(m+l+1)}}}{4\pi} exp\left\{\frac{6\chi(m+1)t}{(m+l+1)}\right\}. \tag{0.50}$$

Subtracting Eq. (7) from Eq. (10) and using Eqs. (40), (41), (42) and assume a barotropic equation of state $p = \gamma\rho$ $(0 \leq \gamma \leq 1)$ we get energy density

$$\rho = \frac{1}{8\pi(1+\gamma)}\left[\frac{-9\chi^2(ml+m-l^2-1)}{(m+l+1)^2} - \frac{1}{2}k_2^{\frac{6(m-l-1)}{(m+l+1)}} exp\left\{\frac{6\chi(m-l-1)t}{(m+l+1)}\right\}\right]$$
$$+ \frac{K^2 k_2^{\frac{6(m+1)}{(m+l+1)}}}{4\pi(1+\gamma)} exp\left\{\frac{6\chi(m+1)t}{(m+l+1)}\right\}. \tag{0.51}$$

Using Eq. (51) in $p = \gamma\rho$ $(0 \leq \gamma \leq 1)$ we get pressure of the barotropic fluid

$$p = \frac{\gamma}{8\pi(1+\gamma)}\left[\frac{-9\chi^2(ml+m-l^2-1)}{(m+l+1)^2} - \frac{1}{2}k_2^{\frac{6(m-l-1)}{(m+l+1)}} exp\left\{\frac{6\chi(m-l-1)t}{(m+l+1)}\right\}\right]$$
$$+ \frac{K^2 k_2^{\frac{6(m+1)}{(m+l+1)}}}{4\pi(1+\gamma)} exp\left\{\frac{6\chi(m+1)t}{(m+l+1)}\right\}. \tag{0.52}$$

Using Eq. (51) in Eq. (16) we get absolute temperature

$$T = T_0\left[\frac{1}{8\pi(1+\gamma)}\left\{\frac{-9\chi^2(ml+m-l^2-1)}{(m+l+1)^2} - \frac{1}{2}k_2^{\frac{6(m-l-1)}{(m+l+1)}} exp\left\{\frac{6\chi(m-l-1)t}{(m+l+1)}\right\}\right\}\right.$$
$$\left. + \frac{K^2 k_2^{\frac{6(m+1)}{(m+l+1)}}}{4\pi(1+\gamma)} exp\left\{\frac{6\chi(m+1)t}{(m+l+1)}\right\}\right]^{\frac{\gamma}{1+\gamma}}. \tag{0.53}$$

Using Eq. (51) in Eq. (17) we get entropy density

$$s = \frac{(1+\gamma)}{T_0}\left[\frac{1}{8\pi(1+\gamma)}\left\{\frac{-9\chi^2(ml+m-l^2-1)}{(m+l+1)^2} - \frac{1}{2}k_2^{\frac{6(m-l-1)}{(m+l+1)}} exp\left\{\frac{6\chi(m-l-1)t}{(m+l+1)}\right\}\right\}\right.$$
$$\left. + \frac{K^2 k_2^{\frac{6(m+1)}{(m+l+1)}}}{4\pi(1+\gamma)} exp\left\{\frac{6\chi(m+1)t}{(m+l+1)}\right\}\right]^{\frac{1}{1+\gamma}}. \tag{0.54}$$

5 Discussions and Conclusions

This paper is devoted to explore the solutions of Bianchi type-II cosmological model with null radiation flow and magnetic field in GR. We use the power law relation between average Hubble parameter H and average scale factor a to find the solution. The assumption of constant deceleration parameter leads to two models of universe, i.e. power law model and exponential model.

- In power law model of the universe, corresponds to $n \neq 0$ with average scale factor $a = (n\chi t + k_1)^{\frac{1}{n}}$ universe exhibits initial singularity of the POINT-type at $t = -\frac{k_1}{n\chi}$. At the initial moment $t = -\frac{k_1}{n\chi}$, the physical and kinematical parameters ρ, X, Θ, σ^2 and H tend to infinity and the magnetic field disappeared but the volume scale factor V vanishes here. The metric functions A, B and C vanishes at this point of singularity. The isotropy condition $\frac{\sigma^2}{\Theta} \to 0$ as $t \to \infty$ is also satisfied. Moreover ρ, X, Θ, σ^2 and H tend to a finite limit as $t \to 0$.
- The exponential model of the universe corresponds to $n = 0$ with average scale factor $a = k_2\ exp\,(\chi t)$. It is non-singular hence there does not exist any physical singularity for this model. The physical parameters ρ, X, Θ, σ^2 and H are all finite for sufficiently large values of t. While metric functions A, B and C do not vanish for this model.

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