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IDENTITIES CONNECTED WITH RAMANUJAN'S CONTINUED FRACTION

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My guide Prof. Bhaskar Srivastava initiated me in this study and I am thankful to him for his continuous help.

Abstract. We give Lambert series and some interesting identities for a continued fraction of Ramanujan.

1 Introduction

Rogers-Ramanujan continued fraction

$$R(q) = 1 + \frac{q}{1+}\,\frac{q^2}{1+}\,\frac{q^3}{1+}\,\frac{q^4}{1+}\cdots = \frac{(q^2;q^5)_\infty (q^3;q^5)_\infty}{(q;q^5)_\infty (q^4;q^5)_\infty} \tag{1.1}$$

was extensively considered by Andrews [6, 7, 8], Hirschhorn [12, 13]. Recently H. M. Srivastava et al.[16] derived some results on q-series and associated continued fractions . In [17] H. M. Srivastava et al. found interesting identities associated with Rogers-Ramanujan continued fraction $R(q)$. In [18] H. M. Srivastava et al. gave some q-identities involving Jacobi's theta-function. Chandrashekar Adiga et al. [2] considered function of order eleven of Rogers-Ramanujan type, gave modular relations and also partition-theoretic interpretation for some modular identities. In [2] Chandrashekar Adiga et al. established two q-series representations of a Ramanujan's continued fraction.They also gave integral representations.
Apart from Rogers-Ramanujan continued fraction, Ramanujan considered many more continued fraction. One of them more interesting continued fraction [10, Entry 12, p. 24] is

$$\frac{(a^2q^3;q^4)_\infty (b^2q^3;q^4)_\infty}{(a^2q;q^4)_\infty (b^2q;q^4)_\infty} = \frac{1}{1-ab+}\,\frac{(a-bq)(b-aq)}{(1-ab)(q^2+1)+}\,\frac{(a-bq^3)(b-aq^3)}{(1-ab)(q^4+1)+}\cdots \tag{1.2}$$

Adiga, Berndt, Bhargava and Watson [1] gave a proof of (1.2). L. Jacobsen [14] gave another simpler proof, though involving time consuming calculation. In the end L. Jacobsen observes that if $|ab| < 1$ and $|q| > 1$ the continued fraction in (1.2) converges to

$$\frac{(a^2/q^3;1/q^4)_\infty (b^2/q^3;1/q^4)_\infty}{(a^2/q;1/q^4)_\infty (b^2/q;1/q^4)_\infty}$$

which further shows the beautiful symmetry in the continued fraction.We make $q \to q^3$ and take $a = q^{5/2}$, $b = q^{1/2}$ in (1.2) to get the continued fraction.

$$P(q) = \frac{(q^4,q^8;q^{12})_\infty}{(q^2,q^{10};q^{12})_\infty} = \frac{1}{1+}\,\frac{1-q^2}{1-q^3+}\,\frac{(q^{5/2}-q^{7/2})(q^{1/2}-q^{11/2})}{(1-q^3)(q^6+1)+}\,\frac{(q^{5/2}-q^{19/2})(q^{1/2}-q^{23/2})}{(1-q^3)(q^{12}+1)+}\cdots \tag{1.3}$$

In this paper we have found Lambert series and certain identities for continued fraction $P(q)$.

We shall be using the following standard notations throughout this paper.
Let $|q| < 1$

$$(a)_0 = (a;q)_0 = 1,$$
$$(a)_n = (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k),\ n \geq 1,$$
$$(a)_\infty = (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k),$$

Jacobi's triple-product identity in Ramanujan's notation

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty, \quad |ab| < 1,$$

Bilateral basic hypergeometric series

$${}_r\psi_r = \sum_{n=-\infty}^{\infty}\frac{(a_1,...,a_r;q)_n}{(b_1,...,b_r;q)_n}z^n, \quad \left|\frac{b_1,...,b_r}{a_1,...,a_r}\right| < |z| < 1.$$

2 Generalized Lambert Series

The Generalized Lambert series is of the form

$$\sum_{n=-\infty}^{\infty}(-1)^{\epsilon n}q^{\lambda n^2}R(q^n),\ \text{where}\ \ \epsilon = 0 \text{ or } 1,\ \lambda > 0 \tag{2.1}$$

and $R(x)$ is a rational function of x.

We have Ramanujan's ${}_1\psi_1$ summation formula [11, p. 138]

$${}_1\psi_1(a;b;q,z) = \frac{(q,b/a,az,q/az;q)_\infty}{(b,q/a,z,b/az;q)_\infty},\ |b/a| < |z| < 1. \tag{2.2}$$

Bilateral basic hypergeometric series

$${}_1\psi_1(a;b;q,z) = \sum_{n=-\infty}^{\infty}\frac{(a;q)_n}{(b;q)_n}z^n \tag{2.3}$$

For $0 < i \leq 11$, $0 < j \leq 11$ and $i+j \neq 12$, we have

$$\frac{1}{1-q^j}\,{}_1\psi_1(q^j;q^{12+j};q^{12},q^i) = \frac{1}{1-q^j}\frac{(q^{12},q^{12+j}/q^j,q^{i+j},q^{12}/q^{i+j};q^{12})_\infty}{(q^{12+j},q^{12}/q^j,q^i,q^{12+j}/q^{i+j};q^{12})_\infty}$$
$$\frac{1}{1-q^j}\sum_{n=-\infty}^{\infty}\frac{(q^j;q^{12})_n}{(q^{12+j};q^{12})_n}q^{in} = \frac{1}{1-q^j}\frac{(q^{12},q^{12+j}/q^j,q^{i+j},q^{12}/q^{i+j};q^{12})_\infty}{(q^{12+j},q^{12}/q^j,q^i,q^{12+j}/q^{i+j};q^{12})_\infty}.$$
$$\sum_{n=-\infty}^{\infty}\frac{(q^j;q^{12})_n}{(q^j;q^{12})_{n+1}}q^{in} = \frac{(q^{12};q^{12})_\infty^2(q^{i+j};q^{12})_\infty(q^{12-i-j};q^{12})_\infty}{(q^j;q^{12})_\infty(q^{12-j};q^{12})_\infty(q^i;q^{12})_\infty(q^{12-i};q^{12})_\infty}$$
$$\sum_{n=-\infty}^{\infty}\frac{q^{in}}{1-q^{12n+j}} = \frac{(q^{12};q^{12})_\infty^2(q^{i+j};q^{12})_\infty(q^{12-i-j};q^{12})_\infty}{(q^j;q^{12})_\infty(q^{12-j};q^{12})_\infty(q^i;q^{12})_\infty(q^{12-i};q^{12})_\infty} \tag{2.4}$$

We have the identity [9, p. 58],

$$\sum_{n=0}^{\infty}\frac{q^{in}}{1-q^{12n+i}} = \sum_{n=0}^{\infty}q^{12n^2+2in}\frac{1+q^{12n+i}}{1-q^{12n+i}} \tag{2.5}$$

Let

$$P(q) = \frac{S(q)}{T(q)}$$

where

$$T(q)=\frac{1}{(q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty},$$
$$S(q)=\frac{1}{(q^2;q^{12})_\infty(q^{10};q^{12})_\infty(q^{12};q^{12})_\infty}.$$

We have

$$\begin{aligned}(q^{12};q^{12})^3_\infty S(q) &= \sum_{n=-\infty}^{\infty}\frac{q^n}{1-q^{12n+10}},\ \text{for}\ i=1,j=10\\ &= \sum_{n=-\infty}^{\infty}\frac{q^{10n}}{1-q^{12n+1}},\ \text{for}\ i=10,\ j=1\end{aligned}\tag{2.6}$$

$$\begin{aligned}(q^{12};q^{12})^3_\infty T(q) &= \sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+8}},\ \text{for}\ i=2,j=8\\ &= \sum_{n=-\infty}^{\infty}\frac{q^{8n}}{1-q^{12n+2}},\ \text{for}\ i=8,\ j=2\end{aligned}\tag{2.7}$$

$$(q^{12};q^{12})^3_\infty\frac{S^2(q)}{T(q)}=\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+2}},\ \ \text{for}\ i=2,\ j=2.\tag{2.8}$$

$$\begin{aligned}(q^{12};q^{12})^3_\infty S(q) &= \sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+5}},\ \text{for}\ i=2,j=5\\ &= \sum_{n=-\infty}^{\infty}\frac{q^{5n}}{1-q^{12n+2}},\ \text{for}\ i=5,\ j=2\end{aligned}\tag{2.9}$$

$$(q^{12};q^{12})^3_\infty T(q)=\sum_{n=-\infty}^{\infty}\frac{q^{4n}}{1-q^{12n+4}},\ \text{for}\ i=4,\ j=4\tag{2.10}$$

$$\begin{aligned}\frac{(q^{12};q^{12})^2_\infty}{(q^6;q^{12})^2_\infty}P(q) &= \sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+6}},\ \text{for}\ i=2,j=6\\ &= \sum_{n=-\infty}^{\infty}\frac{q^{6n}}{1-q^{12n+2}},\ \text{for}\ i=6,\ j=2\end{aligned}\tag{2.11}$$

$$\begin{aligned}\frac{(q^{12};q^{12})^2_\infty}{(q^6;q^{12})^2_\infty}\frac{1}{P(q)} &= \sum_{n=-\infty}^{\infty}\frac{q^{4n}}{1-q^{12n+6}},\ \text{for}\ i=4,j=6\\ &= \sum_{n=-\infty}^{\infty}\frac{q^{6n}}{1-q^{12n+4}},\ \text{for}\ i=6,\ j=4\end{aligned}\tag{2.12}$$

By equation (2.5), equation (2.8) becomes

$$(q^{12};q^{12})^3_\infty\frac{S^2(q)}{T(q)}=\sum_{n=0}^{\infty}q^{12n^2+4n}\frac{1+q^{12n+2}}{1-q^{12n+2}}\tag{2.13}$$

By equation (2.6) and (2.7)

$$P(q)=\frac{\sum_{n=-\infty}^{\infty}\frac{q^n}{1-q^{12n+10}}}{\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+8}}}\tag{2.14}$$

$$P(q)=\frac{\sum_{n=-\infty}^{\infty}\frac{q^{10n}}{1-q^{12n+1}}}{\sum_{n=-\infty}^{\infty}\frac{q^{8n}}{1-q^{12n+2}}} \tag{2.15}$$

By equation (2.7) and (2.8)

$$P^2(q)=\frac{\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+2}}}{\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+8}}} \tag{2.16}$$

$$P^2(q)=\frac{\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+2}}}{\sum_{n=-\infty}^{\infty}\frac{q^{8n}}{1-q^{12n+2}}} \tag{2.17}$$

By equation (2.4) for $i=5,\ j=1$ and $q\to q^2$

$$(q^{12};q^{12})_\infty^3 S(q)=\sum_{n=-\infty}^{\infty}\frac{q^{10n}}{1-q^{24n+2}} \tag{2.18}$$

By equation (2.4) for $i=4,\ j=2$ and $q\to q^2$

$$(q^{12};q^{12})_\infty^3 T(q)=\sum_{n=-\infty}^{\infty}\frac{q^{8n}}{1-q^{24n+4}} \tag{2.19}$$

By equation (2.8) and (2.10)

$$P^{-2}(q)=\frac{\sum_{n=-\infty}^{\infty}\frac{q^{4n}}{1-q^{12n+4}}}{\sum_{n=-\infty}^{\infty}\frac{q^{2n}}{1-q^{12n+2}}} \tag{2.20}$$

with the help of equation (2.5) in (2.20), we have

$$P^{-2}(q)=\frac{\sum_{n=0}^{\infty}q^{12n^2+8n}\frac{1+q^{12n+4}}{1-q^{12n+4}}-\sum_{n=0}^{\infty}q^{12n^2+16n+4}\frac{1+q^{12n+8}}{1-q^{12n+8}}}{\sum_{n=0}^{\infty}q^{12n^2+4n}\frac{1+q^{12n+2}}{1-q^{12n+2}}-\sum_{n=0}^{\infty}q^{12n^2+20n+8}\frac{1+q^{12n+10}}{1-q^{12n+10}}} \tag{2.21}$$

3 Some Identities for $P(q)$

Now we prove interesting identities related to $P(q)$.

$$(i)\quad \sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2-2n}}{(q^2;q^{12})_{n+1}}-\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2+14n+8}}{(q^{10};q^{12})_{n+1}}=(q^{12};q^{12})_\infty P(q) \tag{3.1}$$

$$(ii)\quad \sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}}+2\sum_{n=1}^{\infty}(-1)^n q^{6n^2-4n}=\frac{(q^4;q^4)_\infty}{P(q)} \tag{3.2}$$

$$(iii)\quad \sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n+10}}{(q^6;q^4)_{n+1}}-\sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n+2}(1+q^{24n+8})=\frac{(q^4;q^4)_\infty}{P(q)} \tag{3.3}$$

Proof. (i)

By using quintuple product identity [10, p. 82] of the form

$$\frac{(q;q)_\infty(x^2;q)_\infty(q/x^2;q)_\infty}{(x;q)_\infty(q/x;q)_\infty}=\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{n(3n-1)}{2}}x^{3n}(1+xq^n)$$

Replace $q\to q^{12},\ x\to q^2$ and transfer the term $(q^{12};q^{12})_\infty$ to right hand side, we get

$$P(q)=\frac{1}{(q^{12};q^{12})_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{18n^2}+\sum_{n=-\infty}^{\infty}(-1)^n q^{18n^2+12n+2}\right) \tag{3.4}$$

Replace $n\to-(n+1)$ in both sum of the expression, we get

$$P(q)=-\frac{1}{(q^{12};q^{12})_\infty}\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{18n^2+36n+18}+\sum_{n=-\infty}^{\infty}(-1)^n q^{18n^2+24n+8}\right) \tag{3.5}$$

By Rogers-fine identity [4, p. 564]

$$\sum_{n=0}^{\infty}\frac{(a;q)_n t^n}{(b;q)_{n+1}}=\sum_{n=0}^{\infty}\frac{(a;q)_n(at/b;q)_n b^n t^n q^{n^2}(1-atq^{2n})}{(b;q)_{n+1}(t;q)_{n+1}}. \tag{3.6}$$

Let $q\to q^{12},\ a=q^4/t,\ b=q^2,\ t\to 0$

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2-2n}}{(q^2;q^{12})_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{18n^2}+\sum_{n=0}^{\infty}(-1)^n q^{18n^2+12n+2} \tag{3.7}$$

Replace $n\to-(n+1)$, in both sum of right hand side, we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2-2n}}{(q^2;q^{12})_{n+1}}=-\sum_{n=-1}^{-\infty}(-1)^n q^{18n^2+36n+18}-\sum_{n=-1}^{-\infty}(-1)^n q^{18n^2+24n+8} \tag{3.8}$$

Let $q\to q^{12},\ a=q^{20}/t,\ b=q^{10},\ t\to 0$ again in (3.6), we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2+14n}}{(q^{10};q^{12})_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{18n^2+24n}+\sum_{n=0}^{\infty}(-1)^n q^{18n^2+36n+10} \tag{3.9}$$

Multiplying both side by q^8 we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2+14n+8}}{(q^{10};q^{12})_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{18n^2+24n+8}+\sum_{n=0}^{\infty}(-1)^n q^{18n^2+36n+18} \tag{3.10}$$

Subtracting equation (3.10) from equation (3.8) and using (3.5), we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2-2n}}{(q^2;q^{12})_{n+1}}-\sum_{n=0}^{\infty}\frac{(-1)^n q^{6n^2+14n+8}}{(q^{10};q^{12})_{n+1}}=(q^{12};q^{12})_\infty P(q) \tag{3.11}$$

which proves (i)

□

Proof. (ii)

Let $q\to q^4,\ a=q^4/t,\ b=q^2,\ t\to 0$ in (3.6), we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{6n^2+4n}+\sum_{n=0}^{\infty}(-1)^n q^{6n^2+8n+2} \tag{3.12}$$

Replace $n\to-(n+1)$ in second sum of right hand side, we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{6n^2+4n}-\sum_{n=-1}^{-\infty}(-1)^n q^{6n^2+4n} \tag{3.13}$$

Adding and substracting $\sum_{n=-1}^{-\infty}(-1)^n q^{6n^2+4n}$ in above equation, we get

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}}=\sum_{n=0}^{\infty}(-1)^n q^{6n^2+4n}+\sum_{n=-1}^{-\infty}(-1)^n q^{6n^2+4n} -2\sum_{n=-1}^{-\infty}(-1)^n q^{6n^2+4n} \tag{3.14}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}}=\sum_{n=-\infty}^{\infty}(-1)^n q^{6n^2+4n}-2\sum_{n=1}^{\infty}(-1)^n q^{6n^2+4n} \tag{3.15}$$

By Jacobi Triple Product Identity

$$(x;q)_\infty(q/x;q)_\infty(q;q)_\infty = \sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{n(n-1)}{2}} x^n \tag{3.16}$$

Let $q \to q^{12},\ x \to q^2$ dividing both side by $(q^4;q^4)_\infty$, we have

$$\frac{1}{P(q)} = \frac{1}{(q^4;q^4)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{6n^2+4n} \tag{3.17}$$

Using (3.17) in (3.15), we get

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+2n}}{(q^2;q^4)_{n+1}} + 2\sum_{n=1}^{\infty}(-1)^n q^{6n^2+4n} = \frac{(q^4;q^4)_\infty}{P(q)} \tag{3.18}$$

which proves (ii).

□

Proof. (iii)

Let $q \to q^4,\ a = q^{12}/t,\ b = q^6,\ t \to 0$, again in (3.6), we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n}}{(q^6;q^4)_{n+1}} = \sum_{n=0}^{\infty}(-1)^n q^{6n^2+16n} + \sum_{n=0}^{\infty}(-1)^n q^{6n^2+20n+6} \tag{3.19}$$

Replace $n \to -(n+1)$, in second sum of right hand side, we have

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n}}{(q^6;q^4)_{n+1}} = \sum_{n=0}^{\infty}(-1)^n q^{6n^2+16n} - \sum_{n=-1}^{-\infty}(-1)^n q^{6n^2-8n-8} \tag{3.20}$$

Adding and substracting the term $\sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n-8}$ in above equation, we get

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n}}{(q^6;q^4)_{n+1}} = \sum_{n=0}^{\infty}(-1)^n q^{6n^2+16n} - \sum_{n=-1}^{-\infty}(-1)^n q^{6n^2-8n-8} - \sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n-8} + \sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n-8} \tag{3.21}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n+8}}{(q^6;q^4)_{n+1}} = \sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n}(1+q^{24n+8}) - \sum_{n=-\infty}^{\infty}(-1)^n q^{6n^2-8n} \tag{3.22}$$

Replace $n \to (n-1)$ in (3.17), we have

$$\frac{1}{P(q)} = -\frac{1}{(q^4;q^4)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{6n^2-8n+2} \tag{3.23}$$

Using equation (3.23) in equation (3.22), we get

$$\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n^2+10n+10}}{(q^6;q^4)_{n+1}} - \sum_{n=0}^{\infty}(-1)^n q^{6n^2-8n+2}(1+q^{24n+8}) = \frac{(q^4;q^4)_\infty}{P(q)}$$

which proves (iii).

□

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