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Remainder Cordiality of some Graphs

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Abstract. Let G be a (p,q) graph. Let $f : V(G) \to \{1,2,...,p\}$ be a $1-1$ map. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a remainder cordial labeling of G if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with even integers and number of edges labelled with odd integers. A graph G with admits a remainder cordial labeling is called a remainder cordial graph. In this paper we investigate the remainder cordial behavior of $S(K_{1,n})$, $S(B_{n,n})$, $S(W_n)$ and union of some star related graphs.

1 Introduction

We considered only finite and simple graphs. Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with vertex set is $V(V_1 \cup V_2)$ and edge set is $E(E_1 \cup E_2)$. The graph $W_n = C_n + K_1$ is called a wheel. In a wheel, a vertex of degree 3 is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with the rim and the other incident with the central vertex are called spokes. Ponraj et al. [3], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of path, cycle, star, bistar, complete graph, etc,. In this paper we investigate the remainder cordial labeling behavior of $S(K_{1,n})$, $S(B_{n,n})$, $S(W_n)$, P_n^2 , $P_n^2 \cup K_{1,n}$, $P_n^2 \cup B_{n,n}$, $P_n \cup B_{n,n}$, $P_n \cup K_{1,n}$, $K_{1,n} \cup S(K_{1,n})$, $K_{1,n} \cup S(B_{n,n})$, $S(K_{1,n}) \cup S(B_{n,n})$ etc,. Terms are not defined here follows from Harary [2] and Gallian [1].

2 Remainder cordial labeling

Definition 2.1. Let G be a (p,q) graph. Let $f : V(G) \to \{1,2,\ldots,p\}$ be an injective map. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a remainder cordial labeling of G if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with even integers and number of edges labelled with odd integers. A graph G with a remainder cordial labeling is called a remainder cordial graph.

First we investigate the remainder cordial labeling behavior of the subdivision of the star, $S(K_{1,n})$.

Theorem 2.2. *$S(K_{1,n})$ is remainder cordial for all values of n.*

Proof. Let $V(S(K_{1,n}))$=$\{u, u_i, v_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n}))$=$\{uv_i, u_iv_i : 1 \leq i \leq n\}$. Clearly $S(K_{1,n})$ has $2n+1$ veretices and $2n$ edges. Assign the label 1 to the central vertex u. Then assign the even integers $2,4,\ldots,2n$ to the vertices $v_1, v_2, \ldots, v_n$. Next assign the odd integers $3,5,\ldots,2n+1$ to the vertices $u_1, u_2, \ldots, u_n$. Clearly $e_f(0) = n = e_f(1)$. Hence f is a remainder cordial labeling. □

Next is the subdivision of the bistar, $S(B_{n,n})$.

Theorem 2.3. *$S(B_{n,n})$ is remainder cordial for all values of n.*

Proof. Let $V(S(B_{n,n}))$ = $\{u, w, v, u_i, v_i, w_i, z_i : 1 \le i \le n\}$ and $E(S(B_{n,n}))$= $\{uw, wv, uu_i, u_iv_i, vw_i, w_iz_i : 1 \le i \le n\}$. Clearly $S(B_{n,n})$ has $4n + 3$ vertices and $4n + 2$ edges. We now give the labeling of $S(B_{n,n})$ as follows. Assign the labels $1, 3, 2$ to the vertices u, w, v respectively. Next assign the labels $4, 8, \ldots, 4n$ to the vertices $u_1, u_2, \ldots, u_n$ and assign the labels $5, 9, \ldots, 4n+1$ to the pendent vertices $v_1, v_2, \ldots, v_n$ respectively. We now move to the other side vertices of $S(B_{n,n})$. Assign the labels $6, 10, \ldots, 4n + 2$ to the vertices $w_1, w_2, \ldots, w_n$ and assign the labels $7, 11, \ldots, 4n + 3$ to the pendent vertices $z_1, z_2, \ldots, z_n$ respectively. It is easy to verify that $e_{f(0)} = e_{f(1)} = 2n + 1$. Hence f is a remainder cordial labeling. □

Theorem 2.4. *The graph G obtained by subdividing the pendent edges of the bistar $B_{n,n}$ is remainder cordial.*

Proof. Let $V(G)$=$\{u, v, u_i, x_i, v_i, y_i : 1 \le i \le n\}$ and $E(G)$=$\{uv, ux_i, x_iu_i, vy_i, y_iv_i : 1 \le i \le n\}$. Clearly G has $4n + 2$ vertices and $4n + 1$ edges. We now give the labeling of G as follows. Assign the labels $1, 2$ to the vertices u, v respectively. Next assign the labels $4, 8, \ldots, 4n$ to the vertices $x_1, x_2, \ldots, x_n$ and assign the labels $3, 7, \ldots, 4n-1$ to the pendent vertices $u_1, u_2, \ldots, u_n$ respectively. We now move to the other side vertices of G. Assign the labels $5, 9, \ldots, 4n+1$ to the vertices $v_1, v_2, \ldots, v_n$ and assign the labels $6, 10, \ldots, 4n+2$ to the pendent vertices $y_1, y_2, \ldots, y_n$ respectively. The table 1 establish that this vertex labeling f is a remainder cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$2n + 1$	$2n$
$n \equiv 1 \pmod 4$	$2n + 1$	$2n$
$n \equiv 2 \pmod 4$	$2n + 1$	$2n$
$n \equiv 3 \pmod 4$	$2n + 1$	$2n$

Table 1.

□

Here we investigate the union of star and the bistar.

Theorem 2.5. *$K_{1,n} \cup B_{n,n}$ is remainder cordial.*

Proof. Let $V(K_{1,n})$=$\{u, u_i : 1 \le i \le n\}$ and $V(B_{n,n})$=$\{v, w, v_i, w_i : 1 \le i \le n\}$. Let $E(K_{1,n} \cup B_{n,n}) = \{uu_i, vv_i, ww_i, vw : 1 \le i \le n\}$. Note that $K_{1,n} \cup B_{n,n}$ has $3n + 3$ vertices and $3n + 1$ edges. We now give a remainder cordial labeling as follows. Assign the labels $1, 2$ respectively to the vertices v, w and assign labels $3, 5, \ldots, 2n + 1$ to the vertices $w_1, w_2, \ldots, w_n$. Next assign the labels $4, 6, \ldots, 2n + 2$ to the pendent vertices $v_1, v_2, \ldots, v_n$. We now move to the star $K_{1,n}$. Assign the label $2n + 3$ to the vertex u and assign the labels $2n + 4, 2n + 5, \ldots, 3n + 3$ to the vertices $u_1, u_2, \ldots, u_n$. The table 2 establish that the vertex labeling f given below is a remainder cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
n-odd	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$
n-even	$\frac{3n+2}{2}$	$\frac{3n}{2}$

Table 2.

□

Here we investigate the union of the path and the star.

Theorem 2.6. *$P_n \cup K_{1,n}$ is reminder cordial.*

Proof. Let $V(K_{1,n})$=$\{u, u_i : 1 \le i \le n\}$ and $E(K_{1,n})$=$\{uu_i : 1 \le i \le n\}$. Let P_n be the path $v_1v_2 \ldots v_n$. It is easy to verify that the order and size of $P_n \cup K_{1,n}$ are $2n + 1$ and $2n - 1$ respectively. Fix the label 1 to the central vertex u of the star. Next assign the labels $2, 3, \ldots, n, n + 1$ consecutively to the pendent vertices $u_1, u_2, \ldots, u_n$ of the star. We now move to the path. Assign

Nature of n	$e_f(0)$	$e_f(1)$
n-odd	n	$n-1$
n-even	n	$n-1$

Table 3.

the labels $n+2, n+3, \ldots, 2n+1$ consecutively to the path vertices $v_1, v_2, \ldots, v_n$. The table 3 shows that this vertex labeling f given below is a remainder cordial labeling.

□

Next is the union of the path and the bistar.

Theorem 2.7. *$P_n \cup B_{n,n}$ is remainder cordial.*

Proof. Let P_n be a path $v_1v_2\ldots v_n$ and $V(B_{n,n})$=$\{u, w, u_i, w_i : 1 \leq i \leq n\}$ and $E(B_{n,n})$=$\{uw, uu_i, ww_i : 1 \leq i \leq n\} \cup E(P_n)$. Cearly $P_n \cup B_{n,n}$ has $3n+2$ vertices and $3n$ edges. We describe a remainder cordial labeling as follows. Assign the label 1 and 2 respectively to the vertices u and w. Next assign the even integers $4, 6, \ldots, 2n+2$ to the vertices $u_1, u_2, \ldots, u_n$ and assign the odd integers $3, 5, \ldots, 2n+1$ to the vertices $w_1, w_2, \ldots, w_n$. Now we move to the path. Assign the labels to the Path as into two cases given below.
Case (i). n is even.
Assign the labels $2n+3, 2n+4, \ldots, 2n+2+\frac{n}{2}$ consecutively to the vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$.
Subcase (i). $n \equiv 0 \pmod 4$.
In this case assign the labels $2n+2+\frac{n}{2}+2, 2n+2+\frac{n}{2}+4, \ldots, 2n+2+\frac{n}{2}+\frac{n}{2}$ to the vertices $v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \ldots, v_{\frac{n}{2}+\frac{n}{4}}$ respectively. Finally non-labelled vertices are labelled by $2n+2+\frac{n}{2}+1, 2n+2+\frac{n}{2}+3, \ldots, 2n+2+\frac{n}{2}+\frac{n}{2}-1$.
Subcase (ii). $n \equiv 2 \pmod 4$.
In this case assign the labels $2n+2+\frac{n}{2}+2, 2n+2+\frac{n}{2}+4, \ldots, 2n+2+\frac{n}{2}+\frac{n}{2}-1$ to the vertices $v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \ldots, v_{\frac{n}{2}+\frac{n-2}{4}}$. Finally assign the labels $2n+2+\frac{n}{2}+1, 2n+2\frac{n}{2}+3, \ldots, 3n+2$ to the vertices $v_{\frac{n}{2}+\frac{n-2}{4}+1}, v_{\frac{n}{2}+\frac{n-2}{4}+2}, \ldots, v_n$.
Case(ii). n is odd.
Assign the labels $2n+3, 2n+4, \ldots, 2n+2+\frac{n+1}{2}$ consecutively to the vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$.
Sub case (i). $n \equiv 1 \pmod 4$.
In this case assign the labels $2n+2+\frac{n+1}{2}+2, 2n+2+\frac{n+1}{2}+4, \ldots, 3n+2$ to the vertices $v_{\frac{n+1}{2}+1}, v_{\frac{n+1}{2}+2}, \ldots, v_{\frac{3n+1}{2}}$. Next assign the labels $2n+2+\frac{n+1}{2}+1, 2n+2+\frac{n+1}{2}+3, \ldots, 3n+1$ to the vertices $v_{\frac{3n+1}{4}+1}, v_{\frac{3n+1}{4}+2}, \ldots, v_n$.
Sub case(ii). $n \equiv 3 \pmod 4$.
In this case assign the labels $2n+2+\frac{n+1}{2}+2, 2n+2+\frac{n+1}{2}+4, \ldots, 3n+1$ to the vertices $v_{\frac{n+1}{2}+1}, v_{\frac{n+1}{2}+2}, \ldots, v_{\frac{n+1}{2}+\frac{n-3}{2}}$ respectively. Next assign the labels $2n+2+\frac{n+1}{2}+1, 2n+2+\frac{n+1}{2}+3, \ldots, 3n+2$ to the vertices $v_{\frac{3n-1}{4}+1}, v_{\frac{3n-1}{4}+2}, \ldots, v_n$. □

We investigate the union of the star and the subdivision of the star.

Theorem 2.8. *$K_{1,n} \cup S(K_{1,n})$ is remainder cordial.*

Proof. Let $V((K_{1,n}))$=$\{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n})$=$\{uu_i : 1 \leq i \leq n\}$. Let $V(S(K_{1,n}))$ $=$ $\{v, v_i, w_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n}))$=$\{vv_i, v_iw_i : 1 \leq i \leq n\}$. Clearly $K_{1,n} \cup S(K_{1,n})$ has $3n+2$ veretices and $3n$ edges. We give the remainder cordial labeling as follows. Assign the label 1 to the central vertex v. Then assign the even integers $2, 4, \ldots, 2n$ consecutively to the vertices $v_1, v_2, \ldots, v_n$. Next assign the odd integers $3, 5, \ldots, 2n+1$ to the pendent vertices $w_1, w_2, \ldots, w_n$. We now move to the star. Assign the label $2n+2$ to the central vertex u. Finally assign the labels $2n+3, 2n+4, \ldots, 3n+2$ consecutively to the pendent vertices $u_1, u_2, \ldots, u_n$ of the star. The table 4 given below establish that this vertex labeling f is a remainder cordial labeling.

□

Next is the union of the star and the subdivision of the bistar.

Nature of n	$e_f(0)$	$e_f(1)$
n-even	$\frac{3n}{2}$	$\frac{3n}{2}$
n-odd	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 4.

Theorem 2.9. *$K_{1,n} \cup S(B_{n,n})$ is remainder cordial.*

Proof. Let $V(K_{1,n})$=$\{u, u_i : 1 \leq i \leq n\}$ and $V(S(B_{n,n}))$=$\{v, w, v_i, x_i, w_i, y_i : 1 \leq i \leq n\}$. Let $E(K_{1,n})$= $\{uu_i : 1 \leq i \leq n\}$ and $E(B_{n,n})$=$\{vv_i, v_ix_i, vx, xw, ww_i, w_iy_i : 1 \leq i \leq n\}$. Clearly $K_{1,n} \cup S(B_{n,n})$ has $5n + 4$ vertices and $5n + 2$ edges. Assign the labels $1, 3, 2$ respectively to the vertices v, x, w. Next assign labels $5, 9, \ldots, 4n + 1$ to the pendent vertices $x_1, x_2, \ldots, x_n$ and assign the labels $4, 8, \ldots, 4n$ to the vertices $v_1, v_2, \ldots, v_n$. Next consider the vertices $w_1, w_2, \ldots, w_n$. Assign the labels $6, 10, \ldots, 4n + 2$ to the vertices $w_1, w_2, \ldots, w_n$ and $7, 11, \ldots, 4n + 3$ to the vertices $y_1, y_2, \ldots, y_n$. Next we move to the star $K_{1,n}$. Assign the label $4n+4$ to the central vertex u and assign the labels $4n+5, 4n+6, \ldots, 5n+4$ respectively to the pendent vertices $u_1, u_2, \ldots, u_n$ of the star. Note that this vertex labeling f is a remainder cordial labeling follows from the table 5.

Nature of n	$e_f(0)$	$e_f(1)$
n-even	$\frac{5n+2}{2}$	$\frac{5n+2}{2}$
n-odd	$\frac{5n+1}{2}$	$\frac{5n+3}{2}$

Table 5.

□

Here we investigate the square of the path, P_n^2.

Theorem 2.10. *P_n^2 is remainder cordial.*

Proof. Let P_n be the path $v_1v_2 \ldots v_n$. Clearly P_n^2 has n vertices and $2n - 3$ edges. Assign the labels $1, 2, \ldots, n$ continuously to the vertices $v_1, v_2, \ldots, v_n$. It is easy to verify that this vertex labeling is a remainder cordial labeling. □

Now we investigate the union of the square of the path and the star.

Theorem 2.11. *$P_n^2 \cup K_{1,n}$ is remainder cordial.*

Proof. Let P_n be the path $v_1v_2 \ldots v_n$. Let $V(K_{1,n})$=$\{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n})$=$\{uu_i : 1 \leq i \leq n\}$. Clearly $P_n^2 \cup K_{1,n}$ has $2n+1$ vertices and $3n-3$ edges. We describe a remainder cordial labeling as follows. Assign the labels $1, 2, \ldots, n$ continuously to the vertices $v_1, v_2, \ldots, v_n$ of the square of the path P_n^2. Next assign the label $n + 1$ to the central vertex of the star. Finally assign the labels $n + 2, n + 3, \ldots, 2n + 2$ to the pendent vertices $u_1, u_2, \ldots, u_n$. This vertex labeling f is a remainder cordial labeling follows from table 6.

Nature of n	$e_f(0)$	$e_f(1)$
n-even	$\frac{3n-2}{2}$	$\frac{3n-4}{2}$
n-odd	$\frac{3n-3}{2}$	$\frac{3n-3}{2}$

Table 6.

□

Next is the union of the square of the path and the bistar.

Theorem 2.12. *${P_n}^2 \cup B_{n,n}$ is remainder cordial.*

Proof. Let P_n be a path $v_1v_2\ldots v_n$. Let $V(B_{n,n})$=$\{v, w, v_i, w_i : 1 \le i \le n\}$ and $E(B_{n,n})$=$\{vw, vv_i, ww_i : 1 \le i \le n\}$. It is easy to verify that $P_n^2 \cup B_{n,n}$ has $3n+2$ vertices and $4n-2$ edges. Assign the label $1, 2$ respectively to the vertices v and w. Next assign the labels $4, 6, \ldots, 2n+2$ to the vertices $v_1, v_2, \ldots, v_n$. Assign the labels $3, 5, \ldots, 2n+1$ to the pendent vertices $w_1, w_2, \ldots, w_n$. We now to the square of the path P_n^2. Assign the labels $2n+3, 2n+4, \ldots, 3n+2$ to the pendent vertices $u_1, u_2, \ldots, u_n$. The table 7 given below establish that this vertex labeling f is a remainder cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
n-even	$\frac{4n-2}{2}$	$\frac{4n-2}{2}$
n-odd	$\frac{4n-2}{2}$	$\frac{4n-2}{2}$

Table 7.

□

We now investigate the union of subdivision of the star, $S(K_{1,n})$ and subdivision of the bistar, $S(B_{n,n})$.

Theorem 2.13. *$S(K_{1,n}) \cup S(B_{n,n})$ is remainder cordial.*

Proof. Let $V(S(K_{1,n}))$=$\{u, u_i, v_i : 1 \le i \le n\}$ and $E(S(K_{1,n}))$=$\{uv_i, u_iv_i : 1 \le i \le n\}$. Let $V(S(B_{n,n}))$=$\{x, y, z, x, y_i, z_i, w_i : 1 \le i \le n\}$ and $E(K_{1,n})$=$\{uu_i : 1 \le i \le n\}$ and $E(B_{n,n})$=$\{xy, yz, xx_i, x_iy_i, zz_i, z_iw_i : 1 \le i \le n\}$. Clearly $S(K_{1,n}) \cup S(B_{n,n})$ has $6n+4$ veretices and $6n+2$ edges. First we give the labeling of $S(B_{n,n})$ as follows. Assign the labels $1, 3, 2$ respectively to the vertices x, y, z. Next assign the labels $4, 8, \ldots, 4n$ to the vertices $x_1, x_2, \ldots, x_n$ and assign the labels $3, 5, \ldots, 2n+1$ to the pendent vertices $y_1, y_2, \ldots, y_n$ respectively. We now move to other side vertices of $S(B_{n,n})$. Assign the labels $6, 10, \ldots, 4n+2$ to the vertices $z_1, z_2, \ldots, z_n$ and $7, 11, \ldots, 4n+3$ to the pendent vertices $w_1, w_2, \ldots, w_n$. Second we give the labeling of $S(K_{1,n})$ as follows. Assign the label $4n+4$ to the central vertex u of $S(K_{1,n})$. Next assign the labels $4n+4+2, 4n+4+4, \ldots, 6n+4$ consecutively to the vertices $u_1, u_2, \ldots, u_n$. Finally assign labels $4n+4+1, 4n+4+3, \ldots, 6n+3$ to the pendent vertices $v_1, v_2, \ldots, v_n$. This labeling f is remainder cordial labeling, since $e_f(0) = e_f(1) = 3n+1$. □

We now investigate the subdivision of the wheel, $S(W_n)$.

Theorem 2.14. *$S(W_n)$ is remainder cordial for all even values of n.*

Proof. Let C_n be the cycle $u_1u_2\ldots u_nu_1$ and $V(K_1) = u$. Let $W_n = C_n + K_1$. Let x_i and y_i be the vertices which subdivide the edges uu_i and u_iu_{i+1}. Assign the label 1 to the central vertex. Next assign the labels $2, 4, \ldots, 2n$ to the vertices $u_1, u_2, \ldots, u_n$ and $3, 5, \ldots, 2n+1$ to the vertices $x_1, x_2, \ldots, x_n$. Next assign the labels $2n+5, 2n+6, \ldots, 3n+1$ to the vertices $y_1, y_2, \ldots, y_{n-1}$ and assign $2n+3$ to the y_n. It is easy to verify that $e_{f(0)} = e_{f(1)} = 2n$. Hence f is a remainder cordial labeling. □

For illustration, a remainder cordial labeling of $S(W_8)$ is shown in Figure 1.

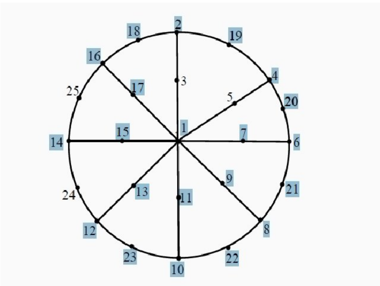

Figure 1.

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