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ON ESTIMATE FOR SECOND AND THIRD COEFFICIENTS FOR A NEW SUBCLASS OF ANALYTIC AND BI-UNIVALENT FUNCTIONS BY USING DIFFERENTIAL OPERATORS

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Abstract. In this work, we firstly introduce a new subclass $B_{\Sigma}^{h,p}(n,\lambda)$ of analytic and bi-univalent functions. Moreover, we estimate the second and third coefficients for functions in this subclass. Our results presented in this paper improve some recent works.

1 Introduction

We denote $\mathcal{A}$ for a class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Assume that $\mathcal{S}$ be the class of functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$.

From Koebe one-quarter theorem [5], every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U}),$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right),$$

where

$$g(w) := f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots. \tag{1.2}$$

If f and f^{-1} are univalent in $\mathbb{U}$, then f is bi-univalent. We denote Σ for the class of bi-univalent functions in $\mathbb{U}$ given by (1.1).

There interest to study the bi-univalent functions class Σ (see [6, 19, 20]) and derive non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. The coefficient estimate problem i.e. bound for $|a_n|$ $(n \in \mathbb{N} - \{2, 3\})$ for each $f \in \Sigma$, is still an open problem.
Recently Frasin and Aouf [6] investigated the following two subclasses of the bi-univalent function class Σ and obtained bounds for the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

Definition 1.1. (see [6]) Let $0 < \alpha \leq 1$ and $0 \leq \lambda \leq 1$. A function $f(z)$ is said to be in the class $B_{\Sigma}(\alpha, \lambda)$ if the following conditions are satisfied:

$$f \in \Sigma \quad and \quad \left|arg[(1-\lambda)\frac{f(z)}{z} + \lambda f'(z)]\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}),$$

and

$$\left|arg[(1-\lambda)\frac{g(w)}{w} + \lambda g'(w)]\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}),$$

where the function g is given by (1.2).

Theorem 1.2. *(see [6]) If $f(z)$ is in the class $B_\Sigma(\alpha,\lambda)$, then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{(\lambda+1)^2+\alpha(1+2\lambda-\lambda^2)}}, \qquad |a_3| \leq \frac{4\alpha^2}{(1+\lambda)^2}+\frac{2\alpha}{(1+2\lambda)}.$$

Definition 1.3. (see [6]) Let $0 \leq \beta < 1$ and $0 \leq \lambda \leq 1$. We say that a function $f(z)$ is in the class $B_\Sigma(\beta,\lambda)$ if the following conditions are satisfied:

$$f \in \Sigma \quad and \quad \mathfrak{Re}[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)] > \beta \qquad (z \in \mathbb{U}),$$

and

$$\mathfrak{Re}[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)] > \beta \qquad (w \in \mathbb{U}),$$

where the function g is given by (1.2).

Theorem 1.4. *(see [6]) If $f(z)$ is in the class $B_\Sigma(\beta,\lambda)$, then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{2\lambda+1}}, \qquad |a_3| \leq \frac{4(1-\beta)^2}{(1+\lambda)^2}+\frac{2(1-\beta)}{(1+2\lambda)}.$$

In 1983, Salagean [8] defined differential operator $D^k : \mathcal{A} \to \mathcal{A}$ as

$$D^0 f(z) = f(z),$$

$$D^1 f(z) = Df(z) = zf'(z),$$

$$D^k f(z) = D(D^{k-1}f(z)) = z(D^{k-1}f(z))', \qquad k \in \mathbb{N} = \{1,2,3,\cdots\}.$$

We note that

$$D^k f(z) = z + \sum_{n=2}^{\infty} n^k a_n z^n, \qquad k \in \mathbb{N}_0 = \{0\} \cup \mathbb{N}. \tag{1.3}$$

By using Salagean differential operator, Porwal and Darus [7] introduced the following two subclasses of the bi-univalent function.

Definition 1.5. (see [7]) Let $n \in \mathbb{N}_0$, $0 < \alpha \leq 1$ and $\lambda \geq 1$. A function $f(z)$ is called in the class $B_\Sigma(n,\alpha,\lambda)$ if the following conditions are satisfied:

$$f \in \Sigma \quad and \qquad \left|arg\left[\frac{(1-\lambda)D^n f(z)+\lambda D^{n+1}f(z)}{z}\right]\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}),$$

and

$$\left|arg\left[\frac{(1-\lambda)D^n g(w)+\lambda D^{n+1}g(w)}{w}\right]\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}),$$

where the function g is given by (1.2).

Theorem 1.6. *(see [7]) If $f(z)$ is in the class $B_\Sigma(n,\alpha,\lambda)$, then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{4^n(1+\lambda)^2+\alpha[2.3^n(1+2\lambda)-4^n(1+\lambda)^2]}},$$

and

$$|a_3| \leq \frac{2\alpha}{[(1-\lambda)3^n+\lambda 3^{n+1}]}+\frac{4\alpha^2}{[(1-\lambda)2^n+\lambda 2^{n+1}]^2}.$$

Definition 1.7. (see [7]) Let $n \in \mathbb{N}_0$, $0 \leq \beta < 1$ and $\lambda \geq 1$. Under the following conditions, a function $f(z)$ is said to be in the class $B_\Sigma(n, \beta, \lambda)$:

$$f \in \Sigma \quad and \qquad \mathfrak{Re}\left(\frac{(1-\lambda)D^n f(z) + \lambda D^{n+1} f(z)}{z}\right) > \beta \qquad (z \in \mathbb{U}),$$

and

$$\mathfrak{Re}\left(\frac{(1-\lambda)D^n g(w) + \lambda D^{n+1} g(w)}{w}\right) > \beta \qquad (w \in \mathbb{U}),$$

where the function g is given by (1.2).

Theorem 1.8. *(see [7]) If $f(z)$ is in the class $B_\Sigma(n, \beta, \lambda)$, then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]}},$$

and

$$|a_3| \leq \frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]} + \frac{4(1-\beta)^2}{[(1-\lambda)2^n + \lambda 2^{n+1}]^2}.$$

The aim of this paper is to investigate the bi-univalent function class $B_\Sigma^{h,p}(n, \lambda)$ introduced in Definition 2.1 and derive coefficient estimates on the first two Taylor-Maclaurin coefficient $|a_2|$ and $|a_3|$. Our results for the bi-univalent function class $f \in B_\Sigma^{h,p}(n, \lambda)$ would generalize and improve some recent works Srivastava [9], Frasin and Aouf [6] and Porwal and Darus [7].

2 The subclass $B_\Sigma^{h,p}(n, \lambda)$

In this section, the general subclass $B_\Sigma^{h,p}(n, \lambda)$ is introduced and investigated.

Definition 2.1. Assume that the functions $h, p : \mathbb{U} \to \mathbb{C}$ be so constrained that

$$\min\{\mathfrak{Re}(h(z)), \mathfrak{Re}(p(z))\} > 0 \quad (z \in \mathbb{U}) \quad and \quad h(0) = p(0) = 1.$$

Let $n \in \mathbb{N}_0$ and $\lambda \geq 1$. We say that a function $f \in \mathcal{A}$ given by (1.1) is in the class $B_\Sigma^{h,p}(n, \lambda)$ if the following conditions are satisfied:

$$f \in \Sigma \quad and \quad \left(\frac{(1-\lambda)D^n f(z) + \lambda D^{n+1} f(z)}{z}\right) \in h(\mathbb{U}) \qquad (z \in \mathbb{U}), \tag{2.1}$$

and

$$\left(\frac{(1-\lambda)D^n g(w) + \lambda D^{n+1} g(w)}{w}\right) \in p(\mathbb{U}) \qquad (w \in \mathbb{U}), \tag{2.2}$$

where the function g is defined by (1.2).

Remark 2.2. There are many selections of h and p which would provide interesting subclasses of class $B_\Sigma^{h,p}(n, \lambda)$. For example, if we set

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^\alpha \qquad (0 < \alpha \leq 1, \lambda \geq 1, z \in \mathbb{U}),$$

it can be easily verified that two functions $h(z)$ and $p(z)$ satisfy the hypotheses of Definition 2.1. If we have $f \in B_\Sigma^{h,p}(n, \lambda)$, then

$$f \in \Sigma \quad and \quad \left| arg\left[\frac{(1-\lambda)D^n f(z) + \lambda D^{n+1} f(z)}{z}\right]\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}),$$

and

$$\left|arg\left[\frac{(1-\lambda)D^n g(w)+\lambda D^{n+1}g(w)}{w}\right]\right| < \frac{\alpha\pi}{2} \qquad (w\in\mathbb{U}),$$

where the function g is given by (1.2).
Therefore, in this case the class $B_\Sigma^{h,p}(n,\lambda)$ reduce to class in Definition 1.5 and if we take $n=0$, it decrease to class in Definition 1.1.

If we put

$$h(z)=p(z)=\frac{1+(1-2\beta)z}{1-z} \qquad (0\le\beta<1, \lambda\ge 1, z\in\mathbb{U}),$$

then the conditions of Definition 2.1 are satisfied for both functions $h(z)$ and $p(z)$. If $f\in B_\Sigma^{h,p}(n,\lambda)$, then

$$f\in\Sigma \quad and \quad \mathfrak{Re}\left(\frac{(1-\lambda)D^n f(z)+\lambda D^{n+1}f(z)}{z}\right) > \beta \qquad (z\in\mathbb{U}),$$

and

$$\mathfrak{Re}\left(\frac{(1-\lambda)D^n g(w)+\lambda D^{n+1}g(w)}{w}\right) > \beta \qquad (w\in\mathbb{U}),$$

where the function g is defined by (1.2).

Therefore, in this case the class $B_\Sigma^{h,p}(n,\lambda)$ reduce to class in Definition 1.6 and if we set $n=0$, it decrease to class in Definition 1.3.

2.1 Coefficient Estimates

We are now ready to express the bounds for the coefficients $|a_2|$ and $|a_3|$ for subclass $B_\Sigma^{h,p}(n,\lambda)$.

Theorem 2.3. *If $f(z)$ be in the class $B_\Sigma^{h,p}(n,\lambda)$, then*

$$|a_2| \le \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2[(1-\lambda)2^n+\lambda 2^{n+1}]^2}}, \sqrt{\frac{|h''(0)|+|p''(0)|}{4[(1-\lambda)3^n+\lambda 3^{n+1}]}}\right\}, \tag{2.3}$$

and

$$|a_3| \le \min\left\{\frac{|h''(0)|+|p''(0)|}{4[(1-\lambda)3^n+\lambda 3^{n+1}]} + \frac{|h'(0)|^2+|p'(0)|^2}{2[(1-\lambda)2^n+\lambda 2^{n+1}]^2}, \frac{|h''(0)|}{2[(1-\lambda)3^n+\lambda 3^{n+1}]}\right\}. \tag{2.4}$$

Proof. The main idea of the proof is to get the desired bounds for the coefficient $|a_2|$ and $|a_3|$. Consider relations (2.1) and (2.2). Now, we have:

$$\left(\frac{(1-\lambda)D^n f(z)+\lambda D^{n+1}f(z)}{z}\right) = h(z) \qquad (\lambda\ge 1, z\in\mathbb{U}), \tag{2.5}$$

and

$$\left(\frac{(1-\lambda)D^n g(w)+\lambda D^{n+1}g(w)}{w}\right) = p(w) \qquad (\lambda\ge 1, w\in\mathbb{U}), \tag{2.6}$$

where two functions h and p satisfy the conditions of Definition 2.1, respectively. With respect to the following Taylor-Maclaurin series expansions for the functions h and p, we get:

$$h(z)=1+h_1 z+h_2 z^2+h_3 z^3+\cdots, \tag{2.7}$$

and

$$p(w) = 1 + p_1 w + p_2 w^2 + p_3 w^3 + \cdots . \tag{2.8}$$

Substituting relations (2.7) and (2.8) into (2.5) and (2.6), respectively, yield

$$[(1-\lambda)2^n + \lambda 2^{n+1}]a_2 = h_1, \tag{2.9}$$

$$[(1-\lambda)3^n + \lambda 3^{n+1}]a_3 = h_2, \tag{2.10}$$

$$-[(1-\lambda)2^n + \lambda 2^{n+1}]a_2 = p_1, \tag{2.11}$$

and

$$[(1-\lambda)3^n + \lambda 3^{n+1}](2a_2^2 - a_3) = p_2. \tag{2.12}$$

Comparing the coefficients (2.9) and (2.11), we obtain

$$h_1 = -p_1, \tag{2.13}$$

and

$$2[(1-\lambda)2^n + \lambda 2^{n+1}]^2 a_2^2 = h_1^2 + p_1^2. \tag{2.14}$$

Adding (2.10) and (2.12) give us the following relation:

$$2[(1-\lambda)3^n + \lambda 3^{n+1}]a_2^2 = p_2 + h_2. \tag{2.15}$$

Therefore, considering relations (2.14) and (2.15), we have:

$$a_2^2 = \frac{h_1^2 + p_1^2}{2[(1-\lambda)2^n + \lambda 2^{n+1}]^2}, \tag{2.16}$$

and

$$a_2^2 = \frac{p_2 + h_2}{2[(1-\lambda)3^n + \lambda 3^{n+1}]}, \tag{2.17}$$

respectively. So, we find from the equations (2.16) and (2.17), that

$$|a_2|^2 \leq \frac{|h'(0)|^2 + |p'(0)|^2}{2[(1-\lambda)2^n + \lambda 2^{n+1}]^2},$$

and

$$|a_2|^2 \leq \frac{|h''(0)| + |p''(0)|}{4[(1-\lambda)3^n + \lambda 3^{n+1}]},$$

respectively. Hence, we derive the desired bound on the coefficient $|a_2|$ as asserted in (2.3).

The proof is completed by finding the bound for the coefficient $|a_3|$. Subtracting (2.12) from (2.10) yields

$$2[(1-\lambda)3^n + \lambda 3^{n+1}]a_3 - 2[(1-\lambda)3^n + \lambda 3^{n+1}]a_2^2 = h_2 - p_2. \tag{2.18}$$

Put the value of a_2^2 from (2.16) into (2.18). So, it follows that

$$a_3 = \frac{h_1^2 + p_1^2}{2[(1-\lambda)2^n + \lambda 2^{n+1}]^2} + \frac{h_2 - p_2}{2[(1-\lambda)3^n + \lambda 3^{n+1}]}.$$

Therefore, we conclude the following bound:

$$|a_3| \leq \frac{|h'(0)|^2 + |p'(0)|^2}{2[(1-\lambda)2^n + \lambda 2^{n+1}]^2} + \frac{|h''(0)| + |p''(0)|}{4[(1-\lambda)3^n + \lambda 3^{n+1}]}. \tag{2.19}$$

By taking the value of a_2^2 from (2.17) into (2.18), it follows that

$$a_3 = \frac{(p_2 + h_2)}{2[(1-\lambda)3^n + \lambda 3^{n+1}]} + \frac{(h_2 - p_2)}{2[(1-\lambda)3^n + \lambda 3^{n+1}]} = \frac{h_2}{[(1-\lambda)3^n + \lambda 3^{n+1}]}.$$

Hence, we get:

$$|a_3| \le \frac{|h''(0)|}{2[(1-\lambda)3^n + \lambda 3^{n+1}]}. \tag{2.20}$$

Now, we obtain from (2.19) and (2.20) the desired estimate for the coefficient $|a_3|$ as asserted in (2.4) and the proof is completed. □

3 Conclusions

If we set

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \qquad (0 < \alpha \le 1,\ z \in \mathbb{U}),$$

in Theorem 2.3, then Corollary 3.1 can be obtained.

Corollary 3.1. *Let the function* $f(z)$ *given by* (1.1) *be in the class* $B_\Sigma(n, \alpha, \lambda)$. *Then*

$$|a_2| \le \min\left\{\frac{2\alpha}{[(1-\lambda)2^n + \lambda 2^{n+1}]}, \sqrt{\frac{2}{[(1-\lambda)3^n + \lambda 3^{n+1}]}}\alpha\right\},$$

and

$$|a_3| \le \frac{2\alpha^2}{[(1-\lambda)3^n + \lambda 3^{n+1}]}.$$

Remark 3.2. For the coefficient $|a_3|$, it is easily seen that

$$\frac{2\alpha^2}{[(1-\lambda)3^n + \lambda 3^{n+1}]} \le \frac{2\alpha}{[(1-\lambda)3^n + \lambda 3^{n+1}]} + \frac{4\alpha^2}{[(1-\lambda)2^n + \lambda 2^{n+1}]^2}.$$

So, it is clear that Corollary 3.1 is an improvement of Theorem 1.6.

Set $n = 0$ in Corollary 3.1. So, we have the following corollary.

Corollary 3.3. *Let the function* $f(z)$ *given by* (1.1) *be in the class* $B_\Sigma(\alpha, \lambda)$. *Then*

$$|a_2| \le \begin{cases} \sqrt{\dfrac{2}{2\lambda+1}}\alpha, & 1 \le \lambda < 1+\sqrt{2} \\[2ex] \dfrac{2\alpha}{\lambda+1}, & \lambda \ge 1+\sqrt{2} \end{cases}$$

and

$$|a_3| \le \frac{2\alpha^2}{2\lambda+1}.$$

Remark 3.4. Corollary 3.3 improves Theorem 1.2.

If we take $\lambda = 1$ in Corollary 3.3, then we get the following corollary.

Corollary 3.5. *Let the function $f(z)$ given by (1.1) be in the class $H_{\Sigma}^{\alpha}(0 < \alpha \leq 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2}{3}}\alpha,$$

and

$$|a_3| \leq \frac{2\alpha^2}{3}.$$

Remark 3.6. Corollary 3.5 provides a refinement of a result which obtained by Srivastava [9, Theorem 1].

Put

$$h(z) = p(z) = \frac{1+(1-2\beta)z}{1-z} \qquad (0 \leq \beta < 1,\ z \in \mathbb{U}),$$

in Theorem 2.3. Then, Corollary 3.7 can be easily concluded.

Corollary 3.7. *Let the function $f(z)$ given by (1.1) be in the class $B_{\Sigma}(n, \beta, \lambda)$. Then*

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{[(1-\lambda)2^n + \lambda 2^{n+1}]}, \sqrt{\frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]}}\right\},$$

and

$$|a_3| \leq \frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]}.$$

Remark 3.8. For the coefficient $|a_3|$, it can be easily seen that

$$\frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]} \leq \frac{2(1-\beta)}{[(1-\lambda)3^n + \lambda 3^{n+1}]} + \frac{4(1-\beta)^2}{[(1-\lambda)2^n + \lambda 2^{n+1}]^2}. \tag{3.1}$$

Hence, Corollary 3.7 improves Theorem 1.8 with regard to relation (3.1).

By setting $n = 0$ in Corollary 3.7, we get:

Corollary 3.9. *Let the function $f(z)$ given by (1.1) be in the class $B_{\Sigma}(\beta, \lambda)$ where $\lambda \geq 1$. Then*

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{\lambda+1}, \sqrt{\frac{2(1-\beta)}{2\lambda+1}}\right\},$$

and

$$|a_3| \leq \frac{2(1-\beta)}{2\lambda+1}.$$

Remark 3.10. Theorem 1.4 is improved by Corollary 3.9.

If we put $\lambda = 1$ in Corollary 3.9, the following corollary can be obtained.

Corollary 3.11. *Let the function $f(z)$ given by (1.1) be in the class $H_{\Sigma}(\beta)(0 \leq \beta < 1)$. Then*

$$|a_2| \leq \begin{cases} \sqrt{\frac{2(1-\beta)}{3}}, & 0 \leq \beta \leq \frac{1}{3} \\ (1-\beta), & \frac{1}{3} \leq \beta < 1 \end{cases}$$

and

$$|a_3| \leq \frac{2(1-\beta)}{3}.$$

Remark 3.12. The bounds on $|a_2|$ and $|a_3|$ given in Corollary 3.11 are better than those given by Srivastava [9, Theorem 2].

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