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PERMUTING TRI-DERIVATIONS ON HYPERRINGS

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Abstract In this paper we describe permuting tri-derivations in Krasner hyperrings. In this way we derive some important properties of permuting tri-derivations.

1 Introduction

Hyperstructures represent a natural extension of classical algebraic structures and they were introduced by the French mathematician Marty [13] at the $8th$ congress Scandinavian Mathematicians. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. In [15], Mittas introduced the concept of canonical hypergroups. Corsini [4] introduced and studied the Canonical Hypergroups [6], Feebly Canonical Hypergroups [5], Quasi- Canonical Hypergroups [7]. In [12], Krasner introduced the concept of hyperrings and hyperfields. G.G Massouros studied the theory of hypercompositional structures into the theory of automata (see [14]). Asokkumar [1] defined the idempotent elements of Krasner hyperrings. Babaei et al. studied R-parts in hyperrings (see [3]).

The concept of derivations in rings plays a significant role in algebra. After Posner [17], many papers concerning derivations have appeared in the literature. In [16], Ozturk presented permuting tri-derivations in prime and semi-prime rings. For more information for permuting tri derivations see [10] and [11]. In [18], Vougiouklis defined a hyperoperation called theta hyperoperation and studied H_v-structures. Jan Chvalina et al. [8], introduced a hyperoperation $*$ on a differential ring R so that $(R, *)$ is a hypergroup.

In [2], the author introduced derivations in Krasner hyperrings and in [9], the author studied symmetric bi-derivation in Krasner hyperrings. So in this paper, we aim to generalize some results given in [2] and [9]. In this way, we introduce the notion of permuting tri-derivations on Krasner hyperrings and some properties of them are investigated.

2 Preliminaries

In this section, for the sake of completeness we remind some definitions used in the sequel.

A hyperoperation on a nonempty set H is a function $\circ : H \times H \to \wp(H)^*$ where $\wp(H)$ is the power set of H and $(H, \circ)$ is called a hypergroupoid. For nonempty subsets A and B of H and $x \in H$, let

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\} \quad and \quad x \circ A = \{x\} \circ A.$$

An element $e \in H$ is called an identity of $(H, \circ)$ if $x \in x \circ e \cap e \circ x$, for all $x \in H$. If e is a scalar identity of $(H, \circ)$, then e is the unique identity of $(H, \circ)$. The hypergroupoid is said to be commutative if $x \circ y = y \circ x$ for all $x, y \in H$.

A hypergroupoid is called a semihypergroup if $(x \circ y) \circ z = x \circ (y \circ z)$, for all $x, y, z \in H$.

A semihypergroup is called a hypergroup if $H \circ x = x \circ H = H$, for all $x \in H$.

An element y of a hypergroupoid $(H, \circ)$ is called an inverse of $x \in H$ if $(x \circ y) \cap (y \circ x)$ contains at least one identity. A hypergroup $(H, \circ)$ is said to be regular if every element of $(H, \circ)$ has at least one inverse. A regular hypergroup $(H, \circ)$ is said to be reversible if for all $x, y, z \in H$, $x \in y \circ z \Rightarrow z \in y^{'} \circ x$ and $y \in x \circ z^{'}$, for some inverse $y^{'}$ of y and some inverse $z^{'}$ of z.

Definition 2.1. A non-empty set R with a hyperaddition $+$ and a multiplication . is called additive hyperring or Krasner hyperring if it satisfies the following:

(1) $(R, +)$ is a canonical hypergroup, i.e.,

(i) for every $x, y, z \in R$, $x + (y + z) = (x + y) + z$,

(ii) for every $x, y \in R$, $x + y = y + x$,

(iii) there exists $0 \in R$ such that $0 + x = x$ for all $x \in R$,

(iv) for every $x \in R$ there exists an unique element denoted by $-x \in R$ such that $0 \in x + (-x)$

(v) for every $x, y, z \in R$, $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$.

(2) $(R, .)$ is a semigroup having 0 as a bilaterally absorbing element, i.e.,

(i) for every $x, y, z \in R$, $(x.y).z = x.(y.z)$,

(ii) $x.0 = 0.x = 0$ for all $x \in R$.

(3) The multiplication . is distributive with respect to the hyperoperation $+$. i.e., for every $x, y, z \in R$, $x.(y + z) = x.y + x.z$ and $(x + y).z = x.z + y.z$.

A non-empty subset I of a canonical hypergroup R is called a canonical subhypergroup of R if I itself is a canonical hypergroup under the same hyperoperation as that of R. Equivalently, a non-empty subset I of a canonical hypergroup R is a canonical subhypergroup of R if for every $x, y \in I$, $xy \subseteq I$. Here after we denote xy instead of $x.y$. Moreover, for $A, B \subseteq R$ and $x \in R$, by $A + B$ we mean the set $\cup_{a \in A, b \in B}(a + b)$ and $AB = \cup_{a \in A, b \in B}(ab)$, $A + x = A + \{x\}$, $x + B = \{x\} + B$ and also $-A = \{-a : a \in A\}$. The following elementary facts in a hyperring easily follow from the axioms:

(i) $-(-a) = a$ for every $a \in R$;

(ii) 0 is the unique element such that for every $a \in R$, there is an element $-a \in R$ with the property $0 \in a + (-a)$ and $-0 = 0$;

(iii) $-(a + b) = -a - b$ for all $a, b \in R$;

(iv) $-(ab) = (-a)b = a(-b)$ for all $a, b \in R$.

In a hyperring R, if there exists an element $1 \in R$ such that $1a = a1 = a$ for every $a \in R$, then the element 1 is called the identity element of the hyperring R. In fact, the element 1 is unique. Further, if $ab = ba$ for every $a, b \in R$ then the hyperring R is called a commutative hyperring. Throughout this paper, by a hyperring we mean the Krasner hyperring.

Example 2.2. The set $R = \{0, 1\}$ with the following hyperoperations is a hyperring.

$+$	0	1
0	$\{0\}$	$\{1\}$
1	$\{1\}$	$\{0, 1\}$

.	0	1
0	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{1\}$

Definition 2.3. Let R be a hyperring. A non-empty subset S of R is called a subhyperring of R if $x - y \subseteq S$ and $xy \in S$ for all $x, y \in S$.

Definition 2.4. Let R be a hyperring and I be a non-empty subset of R. I is called a left (resp. right) hyperideal of R if

(i) $(I, +)$ is a canonical subhypergroup of R, i.e., for every $x, y \in I$, $x - y \subseteq I$

(ii) for every $a \in I$, $r \in R$, $ra \subseteq I$ (resp. $ar \subseteq I$). A hyperideal of R is one which is a left as well as a right hyperideal of R.

Definition 2.5. A hyperring R is said to be prime hyperring if $aRb = 0$ for $a, b \in R$ implies either $a = 0$ or $b = 0$.

Definition 2.6. A hyperring R is said to be reduced hyperring if it has no nilpotent elements. That is, if $x^n = 0$ for all $x \in R$ and a natural number n, then $x = 0$.

Definition 2.7. A hyperring R is said to be $2-$torsion free if $0 \in x + x$ for $x \in R$ implies $x = 0$.

3 Permuting tri-derivations of hyperrings and examples

In this section we define permuting tri-derivation and strong permuting tri-derivation of hyperrings and give examples.

Definition 3.1. Let R be a hyperring. A mapping $D : R \times R \times R \to R$ is called permuting if it satisfies the condition $D(x, y, z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$ for all $x, y, z \in R$.

Definition 3.2. Let R be a hyperring. A map $D : R \times R \times R \to R$ is said to be a permuting tri-derivation of R if D satisfies:

(i) $D(x + w, y, z) \subseteq D(x, y, z) + D(w, y, z)$

(ii) $D(xw, y, z) \in D(x, y, z)w + xD(w, y, z)$

for all $x, y, z, w \in R$.

The hyperring R equipped with a permuting tri-derivation D is called a $D-$ differential hyperring. If the map D is such that $D(x + w, y, z) = D(x, y, z) + D(w, y, z)$ for all $x, y, z, w \in R$ and satisfies the condition (ii), then D is called a strong permuting tri-derivation of R. In this case, the hyperring is called strongly $D-$ differential hyperring.

Proposition 3.3. *Let R be a hyperring and $D : R \times R \times R \to R$ be a permuting tri-derivation of R. Then*

(i) $D(a, b, 0) = 0$, $\forall a, b \in R$.

(ii) $D(-a, b, c) = -D(a, b, c)$, $\forall a, b, c \in R$.

(iii) if 1 is the identity element of R, then $D(1, a, b) \in D(1, a, b) + D(1, a, b)$, $\forall a, b \in R$.

Proof.

(i) $D(a, b, 0) = D(a, b, 0.0) \in D(a, b, 0).0 + 0.D(a, b, 0)$ and so $D(a, b, 0) = 0$.

(ii) $\forall a, b, c \in R$, $0 = D(a, b, 0) = D(a, b, c - c) \subseteq D(a, b, c) + D(a, b, -c)$. That is $D(a, b, c) \in 0 - D(a, b, -c)$. Hence we get $D(a, b, c) = -D(a, b, -c)$. Therefore we obtain $-D(a, b, c) = -(-D(a, b, -c)) = D(a, b, -c)$.

(iii) $D(1, a, b) = D(1.1, a, b) \in D(1, a, b).1 + 1.D(1, a, b) = D(1, a, b) + D(1, a, b)$, $\forall a, b \in R$. Therefore we obtain $D(1, a, b) \in D(1, a, b) + D(1, a, b)$.

Example 3.4. Consider the hyperring $R = \{0, a, b\}$ with the hyperaddition and the multplication defined as follows.

$+$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{a\}$	$\{a,b\}$	R
b	$\{b\}$	R	$\{a,b\}$

$.$	0	a	b
0	0	0	0
a	0	b	a
b	0	a	b

Define a map $D : R\times R\times R \to R$ by $D(0,0,0) = 0$, $D(a,0,0) = D(0,a,0) = D(0,0,a) = D(b,0,0) = D(0,b,0) = D(0,0,b) = 0$, $D(a,a,0) = D(a,0,a) = D(0,a,a) = b$, $D(b,b,0) = D(b,0,b) = D(0,b,b) = a$, $D(a,b,0) = D(a,0,b) = D(0,a,b) = D(b,a,0) = D(b,0,a) = D(0,b,a) = a$, $D(b,b,a) = D(b,a,b) = D(a,b,b) = a$, $D(a,a,b) = D(a,b,a) = D(b,a,a) = b$, $D(a,a,a) = D(b,b,b) = a$. Clearly, D is a strong permuting tri-derivation of R.

Example 3.5. Let R be a commutative hyperring and $M(R) = \left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} : a,b\in R\right\}$ be a collection of 2×2 matrices over R. A hyperaddition $\oplus$ is defined on $M(R)$ by $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \oplus \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} = \left\{\begin{pmatrix}0 & x\\ 0 & y\end{pmatrix} : x\in a+c, y\in b+d\right\}$ for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \in M(R)$. Clearly, this hyperaddition is well-defined and $(M(R),\oplus)$ is a canonical hypergroup. The matrix $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ is the additive identity of $M(R)$. Also for each matrix $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \in M(R)$, there exists a unique matrix $\begin{pmatrix}0 & -a\\ 0 & -b\end{pmatrix} \in M(R)$ such that $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} \in \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \oplus \begin{pmatrix}0 & -a\\ 0 & -b\end{pmatrix}$.

Now a multiplication $\otimes$ is defined on $M(R)$ by $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} = \begin{pmatrix}0 & ad\\ 0 & bd\end{pmatrix}$ for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \in M(R)$. Clearly, the multiplication $\otimes$ is well defined and associative. Therefore $(M(R),\otimes)$ is a semigroup.

Let $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix} \in M(R)$. Then

$\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \left\{\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \oplus \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right\} = \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \left\{\begin{pmatrix}0 & r\\ 0 & s\end{pmatrix} : r\in c+e, s\in d+f\right\}$ and

$\left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}\right\} \oplus \left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right\} = \begin{pmatrix}0 & ad\\ 0 & bd\end{pmatrix} \oplus \begin{pmatrix}0 & af\\ 0 & bf\end{pmatrix}$

$= \left\{\begin{pmatrix}0 & l\\ 0 & m\end{pmatrix} : l\in ad+af, m\in bd+bf\right\}$. So, we have

$$\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \left\{\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \oplus \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right\} = \left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}\right\} \oplus \left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right\}.$$

Similarly we have

$$\left\{\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \oplus \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right\} \otimes \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} = \left\{\begin{pmatrix}0 & c\\ 0 & d\end{pmatrix} \otimes \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}\right\} \oplus \left\{\begin{pmatrix}0 & e\\ 0 & f\end{pmatrix} \otimes \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}\right\}.$$

Thus $M(R)$ is a Krasner hyperring.

Now define a function D on $M(R)$ by $D\left(\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right) = \begin{pmatrix}0 & bdf\\ 0 & 0\end{pmatrix}$. Clearly this map is well defined. Now we will show that D is a permuting tri-derivation. For all

$\begin{pmatrix}0 & a\\0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}, \begin{pmatrix}0 & g\\0 & h\end{pmatrix} \in M(R)$

$D\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix} \oplus \begin{pmatrix}0 & g\\0 & h\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) = D\left(\left\{\begin{pmatrix}0 & r\\0 & s\end{pmatrix} : r \in a+g, s \in b+h\right\}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right)$

$= \left\{\begin{pmatrix}0 & sdf\\0 & 0\end{pmatrix} : s \in b+h\right\}$

and

$D\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) \oplus D\left(\begin{pmatrix}0 & g\\0 & h\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) =$

$\begin{pmatrix}0 & 0\\0 & bdf\end{pmatrix} \oplus \begin{pmatrix}0 & 0\\0 & hdf\end{pmatrix} = \left\{\begin{pmatrix}0 & 0\\0 & l\end{pmatrix} : l \in bdf + hdf\right\}.$

Also

$D\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & g\\0 & h\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) = D\left(\begin{pmatrix}0 & ag\\0 & bh\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) = \begin{pmatrix}0 & bhdf\\0 & 0\end{pmatrix}$

and

$\left\{D\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) \otimes \begin{pmatrix}0 & g\\0 & h\end{pmatrix}\right\} \oplus \left\{\begin{pmatrix}0 & a\\0 & b\end{pmatrix} \otimes D\left(\begin{pmatrix}0 & g\\0 & h\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right)\right\}$

$= \left\{\begin{pmatrix}0 & bdf\\0 & 0\end{pmatrix} \otimes \begin{pmatrix}0 & g\\0 & h\end{pmatrix}\right\} \oplus \left\{\begin{pmatrix}0 & a\\0 & b\end{pmatrix} \otimes \begin{pmatrix}0 & hdf\\0 & 0\end{pmatrix}\right\}$

$= \begin{pmatrix}0 & bdfh\\0 & 0\end{pmatrix} \oplus \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix} = \begin{pmatrix}0 & bdfh\\0 & 0\end{pmatrix}.$

Thus D is a permuting tri-derivation on $M(R)$. Here D is a strong permuting tri-derivation.

Definition 3.6. Let R be a hyperring and $D : R \times R \times R \to R$ be a permuting tri-derivation. A mapping $d : R \to R$ defined by $d(x) = D(x, x, x)$ is called the trace of D.
It is clear that, in case $D : R \times R \times R \to R$ be a permuting tri-mapping, the trace of D satisfies the following relation

$$d(x+y) = D(x+y, x+y, x+y) \subseteq d(x) + D(x,x,y) + D(x,y,x) + D(x,y,y) + D(y,x,x) + D(y,x,y) + D(y,y,x) + d(y)$$

and $d(0) = D(0,0,0)$. If D is strong permuting tri-derivation, we have

$$d(x+y) = d(x) + D(x,x,y) + D(x,y,x) + D(x,y,y) + D(y,x,x) + D(y,x,y) + D(y,y,x) + d(y).$$

Since

$$0 = d(0) = d(x+(-x)) \subseteq dx + D(x,x,-x) + D(x,-x,x) + D(x,-x,-x) + D(-x,x,x) + D(-x,x,-x) + D(-x,-x,x) + d(-x) = -d(x) + d(-x),$$

we have $d(-x) \in 0 - (-d(x))$. Therefore we obtain $d(-x) = d(x)$.

Proposition 3.7. *Let R be a hyperring. D be a permuting tri-derivation of R and a, b be fixed elements of R. Then $S = \{x \in R : D(x, a, b) = 0\}$ is a subhyperring of R.*

Proof. S is nonempty since $D(0, a, b) = 0$. So we get $D(x, a, b) = 0$ and $D(y, a, b) = 0$ for $x, y \in S$. Hence we have $D(x + y, a, b) \subseteq D(x, a, b) + D(y, a, b)$. In addition to this for any $x \in S$, $D(-x, a, b) = -D(x, a, b) = 0$. Also, $D(xy, a, b) \in D(x, a, b)y + xD(y, a, b) = 0$. Thus for any $x, y \in S$, $x + y \subseteq S$, $-x \in S$, $xy \in S$. Therefore S is a subhyperring of R.

Proposition 3.8. *Let D be a permuting tri-derivation of a prime hyperring R and $a \in R$ such that $aD(x, y, z) = 0$ (or $D(x, y, z)a = 0$) for all $x, y, z \in R$. Then either $a = 0$ or $D = 0$.*

Proof. Assume that $aD(x,y,z) = 0$ for all $x, y, z \in R$, then we have

$$0 = aD(xt, y, z) \in axD(t, y, z) + aD(x, y, z)t = axD(t, y, z)$$

That is, $axD(t, y, z) = 0$. Since R is a prime hyperring we obtain $a = 0$ or $D(t, y, z) = 0$. If $a \neq 0$, then we have $D(t, y, z) = 0$. That is, $D = 0$.

Suppose that $D(x, y, z)a = 0$ for all $x, y, z \in R$, then

$$0 = D(xt, y, z)a \in xD(t, y, z)a + D(x, y, z)ta = D(x, y, z)ta$$

That is, $D(x, y, z)ta = 0$. Since R is a prime hyperring we obtain $a = 0$ or $D(x, y, z) = 0$. If $a \neq 0$, then we have $D(x, y, z) = 0$. That is, $D = 0$.

Proposition 3.9. *Let R be a prime hyperring with $charR \neq 2, 3$ and D be a strong permuting tri-derivation with trace d of R and $a \in R$ such that $ad(x) = 0$ (or $d(x)a = 0$) for all $x \in R$. Then either $a = 0$ or $D = 0$.*

Proof. Assume that $ad(x) = 0$ for all $x \in R$. Replacing x by $x + y$ we get

$$0 = ad(x + y) = ad(x) + 3aD(x, x, y) + 3aD(x, y, y) + ad(y).$$

Since $charR \neq 3$ we obtain

$$aD(x, x, y) + aD(x, y, y) = 0. \tag{3.1}$$

Writing $-x$ for x in (3.1) we have

$$aD(x, y, y) = 0. \tag{3.2}$$

Replacing x by xy in (3.2) we conclude $axd(y) = 0$. Since R is prime hyperring we obtain $a = 0$ or $d(y) = 0$ for all $x \in R$. Consequently $a = 0$ or $D = 0$.

Theorem 3.10. *Let D be a permuting tri-derivation of $2-$torsion free reduced hyperring R. If $D(D(x, y, z), y, z) = 0$ for all $x, y, z \in R$ then $D = 0$.*

Proof. Let $D(D(x, y, z), y, z) = 0$ for all $x, y, z \in R$. Replacing x by xt, $t \in R$, we obtain

$$\begin{aligned} 0 = D(D(xt, y, z), y, z) &\in D(D(x, y, z)t + xD(t, y, z), y, z) \in D(D(x, y, z)t, y, z) + D(xD(t, y, z), y, z) \\ &\in D(D(x, y, z)t, y, z) + D(xD(t, y, z), y, z) \in D(x, y, z)D(t, y, z) + D(D(x, y, z), y, z)t + \\ &\qquad D(x, y, z)D(t, y, z) + xD(D(t, y, z), y, z) \\ &= D(x, y, z)D(t, y, z) + D(x, y, z)D(t, y, z) \end{aligned}$$

Since R is $2-$torsion free hyperring we get $D(x, y, z)D(t, y, z) = 0$. If we take x instead of t we have $D(x, y, z)^2 = 0$ for all $x, y, z \in R$. Since R is reduced hyperring we have $D(x, y, z) = 0$ for all $x, y, z \in R$. Hence we obtain $D = 0$.

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