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Some New Classes of Multiplier Ideal Convergent Triple Sequence Spaces of Fuzzy Numbers Defined by Orlicz Functions

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Abstract In this article, some new classes of multiplier ideal convergent triple sequence spaces of fuzzy numbers defined by an Orlicz function and a multiplier sequence are introduced. The multiplier problem is characterized.We also make an effort to prove some algebraic and topological properties such as closed property, completeness, solid, monotone, symmetric, sequence algebra, convergence free etc. of these spaces. Moreover some inclusion relation between these spaces are established.

1 Introduction

The fuzzy set theory extended the basic mathematical concept of a set. After the pioneering work done on fuzzy set theory by Zadeh [35] in 1965, a huge number of research papers have been appeared on fuzzy theory and its applications as well as fuzzy analogues of the classical theories. Fuzzy set theory is a powerful hand set for modeling, uncertainty and vagueness in various problems arising in the field of science and engineering. Several mathematicians have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. In fact the fuzzy set theory has become an active area of research in science and engineering for the last half century. While studying fuzzy topological spaces, we face many situations where we need to deal with convergence of fuzzy numbers. Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. Matloka [12] introduced bounded and convergent sequences of fuzzy numbers and studied some of their properties. Nanda [13] studied the sequences of fuzzy numbers and showed that the set of all convergent sequences of fuzzy numbers forms a complete metric space.

The summability theory of multiple sequences was studied by Agnew [1] and he derived certain theorems for double sequences. At the initial stage, the different types of notions of triple sequences were introduced and investigated by Sahiner et al. [20] and Sahiner and Tripathy [21]. Recently statistical convergence of triple sequences on probabilistic normed space was introduced by Savas and Esi [24]. Later on, Esi [5] has introduced statistical convergence of triple sequences in topological groups. More works on triple sequences are found in Kumar et. al. [9], Dutta et. al. [3], Tripathy and Goswami [28], Nath and Roy [14-16], Saha et. al. [18], Saha and Roy [19] and so on.

The notion of ideal convergence depends on the structure of the ideal I of the subset of the set of natural numbers. The concept of ideal convergence for single sequences was introduced by Kostyrko, Salat and Wilczyaski [8] in 2000-2001. Later on it was further developed by Salat et al. [22-23], Tripathy and Sen [32], Tripathy and Tripathy [34], Kumar et. al. [9], Das et al. [2], Tripathy and Hazarika [29] and many others.

An Orlicz function M is a function $M : [0, \infty) \to [0, \infty)$ such that it is continuous, non-decreasing and convex with $M(0) = 0, M(x) > 0 \; for \; x > 0 \; and \; M(x) \to \infty \; as \; x \to \infty$.

An Orlicz function may be bounded or unbounded. For example, $M(x) = x^p, (0 < p \leq 1)$ is unbounded and $M(x) = \dfrac{x}{x+1}$ is bounded .

The scope for the studies on sequence spaces was extended by using the concept of Orlicz function. The study of Orlicz sequence spaces was initiated with certain specific purpose in Banach space theory. Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct the sequence space, which becomes a Banach space, with the norm

$$||x|| = inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}.$$

Parashar and Choudhary [17] have introduced and discussed some properties of the sequence spaces defined by an Orlicz function M which generalized the well-known Orlicz sequence space ℓ_M. Some works on Orlicz sequence spaces can be found in ([4], [25], [27], [29], [31]).

The scope for the studies on sequence spaces was extended further by using the concept of multiplier sequences. The notion of multiplier sequences was first studied by Goes and Goes [7]. Goes and Goes defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, by using multiplier sequences $\left(k^{-1}\right)$ and (k) respectively. Tripathy and Sen [33], Tripathy and Mahanta [30] used a general multiplier sequence (λ_k) of non-zero scalars for their studies on sequence spaces associated with multiplier sequences. Sen and Roy [26] used a general multiplier sequence (λ_{nk}) of non-zero scalars on sequence spaces associated with multiplier sequences. In this article we shall consider a general multiplier triple sequence $\Lambda = (\lambda_{ijk})$ of non-zero real numbers.

2 PRELIMINARIES AND BACKGROUND

Throughout the article, N, R and C denote the sets of natural, real and complex numbers respectively and *w,c,*c_0, ℓ_∞ denote the spaces of all, convergent, null and bounded sequences respectively.

Let X be a nonempty set. A non-void class $I \subseteq 2^x$ (power set of X) is said to be an ideal if I satisfies the following conditions:
(i) $A, B \in I \Rightarrow A \cup B \in I$ and (ii) $A \in I$ and $B \subseteq A \Rightarrow B \in I.$

A non-empty family of sets $F \subseteq 2^x$ is said to be a filter on X if
(i) $\emptyset \notin F,$ (ii) $A, B \in F \Rightarrow A \cap B \in F$ and (iii) $A \in F$ and $A \subseteq B \Rightarrow B \in F.$

For any ideal I, there is a filter $F(I)$ given by $F(I) = \{K \subseteq N : N/K \in I\}$.
An ideal $I \subseteq 2^x$ is said to be $non-trivial$ if $I \neq \emptyset$ and $X \notin I.$

A subset E of $N \times N \times N$ is said to have density or asymptotic density $\delta_3(E)$, if the limit given by

$$\delta_3(E) = \lim_{p,q,r\to\infty} \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} \chi_E(i,j,k)$$

exists, where χ_E is the characteristic function of E.

The notion of the ideals of $2^{N\times N}$ are introduced by Tripathy and Tripathy [34]. Throughout the article, the ideals of $2^{N\times N\times N}$ will be denoted by I_3.

Example 2.1. Let $I_3(P)$ be the class of all subsets of $N \times N \times N$ such that $D \in I_3(P)$ implies there exists $n_0, l_0, k_0 \in N$ such that

$$D \subseteq N \times N \times N - \{(n,l,k) \in N \times N \times N : n \geq n_0, l \geq l_0, k \geq k_0\}.$$

Then $I_3(P)$ is an ideal of $2^{N\times N\times N}$.

A fuzzy real number X is a fuzzy set on R i.e. a mapping $X : R \to L(= [0,1])$ associating each real number $t \in R$ having grade of membership $X(t)$. Every real number r can be expressed as a fuzzy number $\bar{r}$ as:

$$\bar{r}(t) = \begin{cases} 1, \ if\ t = r \\ 0, \ otherwise \end{cases}$$

The α-level set of a fuzzy number $X, 0 < \alpha \leq 1$, is defined and denoted as

$$[X]^\alpha = \{t \in R : X(t) \geq \alpha\}.$$

A fuzzy number X is said to be convex if $X(t) \geq X(s) \wedge X(r) = min(X(s), X(r))$, where $s < t < r$.
A fuzzy number X is called normal if there exists $t_0 \in R$ such that $X(t_0) = 1$.
If for each $\epsilon > 0, X^{-1}[0, a + \epsilon))$, for all $a \in L$ is open in the usual topology of R, then a fuzzy number X is called upper semi-continuous. The set of all upper semi continuous, normal, convex fuzzy numbers is denoted by $R(L)$, whose additive and multiplicative identities are $\bar{0}$ and $\bar{1}$ respectively.

If D denotes the set of all closed bounded intervals $X = \lfloor X^L, X^R \rfloor$ on the real line R and if $d(X, Y) = max(|X^L - X^R|, |Y^L - Y^R|)$, then (D, d) is a complete metric space.
Also $\bar{d} : R(L) \times R(L) \to R$ defined by $\bar{d}(X, Y) = \sup\limits_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha),\ for\ X, Y \in R(L)$ is also a metric on $R(L)$.

Throughout ${}_3(w^F), {}_3(\ell_\infty^F), {}_3(c^F), {}_3(c_0^F)$ denote the spaces of all, bounded, convergent in Pringsheim's sense and null in Pringsheim's sense fuzzy real-valued triple sequences respectively. A triple sequence can be defined as a function $x : N \times N \times N \to R(C)$.

A fuzzy real-valued triple sequence $X = \langle X_{ijk} \rangle$ is a triple infinite array of fuzzy real numbers X_{ijk} for all $i, j, k \in N$ and is denoted by $\langle X_{ijk} \rangle$ where $X_{ijk} \in R(L)$.

A fuzzy real-valued triple sequence $X = \langle X_{ijk} \rangle$ is said to be convergent in Pringsheim's sense to the fuzzy real number X, if for every $\varepsilon > 0$, there exist $i_0 = i_0(\varepsilon), j_0 = j_0(\varepsilon), k_0 = k_0(\varepsilon) \in N$, such that $\bar{d}(X_{ijk}, X) < \varepsilon$ for all $i \geq i_0, j \geq j_0, k \geq k_0$.

A fuzzy real-valued triple sequence $X = \langle X_{ijk} \rangle$ is said to be $I_3 - convergent$ to the fuzzy number X_0 , if for all $\epsilon > 0, \{(n, l, k) \in N \times N \times N : \bar{d}(X_{nlk}, X_0) \geq \varepsilon\} \in I_3$ and we write $I_3 - \lim X_{ijk} = X_0$.

A fuzzy real-valued triple sequence $X = \langle X_{ijk} \rangle$ is said to be $I_3 - bounded$ if there exists a real number μ such that $\{(i, j, k) \in N \times N \times N : \bar{d}(X_{ijk}, \bar{0}) > \mu\} \in I_3$.

A fuzzy real-valued triple sequence space E^F is said to be solid or normal if $\langle Y_{ijk} \rangle \in E^F$ whenever $\langle X_{ijk} \rangle \in E^F$ and $\bar{d}(Y_{ijk}, \bar{0}) \leq \bar{d}(X_{ijk}, \bar{0})$ for all $i, j, k \in N$.

A K-step space of a fuzzy real valued triple sequence space E^F is a sequence space
$\lambda_K^{E^F} = \{(X_{i_n j_n k_n}) \in {}_3(w^F) : (X_{ijk}) \in E^F\}$.

A canonical pre-image of a sequence $(X_{i_n j_n k_n}) \in E^F$ is a sequence $\langle Y_{ijk} \rangle \in {}_3(w^F)$ defined by:

$$Y_{ijk} = \begin{cases} X_{ijk}, & \text{if } (i, j, k) \in K \\ \bar{0}, & \text{otherwise} \end{cases}$$

A canonical pre-image of a step space $\lambda_K^{E^F}$ is a set of canonical pre-images of all elements in $\lambda_K^{E^F}$, that is Y is in canonical pre-image $\lambda_K^{E^F}$ if and only if Y is canonical pre-image of some

$X \in \lambda_K^{E^F}$.
A fuzzy real-valued triple sequence space E^F is said to be monotone if E^F contains the canonical pre-image of all its step spaces.
A fuzzy real-valued triple sequence space E^F is said to be symmetric if $\langle X_{\pi(ijk)} \rangle \in E^F$, whenever $\langle X_{ijk} \rangle \in E^F$ where π is a permutation on $N \times N \times N$.
A fuzzy real-valued triple sequence space E^F is said to be sequence algebra if $\langle X_{ijk} \bigotimes Y_{ijk} \rangle \in E^F$, whenever $\langle X_{ijk} \rangle, \langle Y_{ijk} \rangle \in E^F$.
A fuzzy real-valued triple sequence space E^F is said to be convergence free if $\langle Y_{ijk} \rangle \in E^F$ whenever $\langle X_{ijk} \rangle \in E^F$ and $X_{ijk} = \bar{0}$ implies $Y_{ijk} = \bar{0}$.

Let ${}_3l_\infty$ denote the set of all bounded triple sequences of fuzzy numbers.

Let $\Lambda = \langle \lambda_{ijk} \rangle$ be a triple sequence of non-zero scalars. For a fuzzy real-valued triple sequence space E^F, the multiplier sequence space $E^F(\Lambda)$ associated with the multiplier double sequence Λ is defined as $E^F(\Lambda) = \{\langle X_{ijk} \rangle : \langle \lambda_{ijk} X_{ijk} \rangle \in E^F\}$.

A multiplier from a fuzzy real-valued double sequence space D^F into another fuzzy-real valued double sequence space E^F is a real sequence $u = \langle u_{ijk} \rangle$ such that $uX = \langle u_{ijk} X_{ijk} \rangle \in E^F$, whenever $X = \langle X_{ijk} \rangle \in D^F$.

The linear space of all such multipliers will be denoted by $m(D^F, E^F)$. Bounded multipliers will be denoted by $M(D^F, E^F)$. Hence $M(D^F, E^F) = {}_2(\ell_\infty^F) \cap m(D^F, E^F)$.

Let $\Lambda = \langle \lambda_{ijk} \rangle$ be a multiplier sequence and $p = \langle p_{ijk} \rangle$ be a triple sequence of bounded strictly positive numbers. We introduce the following fuzzy $I - convergent$ triple sequence spaces:

$$ {}_3(c^{I(F)})(M, \Lambda, P) = \left\{ X = \langle X_{ijk} \rangle : I_3 - \lim \left[M \left(\frac{\bar{d}(\lambda_{ijk} X_{ijk}, X_0)}{\rho} \right) \right]^{p_{ijk}} = 0,\ for\ some\ \rho > 0\ and\ X_0 \in R(L) \right\}, $$

$$ {}_3(c_0^{I(F)})(M, \Lambda, P) = \left\{ X = \langle X_{ijk} \rangle : I_3 - \lim \left[M \left(\frac{\bar{d}(\lambda_{ijk} X_{ijk}, \bar{0})}{\rho} \right) \right]^{p_{ijk}} < \infty,\ for\ some\ \rho > 0 \right\}, $$

$$ {}_3(\ell_\infty^{(F)})(M, \Lambda, P) = \left\{ X = \langle X_{ijk} \rangle : \sup_{i,j,k} \left[M \left(\frac{\bar{d}(\lambda_{ijk} X_{ijk}, \bar{0})}{\rho} \right) \right]^{p_{ijk}} < \infty \right\}, $$

$$ {}_3(\ell_\infty^{I(F)})(M, \Lambda, P) = \left\{ \begin{array}{c} X = \langle X_{ijk} \rangle : \ there\ \ exists\ \ a\ \ real\ \ number\ \ \mu > 0\ \ such\ \ that \\ \left\{ (i,j,k) \in N \times N \times N : \left[M \left(\frac{\bar{d}(\lambda_{ijk} X_{ijk}, \bar{0})}{\rho} \right) \right]^{p_{ijk}} > \mu \right\} \in I_3, for\ some\ \rho > 0 \end{array} \right\} $$

Also we define

$$ {}_3(m^{I(F)})(M, \Lambda, p) = {}_3\ (c^{I(F)})(M, \Lambda, p) \cap\ {}_3(\ell_\infty^{(F)})(M, \Lambda, p) $$

and

$$ {}_3(m_0^{I(F)})(M, \Lambda, p) = {}_3\ (c_0^{I(F)})(M, \Lambda, p) \cap\ {}_3(\ell_\infty^{(F)})(M, \Lambda, p). $$

Let Z denote any one of ${}_3(c^{I(F)}),\ {}_3(c_0^{I(F)})$ and ${}_3(\ell_\infty^{I(F)})$. On giving particular values to M and p , we get the following sequence spaces from the above sequence spaces :
(i) If $p_{ijk} = 1$, for all $i, j, k \in N$, then we obtain $Z(M, \Lambda)$ instead of $Z(M, \Lambda, p)$.
(ii) If $M(x) = x$, then $Z(M, \Lambda, p)$ becomes $Z(\Lambda, p)$.
(iii)If $M(x) = x$, $p_{ijk} = 1$, for all $i, j, k \in N$, we obtain $Z(\Lambda)$ instead of $Z(M, \Lambda, p)$.

Let $\langle X_{ijk} \rangle$ and $\langle Y_{ijk} \rangle$ be two fuzzy real valued triple sequences. Then we say that $X_{ijk} = Y_{ijk}$ for almost all i, j and k relative to I_3 (in short a.a. i, j and k r. I_3) if the set
$\{(i, j, k) \in N \times N \times N : X_{ijk} \neq Y_{ijk}\} \in I_3$.

Note: Let $p = \langle p_{ijk} \rangle$ be a triple sequence of bounded positive numbers and $H = \sup\limits_{i,j,k}\ p_{ijk} < \infty$. Then for sequences $\langle a_{ijk} \rangle$ and $\langle b_{ijk} \rangle$ of complex numbers, we have the following inequality:

$$|a_{ijk} + b_{ijk}|^{p_{nkl}} \leq D\left(|a_{ijk}|^{p_{ijk}} + |b_{ijk}|^{p_{ijk}}\right),\ where\ D = \max(1, 2^{H-1}).$$

To prove some results in the paper, the following existing result will be used.

Remark 2.2. Every normal sequence space is monotone.

3 MAIN RESULTS

Theorem 3.1. *Let $p = \langle p_{ijk} \rangle$ be a triple sequence of bounded positive real numbers. If $\Lambda = \langle \lambda_{ijk} \rangle$ is a given multiplier sequence and M is an Orlicz function, then the classes of sequences ${}_3(m^{I(F)})(M, \Lambda, p)$ and ${}_3(m_0^{I(F)})(M, \Lambda, p)$ are closed under the operations of addition and scalar multiplication.*

Proof. We prove the result for the space ${}_3(m_0^{I(F)})(M, \Lambda, p)$ and the result for the other space can be established in a similar manner.
Let $\langle X_{nlk} \rangle, \langle Y_{nlk} \rangle \in\ {}_3(m_0^{I(F)})(M, \Lambda, p)$.
Then there exist positive numbers ρ_1 and ρ_2 such that the sets

$$A = \left\{(i, j, k) \in N \times N \times N : \left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk}, \bar{0})}{\rho_1}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \in I_3$$

and

$$B = \left\{(i, j, k) \in N \times N \times N : \left[M\left(\frac{\bar{d}(\lambda_{ijk}Y_{ijk}, \bar{0})}{\rho_2}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \in I_3.$$

Let α, β be two scalars and let $\rho = \max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. M, being continuous we have the following inequality :

$$\left[M\left(\frac{\bar{d}(\alpha\lambda_{ijk}X_{nlk} + \beta\lambda_{ijk}Y_{nlk}, \bar{0})}{\rho}\right)\right]^{p_{ijk}} \leq D\left\{\left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{nlk}, \bar{0})}{\rho_1}\right)\right]^{p_{ijk}} + \left[M\left(\frac{\bar{d}(\lambda_{ijk}Y_{nlk}, \bar{0})}{\rho_2}\right)\right]^{p_{ijk}}\right\},$$

where $D = \max(1, 2^{H-1}),\ H = \sup\limits_{i,j,k}\ p_{ijk} < \infty$.
From the above inequality, we obtained:

$$\left\{(i,j,k)\in N\times N\times N:\left[M\left(\frac{\bar{d}(\alpha\lambda_{ijk}X_{ijk}+\beta\lambda_{ijk}Y_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}}\geq\frac{\varepsilon}{2}\right\}\subseteq$$

$$\left\{(i,j,k)\in N\times N\times N:D\left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_1}\right)\right]^{p_{ijk}}\geq\frac{\varepsilon}{2}\right\}\cup$$

$$\left\{(i,j,k)\in N\times N\times N:D\left[M\left(\frac{\bar{d}(\lambda_{ijk}Y_{ijk},\bar{0})}{\rho_2}\right)\right]^{p_{ijk}}\geq\frac{\varepsilon}{2}\right\}\in I_3.$$

$\therefore(\alpha X_{ijk+\beta Y_{ijk}})\in\ _3(m_0^{I(F)})(M,\Lambda,p).$

This completes the proof.

Theorem 3.2. *Let* $\sup\limits_{i,j,k}\ p_{ijk}<\infty$. *Then the following statements are equivalent:*
(i) $\langle X_{ijk}\rangle\in\ _3(c^{I(F)})(M,\Lambda,p)$.
(ii) There exists a sequence $\langle Y_{ijk}\rangle\in\ _3(c^{(F)})(M,\Lambda,p)$ *such that* $X_{ijk}=Y_{ijk}$ *for a.a. i,j and k r.* I_3.
(iii) There exists a subset $P=\{(i_n,j_m,k_l)\in N\times N\times N:n,m,l\in N\}$ *of* $N\times N\times N$ *such that* $P\in F(I_3)$ *and* $(X_{i_nj_mk_l})\in\ _3(c^{(F)})(M,\Lambda,p)$.

Proof. $(i)\Rightarrow(ii)$. Let $\langle X_{ijk}\rangle\in\ _3(c^{I(F)})(M,\Lambda,p)$.
Then there exists some $\rho>0$ and $X_0\in R(L)$ such that

$$I_3-\lim\left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},X_0)}{\rho}\right)\right]^{p_{ijk}}=0$$

So for a given $\varepsilon>0$, we have the set

$$\left\{(i,j,k)\in N\times N\times N:\left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},X_0)}{\rho}\right)\right]^{p_{ijk}}\geq\varepsilon\right\}\in I_3.$$

Let us consider the increasing sequences $(S_m),(T_m)$ and (U_m) of natural numbers such that if $p>S_m,q>T_m$ and $r>U_m$, then the set

$$\left\{(i,j,k)\in N\times N\times N:i\leq p,j\leq q,k\leq r\ and\ \left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},X_0)}{\rho}\right)\right]^{p_{ijk}}\geq\frac{1}{m}\right\}\in I_3.$$

We define the sequence $\langle Y_{ijk}\rangle$ as follows:

$$Y_{ijk}=X_{ijk},\ if\ i\leq S_1\ or\ j\leq T_1\ or\ k\leq U_1.$$

Also for all (i,j,k) with $S_m<i\leq S_{m+1}$ or $T_m<j\leq T_{m+1}$ or $U_m<k\leq U_{m+1}$, let

$$Y_{ijk}=X_{ijk},\ if\ \left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},X_0)}{\rho}\right)\right]^{p_{ijk}}<\frac{1}{m},$$

otherwise $Y_{ijk}=\lambda_{ijk}^{-1}X_0$.
We show that $\langle Y_{ijk}\rangle\in\ _3(c^{(F)})(M,\Lambda,p)$.
Let $\epsilon>0$ and m be chosen such that $\varepsilon>\frac{1}{m}$.
We see. for $i>S_m$, $j>T_m$ and $k>U_m$,

$$\left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},X_0)}{\rho}\right)\right]^{p_{ijk}}<\varepsilon.$$

Hence $\langle Y_{ijk}\rangle \in {}_3(c^{(F)})(\Lambda, p)$.
Suppose that $S_m < i \leq S_{m+1}$, $T_m < j \leq T_{m+1}$ and $U_m < k \leq U_{m+1}$, then the set

$$A = \{(i,j,k) \in N \times N \times N : X_{ijk} \neq Y_{ijk}\} \subseteq \left\{(i,j,k) \in N \times N \times N : \left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk}, X_0)}{\rho}\right)\right]^{p_{ijk}} \geq \frac{1}{m}\right\} \in I_3.$$

Hence $A \in I_3$ and so $X_{ijk} = Y_{ijk}$ for a.a. *i,j* and $k \in I_3$.
$(ii) \Rightarrow (iii)$. Suppose there exists a sequence $\langle Y_{ijk}\rangle \in {}_3(c^{(F)})(M, \Lambda, p)$ such that $X_{ijk} = Y_{ijk}$ for a.a. *i,j* and $k \in I_3$.
Let $P = \{(i,j,k) \in N \times N \times N : X_{ijk} = Y_{ijk}\}$.
Then $P \in F(I_3)$.
P can be enumerated as $P = \{(i_n, j_m, k_l) \in N \times N \times N : n, m, l \in N\}$, on neglecting the rows and columns those contain finite number of elements.
Then $(X_{i_n j_m k_l}) \in {}_3(c^{(F)})(M, \Lambda, p)$.
$(iii) \Rightarrow (i)$. The result (i) follows immediately from (iii).

Proposition 3.3. *Let the double sequence* $p = \langle p_{ijk}\rangle$ *be bounded. Then* ${}_3(c_0^{I(F)})(M, \Lambda, p) \subset {}_3(c^{I(F)})(M, \Lambda, p) \subset {}_3(\ell_\infty^{I(F)})(M, \Lambda, p)$ *and the inclusions are strict.*

Proof. The inclusion ${}_3(c_0^{I(F)})(M, \Lambda, p) \subset {}_3(c^{I(F)})(M, \Lambda, p) \subset {}_3(\ell_\infty^{I(F)})(M, \Lambda, p)$ is obvious. In order to show that the inclusion ${}_3(c^{I(F)})(M, \Lambda, p) \subset {}_3(\ell_\infty^{I(F)})(M, \Lambda, p)$ is strict, we consider the following example.

Example 3.4. Let $I_3(P)$ denote the class of all subsets of $N \times N \times N$ such that $A \in I_3(P)$ implies there exits $i_0, j_0, k_0 \in N$ such that

$$A \subseteq N \times N \times N - \{(i,j,k) \in N \times N \times N : i \geq i_0, j \geq j_0, k \geq k_0\}.$$

Let $M(x) = x^2$ and $i_0, j_0, k_0 \in N$ be fixed such that

$$p_{ijk} = \begin{cases} \frac{1}{2}, & \text{for } 1 \leq i \leq i_0, 1 \leq j \leq j_0, 1 \leq k \leq k_0 \\ 2, & \text{otherwise} \end{cases}$$

Consider the sequence $\langle X_{ijk}\rangle$ defined by :
$X_{ijk} = \bar{1}$, for $1 \leq i \leq i_0, 1 \leq j \leq j_0, 1 \leq k \leq k_0$.
For $i > i_0, j > j_0, k > k_0$ and $(i+j+k)$ even,

$$X_{ijk}(t) = \begin{cases} \frac{it-2i+1}{i+1}, & \text{for } 2 - i^{-1} \leq t \leq 3 \\ 4 - t, & \text{for } 3 < t \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise

$$X_{ijk}(t) = \begin{cases} \frac{it-1}{2i-1}, & \text{for } i^{-1} \leq t \leq 2 \\ 3 - t, & \text{for } 2 < t \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Then taking $\lambda_{ijk} = \frac{1}{i}$, for all $i, j, k \in N$, we have $\langle X_{ijk}\rangle \in {}_3(\ell_\infty^{I(F)})(M, \Lambda, p)$ but $\langle X_{ijk}\rangle \notin {}_3(c^{I(F)})(M, \Lambda, p)$.
Hence the inclusion ${}_3(c^{I(F)})(M, \Lambda, p) \subset {}_3(\ell_\infty^{I(F)})(M, \Lambda, p)$ is strict.

We state the following result without proof, since it can be established writing standard technique.

Theorem 3.5. *If* $H = \sup_{i,j,k} p_{ijk} < \infty$,*then the classes of sequences* ${}_3(m^{I(F)})(M,\Lambda,p)$ *and* ${}_3(m_0^{I(F)})(M,\Lambda,p)$ *are complete metric spaces with respect to the metric* τ *defined by*

$$\tau(X,Y) = \inf\left\{\rho^{\frac{p_{ijk}}{J}} > 0 : \sup_{i,j,k}\left[M\left(\frac{\bar{d}\left(\lambda_{ijk}X_{ijk}, \lambda_{ijk}Y_{ijk}\right)}{\rho}\right)\right] \leq 1,\ \rho > 0\right\},$$

where

$$J = max(1,H).$$

Theorem 3.6. *Let* M_1 *and* M_2 *be two Orlicz functions and* $\Lambda = \langle\lambda_{ijk}\rangle$ *be a given multiplier sequence, then*
(i) $Z(M_1,\Lambda,p) \cap Z(M_2,\Lambda,p) \subseteq Z(M_1+M_2,\Lambda,p)$
(ii)$Z(M_2,\Lambda,p) \subseteq Z(M_1 \circ M_2,\Lambda,p)$, *for* $Z = {}_3(c_0^{I(F)}),\ {}_3(c^{I(F)}),\ {}_3(\ell_\infty^{I(F)})$.

Proof. We prove the result for the case $Z = {}_3(C_0^{I(F)})$.

(i) Let $\langle X_{ijk}\rangle \in {}_3(c_0^{I(F)})(M_1,\Lambda,p) \cap {}_3(c_0^{I(F)})(M_2,\Lambda,p)$.
Then $\exists\ \rho_1,\rho_2 > 0$ such that the sets

$$A = \left\{(i,j,k) \in N\times N\times N : \left[M_1\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_1}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \in I_3$$

and

$$B = \left\{(i,j,k) \in N\times N\times N : \left[M_2\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_2}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \in I_3.$$

Let $\rho = \rho_1+\rho_2$. Since M is continuous, we have the following inequality:

$$\left[(M_1+M_2)\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} \leq D\left[\frac{\rho_1}{\rho_1+\rho_2}M_1\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_1}\right)\right]^{p_{ijk}} + D\left[\frac{\rho_2}{\rho_1+\rho_2}M_2\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_2}\right)\right]^{p_{ijk}},$$

where $D = \max(1,2^{H-1}),\ H = \sup_{i,j,k} p_{ijk}$.
From the above relation, we obtain

$$\left\{(i,j,k) \in N\times N\times N : \left[(M_1+M_2)\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \subseteq$$

$$\left\{(i,j,k) \in N\times N\times N : D\left[\rho M_1\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_1}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \cup$$

$$\left\{(i,j,k) \in N\times N\times N : D\left[\rho M_2\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho_2}\right)\right]^{p_{ijk}} \geq \frac{\varepsilon}{2}\right\} \in I_3.$$

Thus $\langle X_{ijk}\rangle \in {}_3(c_0^{I(F)})(M_1+M_2,\Lambda,p)$.
Similarly we can establish the other cases.

(ii)Let $\varepsilon > 0$ be given. Since M_1 is continuous, so there exists $\eta > 0$ such that $M_1(\eta) = \varepsilon$.
Let $\langle X_{ijk}\rangle \in {}_3(m_0^{I(F)})(M_2,\Lambda,p)$. So there exists $\rho > 0$ such that

$$I_3 - \lim\left[M_2\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} = 0.$$

Then there exists $i_0, j_0, k_0 \in N$ such that

$$\left[M_2\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} < \eta, \text{ for all } i \geq i_0, j \geq j_0, k \geq k_0.$$

$$\Rightarrow \left[(M_1 \circ M_2)\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} < \varepsilon, \text{ for all } i \geq i_0, j \geq j_0, k \geq k_0.$$

$$\therefore \quad I_3 - \lim \left[(M_1 \circ M_2)\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} = 0.$$

$\Rightarrow \langle X_{ijk}\rangle \in {}_3(m_0^{I(F)})(M_1 \circ M_2, p)$.
Similarly the other cases can be established.

Following standard techniques, one can easily prove the following result.

Theorem 3.7. *If $M_1(x) \leq M_2(x)$ for all $x \in [0,\infty)$, then $Z(M_2,p) \subseteq Z(M_1,p)$ for $Z = {}_3(c_0^{I(F)}),\ {}_3(c^{I(F)}),\ {}_3(\ell_\infty^{I(F)})$.*

Theorem 3.8. *The class of sequences ${}_3(m_0^{I(F)})(M,\Lambda,p)$ is normal and monotone.*

Proof. Let $\langle X_{ijk}\rangle \in {}_3(m_0^{I(F)})(M,\Lambda,p)$ and $\langle Y_{ijk}\rangle$ be such that $\bar{d}(Y_{ijk},\bar{0}) \leq \bar{d}(X_{ijk},\bar{0})$, for all $i,j,k \in N$.
Let $\epsilon > 0$ be given. Then the normality of ${}_3(m_0^{I(F)})(M,\Lambda,p)$ follows from the following inclusion relation :
$\left\{(i,j,k) \in N \times N \times N : \left[M\left(\frac{\bar{d}(\lambda_{ijk}X_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} \geq \varepsilon\right\} \supseteq$
$\left\{(i,j,k) \in N \times N \times N : \left[M\left(\frac{\bar{d}(\lambda_{ijk}Y_{ijk},\bar{0})}{\rho}\right)\right]^{p_{ijk}} \geq \varepsilon\right\}.$
Also by **Remark 2.1**, it follows that the space ${}_3(m_0^{I(F)})(M,\Lambda,p)$ is monotone.

Proposition 3.9. *The class of sequences ${}_3(m^{I(F)})(M,\Lambda,p)$ is neither monotone nor solid.*

Proof. The result follows from the following example.

Example 3.10. Let $I_3(\rho) \subset 2^{N\times N\times N}$ denote the class of all subsets of $N \times N \times N$ of zero natural density.
Let $I_3 = I_3(\rho),\ A \in I_3, p_{ijk} = 1$,for all $i,j,k \in N$ and $M(x) = x^2$.
Consider the sequence $\langle X_{ijk}\rangle)$ defined by :
For all $(i,j,k) \notin A$,

$$X_{ijk}(t) = \begin{cases} 1+2(i+j+k)(t-1), & \text{for } 1-\frac{1}{2(i+j+k)} \leq t \leq 1 \\ 1-2(i+j+k)(t-1), & \text{for } 1 < t \leq 1+\frac{1}{2(i+j+k)} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $X_{ijk} = \bar{1}$.
Then taking $\lambda_{ijk} = \frac{1}{i+j+k}$, for all $i,j,k \in N$, we have $\langle X_{ijk}\rangle) \in {}_3(m^{I(F)})(M,\Lambda,p)$.
Let $K = \{2n : n \in N\}$.
Consider the sequence $\langle Y_{ijk}\rangle$ defined by:

$$Y_{ijk} = \begin{cases} X_{ijk}, & \text{if } (i,j,k) \in K \\ \bar{0}, & \text{otherwise} \end{cases}$$

Then $\langle Y_{ijk}\rangle$ belongs to the canonical pre-image of K step space of ${}_3(m^{I(F)})(M,\Lambda,p)$.
But $\langle Y_{ijk}\rangle \notin {}_3(m^{I(F)})(M,\Lambda,p)$.
Hence the class of sequences ${}_3(m^{I(F)})(M,\Lambda,p)$ is not monotone and not solid.

Proposition 3.11. *The class of sequences ${}_3(m^{I(F)})(M,\Lambda,p)$ and ${}_3(m_0^{I(F)})(M,\Lambda,p)$ are not symmetric in general.*

Proof. The result follows from the following example.

Example 3.12. Let $A \in I_3$, $M(x) = x^2$ and for all $x \in [0, \infty)$,

$$p_{ijk} = \begin{cases} 1, & \text{for i even and all } j, k \in N \\ 2, & \text{otherwise} \end{cases}$$

Consider the sequence $\langle X_{ijk} \rangle$ defined by :
For $i = n^2, n \in N$ and for all $j, k \in N$,

$$X_{ijk}(t) = \begin{cases} 1 + \frac{t}{2\sqrt{i}-1}, & \text{for } 1 - 2\sqrt{i} \le t \le 0 \\ 1 - \frac{t}{2\sqrt{i}-1}, & \text{for } 0 < t \le 2\sqrt{i} - 1 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $X_{ijk} = \bar{0}$.
Then taking $\lambda_{ijk} = \frac{1}{i}$, for all $i, j, k \in N$,
$\langle X_{ijk} \rangle \in Z(M, \Lambda, p)$, for $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
Next we consider the rearrangement $\langle Y_{ijk} \rangle$ of $\langle X_{ijk} \rangle$ defined by :
For k odd and for all $i, j \in N$,

$$Y_{ijk}(t) = \begin{cases} 1 + \frac{t}{2i-1}, & \text{for } 1 - 2i \le t \le 0 \\ 1 - \frac{t}{2i-1}, & \text{for } 0 < t \le 2i - 1 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $Y_{ijk} = \bar{0}$.
Then $\langle Y_{ijk} \rangle \notin Z(M, \Lambda, p)$, for $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
Hence the classes of sequences ${}_3(m^{I(F)})(M, \Lambda, p)$ and ${}_3(m_0^{I(F)})(M, \Lambda, p)$ are not symmetric.

Proposition 3.13. *The class of sequences ${}_3(m^{I(F)})(M, \Lambda, p)$ and ${}_3(m_0^{I(F)})(M, \Lambda, p)$ are not sequence algebras.*

Proof. The result follows from the following example.

Example 3.14. Let $A \in I_3$, $M(x) = x^2$ and for all $x \in [0, \infty)$,

$$p_{ijk} = \begin{cases} \frac{1}{3}, & \text{for } (i, j, k) \in A \\ 1, & \text{otherwise} \end{cases}$$

Consider the sequence $\langle X_{ijk} \rangle$ and $\langle Y_{ijk} \rangle$ defined by :
For all $(i, j, k) \notin A$,

$$X_{ijk}(t) = \begin{cases} 1 + \frac{t}{2(i+j+k)^2}, & \text{for } -2(i + j + k)^2 \le t \le 0 \\ 1 - \frac{t}{2(i+j+k)^2}, & \text{for } 0 < t \le 2(i + j + k)^2 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $X_{ijk} = \bar{0}$.
For all $(i, j, k) \notin A$,

$$Y_{ijk}(t) = \begin{cases} 1 + \frac{t-1}{2(i+j+k)^2}, & \text{for } 1 - 2(i + j + k)^2 \le t \le 1 \\ 1 - \frac{t-1}{2(i+j+k)^2}, & \text{for } 1 < t \le 1 + 2(i + j + k)^2 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $Y_{ijk} = \bar{0}$.
Then taking $\lambda_{ijk} = \frac{1}{(i+j+k)^3}$, for all $i, j, k \in N$, we have $\langle X_{ijk} \rangle, \langle Y_{ijk} \rangle \in Z(M, \Lambda, p)$,

for $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
But $(X_{nk} \bigotimes Y_{nk}) \notin Z(M, \Lambda, p)$, $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
Hence the classes of sequences ${}_3(m^{I(F)})(M, \Lambda, p)$ and ${}_3(m_0^{I(F)})(M, \Lambda, p)$ are not sequence algebras.

Proposition 3.15. *The class of sequences* ${}_3(m^{I(F)})(M, \Lambda, p)$ *and* ${}_3(m_0^{I(F)})(M, \Lambda, p)$ *are not convergence free.*

Proof. The result follows from the following example.

Example 3.16. Let $A \in I_3$, $M(x) = x$ and for all $x \in [0, \infty)$,

$$p_{ijk} = \begin{cases} \frac{1}{3}, & \text{for } (i,j,k) \in A \\ 3, & \text{otherwise} \end{cases}$$

Consider the sequence $\langle X_{ijk} \rangle$ defined by :
For all $(i, j, k) \notin A$,

$$X_{ijk}(t) = \begin{cases} 1 + 3(i+j+k)(t-1), & \text{for } 1 - \frac{1}{3(i+j+k)} \leq t \leq 1 \\ 1 - 3(i+j+k)(t-1), & \text{for } 1 < t \leq 1 + \frac{1}{3(i+j+k)} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $X_{ijk} = \bar{0}$.
Then taking $\lambda_{ijk} = \frac{1}{i+j+k}$, for all $i + j + k \in N$, we have $\langle X_{ijk} \rangle \in Z(M, \Lambda, p)$,
for $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
Consider the sequence $\langle Y_{ijk} \rangle$ defined by :
For all $(i, j, k) \notin A$,

$$Y_{ijk}(t) = \begin{cases} 1 + \frac{t-1}{3(i+j+k)^3}, & \text{for } 1 - 3(i+j+k)^3 \leq t \leq 1 \\ 1 - \frac{t-1}{3(i+j+k)^3}, & \text{for } 1 < t \leq 1 + 3(i+j+k)^3 \\ 0, & \text{otherwise} \end{cases}$$

Otherwise $Y_{ijk} = \bar{0}$.
Then $\langle Y_{ijk} \rangle \notin Z(M, \Lambda, p)$, for $Z = {}_3(m_0^{I(F)}),\ {}_3(m^{I(F)})$.
Hence ${}_3(m^{I(F)})(M, \Lambda, p)$ and ${}_3(m_0^{I(F)})(M, \Lambda, p)$ are not convergence free.

4 CONCLUSION

Convergence theory can be applied as a basic tool in measure spaces, sequences of random variables, information theory and so on. In this research article, we have introduced and studied some multiplier ideal convergent triple sequence spaces of fuzzy numbers defined by an Orlicz function. Some basic algebraic and topological properties of these introduced sequence spaces are established and some inclusion relations between these spaces are obtained. Also the multiplier problem is characterized. The introduced notion can be applied for further investigations from different aspects.

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