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LP-Sasakian Manifold Admitting C-Bochner Curvature Tensor

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Abstract. C-Bochner pseudosymmetric LP-Sasakian manifold and LP-Sasakian manifold satisfying $B(\xi, X) \cdot B = 0$, $B(\xi, U) \cdot R = 0$ and $B(\xi, X) \cdot S = 0$ have been studied. Finally an example of LP Sasakian manifold has been constructed.

1 Introduction

In 1989 Matsumoto [11] introduced the notion of Lorentzian para-Sasakian manifolds. The same notion was independently introduced by Mihai and Rosca [13] who obtained several results. As a generalization of spaces of constant curvature, locally symmetric spaces were introduced by Cartan [3]. Every locally symmetric space satisfies $R \cdot R = 0$, whereby the first R stands for the curvature operator which acts as a derivation on the second R which stands for the Riemanian curvature tensor. Manifolds satisfying the condition $R \cdot R = 0$ are called semisymmetric manifolds and were classified by Szabo [21]. The condition of semisymmetry was weakened by Deszcz as pseudosymmetry which are characterized by the condition $R \cdot R = LQ(g, R)$, whereby L is a real function on M and $Q(g, R)$ is the Tachibana tensor of M.

A Riemannian manifold M is said to be pseudosymmetric in the sense of Deszcz [8] if

$$R(X, Y) \cdot R(U, V)Z = L_R((X \wedge Y) \cdot R(U, V)Z), \tag{1.1}$$

holds on $U_R = \{X \in M | R - \frac{r}{n(n-1)} G \neq 0 \quad at \ \ x\}$, where G is the $(0, 4)$ tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$, L_R is some smooth function on U_R and $(X \wedge Y)$ is an endomorphism defined by

$$(X \wedge Y)Z = g(Y, Z)X - g(X, Z)Y. \tag{1.2}$$

A Riemannian manifold M is said to be C-Bochner pseudosymmetric [5] if

$$R(X, Y) \cdot B(U, V)Z = L_B((X \wedge Y) \cdot B(U, V)Z), \tag{1.3}$$

holds on the set $U_B = \{x \in M : B \neq 0 \ \ at \quad x\}$, where L_B is some function on U_B and B is the C-Bochner curvature tensor [5].

Pseudosymmetric LP-Sasakian manifold was studied by De and De [4]. In their article they mainly studied pseudosymmetric, Weyl-pseudosymmetric and Ricci-pseudosymmetric LP-Sasakian manifolds and obtained some interesting results.

Motivated by the above studies we made an attempt to study LP-Sasakian manifold with C-Bochner curvature tensor. The paper is organized as follows: After the preliminaries, in section 3 we studied C-Bochner pseudosymmetric LP-Sasakian manifolds and proved that a $(2n+1)$-dimensional C-Bochner Pseudosymmetric LP-Sasakian manifold is locally isometric to a sphere. In sections 4 and 5 we have proved that LP-Sasakian manifold satisfying $B(\xi, X) \cdot B = 0$ and $B(\xi, X) \cdot R = 0$ are isometric to sphere and hyperbolic space respectively. Section 6 is concerned with LP-Sasakian manifold satisfying $B(\xi, X) \cdot S = 0$. Finally, in the last section we construct an example of LP-Sasakian manifold.

2 Preliminaries

A $(2n+1)$-dimensional differentiable manifold M^{2n+1} is said to be Lorentzian para-Sasakian (shortly, LP-Sasakian) manifold, if it admits a $(1,1)$-tensor field ϕ, a contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfies [11, 12].

$$\phi^2 = X + \eta(X)\xi, \quad \eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi) = 0, \quad g(X,\xi) = \eta(X), \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.2}$$

$$\nabla_X \xi = \phi X, \tag{2.3}$$

$$(\nabla_X \phi)Y = g(X,Y)\xi + \eta(X)Y + 2\eta(X)\eta(Y)\xi, \tag{2.4}$$

for all vector fields $X, Y, Z \in T_pM$. Here ∇ denotes the operator of covariant differentiation with respect to Lorentzian metric g.

Also in LP-Sasakian manifold, the following relations hold [11, 12]:

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.5}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.6}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$R(\xi,X)\xi = X + \eta(X)\xi, \tag{2.8}$$

$$S(X,\xi) = 2n\eta(X). \tag{2.9}$$

for all vector fields X, Y, Z, where S is the Ricci tensor and R is the Riemannian curvature tensor.

C-Bochner curvature tensor on an almost contact metric manifold was defined by Matsumoto and Chuman [10] and is given by

$$\begin{aligned} B(X,Y)Z &= R(X,Y)Z + \frac{1}{2(n+2)}\{S(X,Z)Y - S(Y,Z)X + g(X,Z)QY \\ &- g(Y,Z)QX + S(\phi X,Z)\phi Y - S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y \\ &- g(\phi Y,Z)Q\phi X + 2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z - S(X,Z)\eta(Y)\xi \\ &+ S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX\} \\ &- \frac{\tau+2n}{2(n+2)}\{g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X + 2g(\phi X,Y)\phi Z\} \\ &- \frac{\tau-4}{2(n+2)}\{g(X,Z)Y - g(Y,Z)X\} + \frac{\tau}{2(n+2)}\{g(X,Z)\eta(Y)\xi \\ &- g(Y,Z)\eta(X) + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X\}, \end{aligned} \tag{2.10}$$

where $\tau = \frac{r+2n}{2(n+2)}$, Q is the Ricci operator i.e. $g(QX,Y) = S(X,Y)$ and r is the scalar curvature of the manifold.

Using (2.5)-(2.8), one can get

$$B(X,\xi)Z = H\{\eta(Z)X - g(X,Z)\xi\}, \tag{2.11}$$

$$B(X,Y)\xi = H\{\eta(Y)X - \eta(X)Y\}, \tag{2.12}$$

$$B(\xi,Y)Z = H\{g(Y,Z)\xi - \eta(Z)Y\}, \tag{2.13}$$

$$\eta(B(X,Y)Z) = H\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{2.14}$$

where H is a constant i.e., $H = \{1 - \frac{3n}{n+2} + \frac{\tau-4}{2(n+2)} + \frac{\tau}{2(n+2)}\}$.

3 C-Bochner Pseudosymmetric LP-Sasakian manifold

A $(2n+1)$-dimensional LP-Sasakian manifold M^{2n+1} is said to be C-Bochner pseudosymmetric if

$$(R(X,Y)\cdot B)(U,V)W = L_B[((X \wedge Y)\cdot B)(U,V)W], \tag{3.1}$$

holds on the set $U_B = \{x \in M : B \neq 0\}$ at x, where L_B is some function on U_B.

Let M^{2n+1} be a C-Bochner pseudosymmetric LP-Sasakian manifold. Then from (3.1) we have

$$(R(X,\xi)\cdot B)(U,V)W = L_B[((X\wedge_g \xi)\cdot B)(U,V)W]. \tag{3.2}$$

Using (2.6), the left-hand side of equation (3.2) becomes

$$\begin{aligned} &\{g(\xi,B(U,V)W)\xi - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ +\;& g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ -\;& \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\} = 0. \end{aligned} \tag{3.3}$$

Using (1.2), the right hand side of equation (3.2) turns into

$$\begin{aligned} &L_B\{g(\xi,B(U,V)W)\xi - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ +\;& g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ -\;& \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\} = 0. \end{aligned} \tag{3.4}$$

By virtue of (3.3) and (3.4), (3.2) give rise to

$$\begin{aligned} &(1-L_B)\{g(\xi,B(U,V)W)\xi - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ +\;& g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ -\;& \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\} = 0, \end{aligned} \tag{3.5}$$

which implies $L_B = 1$ or

$$\begin{aligned} &\{g(\xi,B(U,V)W)\xi - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ +\;& g(X,U)B(\xi,V)Z - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ -\;& \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\} = 0. \end{aligned} \tag{3.6}$$

Putting $W = \xi$ in the above equation and simplifying we get

$$B(U,V)X = \{g(X,V)U - g(X,U)V\}. \tag{3.7}$$

Thus, we have the following assertion;

Theorem 3.1. *If a* $(2n+1)$*-dimensional LP-Sasakian manifold* M^{2n+1} *is* C*-Bochner Pseudosymmetric then* M^{2n+1} *is locally isometric to a sphere or* $L_B = 1$.

4 LP-Sasakian manifold satisfying $B(\xi,X)\cdot B = 0$

Let us consider an LP-Sasakian manifold satisfying $B(\xi,X)\cdot B = 0$.
Then we have,

$$\begin{aligned} &B(\xi,X)B(U,V)W - B(B(\xi,X)U,V)W \\ -\;& B(U,B(\xi,X)V)W - B(U,V)B(\xi,X)W = 0. \end{aligned} \tag{4.1}$$

In view of (2.13), (4.1) gives

$$\begin{aligned} &H[g(X,B(U,V)W)\xi - \eta(B(U,V)W)X - g(X,U)B(\xi,V)W \\ +\;& \eta(U)B(X,V)W - g(X,V)B(U,\xi)W + \eta(V)B(U,X)W \\ -\;& g(X,W)B(U,V)\xi + \eta(W)B(U,V)X] = 0. \end{aligned} \tag{4.2}$$

Setting $V = \xi$ in (4.2) and making use of (2.11), we get

$$B(U,X)W = -H\{g(X,W)U - g(U,W)X\}. \tag{4.3}$$

Hence, we can state the following:

Theorem 4.1. *If a* $(2n+1)$*-dimensional LP-Sasakian manifold* M^{2n+1} *satisfies* $B(\xi,X)\cdot B = 0$ *then* M^{2n+1} *is isometric to a hyperbolic space.*

5 LP-Sasakian manifold Satisfying $B(\xi, U) \cdot R = 0$

Suppose M^{2n+1} satisfies $B(\xi, U) \cdot R = 0$. The condition $B(\xi, U) \cdot R = 0$ implies that

$$\begin{aligned} & B(\xi, U)R(X, Y)Z - R(B(\xi, U)X, Y)Z \\ - \quad & R(X, B(\xi, U)Y)Z - R(X, Y)B(\xi, U)Z = 0. \end{aligned} \tag{5.1}$$

By virtue of (2.12), (5.1) turns into

$$\begin{aligned} & H[g(U, R(X, Y)Z)\xi - \eta(R(X, Y)Z)U - g(U, X)R(\xi, Y)Z \\ + \quad & \eta(X)R(U, Y)Z - g(U, Y)R(X, \xi)Z + \eta(Y)R(X, U)Z \\ - \quad & g(U, Z)R(X, Y)\xi + \eta(Z)R(X, Y)U] = 0. \end{aligned} \tag{5.2}$$

Plugging $Z = \xi$ in (5.2) and using (2.7), one can get

$$H\{-g(U, X)Y + g(U, Y)X - R(X, Y)U\} = 0. \tag{5.3}$$

which yields, either $H = 0$ i.e. $\tau = 2n$,

or

$$R(X, Y)U = [g(Y, U)X - g(X, U)Y]. \tag{5.4}$$

Thus, we can state the following theorem;

Theorem 5.1. *An $(2n+1)$-dimensional LP-Sasakian manifold satisfying the condition $B(\xi, X) \cdot R = 0$ is locally isometric to a sphere or $\tau = 2n$.*

6 LP-Sasakian manifolds satisfying $B(\xi, X) \cdot S = 0$

Consider a LP-Sasakian manifold satisfying $B(\xi, X) \cdot S = 0$. Then we have

$$S(B(\xi, X)Y, \xi) + S(Y, B(\xi, X)\xi) = 0. \tag{6.1}$$

Using (2.12) and (2.13) in (6.1), we get

$$S(X, Y) = 2ng(X, Y). \tag{6.2}$$

Thus we can state the following;

Theorem 6.1. *A $(2n + 1)$-dimensional LP-Sasakian manifold satisfying $B(\xi, X) \cdot S = 0$ is an Einstein manifold.*

7 Example

We consider seven dimensional manifold $M = \{x, y, z, u, v, w, t \in R^7\}$, where x, y, z, u, v, w, t are the standard coordinates in R^7. We choose linearly independent global frame fields $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ on M as

$$e_1 = e^t \frac{\partial}{\partial x}, \quad e_2 = e^t \frac{\partial}{\partial y}, \quad e_3 = e^t \frac{\partial}{\partial z}, \quad e_4 = e^t \frac{\partial}{\partial u}, \quad e_5 = e^t \frac{\partial}{\partial v}, \quad e_6 = e^t \frac{\partial}{\partial w}, \quad e_7 = \frac{\partial}{\partial t}.$$

Let g be the Lorentzian metric defined by

$$\begin{aligned} & g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = g(e_4, e_4) = g(e_5, e_5) = g(e_6, e_6) = 1, \quad g(e_7, e_7) = -1, \\ & g(e_i, e_j) = 0 \quad for \quad 1 \le i, j \le 7. \end{aligned}$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_7)$, for any $Z \in \chi(M)$. We define a $(1, 1)$-tensor field ϕ as

$$\begin{aligned} \phi(e_1) = -e_1, \;\; \phi(e_2) = -e_2, \;\; \phi(e_3) = -e_3, \\ \phi(e_4) = -e_4, \;\; \phi(e_5) = -e_5, \;\; \phi(e_6) = -e_6, \;\; \phi(e_7) = 0. \end{aligned} \tag{7.1}$$

The linearity of ϕ and g yields that

$$\eta(e_7) = -1,$$
$$\phi^2(Z) = Z + \eta(Z)\xi,$$
$$g(\phi U, \phi Z) = g(U,Z) + \eta(U)\eta(Z).$$

For any $U, Z \in \chi(M)$, let ∇ be the Levi Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g, then we have

$$\begin{aligned}
[e_1,e_2] = [e_1,e_3] = [e_1,e_4] = [e_1,e_5] = [e_1,e_6] = 0, &\quad [e_1,e_7] = -e_1,\\
[e_2,e_3] = [e_2,e_4] = [e_2,e_5] = [e_2,e_6] = 0, &\quad [e_2,e_7] = -e_2,\\
[e_3,e_4] = [e_3,e_5] = [e_3,e_6] = 0, &\quad [e_3,e_7] = -e_3,\\
[e_4,e_5] = [e_4,e_6] = 0, &\quad [e_4,e_7] = -e_4,\\
[e_5,e_6] = 0, &\quad [e_5,e_7] = -e_5,\\
&\quad [e_6,e_7] = -e_6.
\end{aligned} \tag{7.2}$$

The Koszul's formula is defined by

$$2g(\nabla_X Y, Z) = Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) - g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]).$$

By using the Koszul's formula, we can get the followings

$$\begin{array}{lllllll}
\nabla_{e_1}e_1 = e_7, & \nabla_{e_1}e_2 = 0, & \nabla_{e_1}e_3 = 0, & \nabla_{e_1}e_4 = 0, & \nabla_{e_1}e_5 = 0, & \nabla_{e_1}e_6 = 0, & \nabla_{e_1}e_7 = -e_1,\\
\nabla_{e_2}e_1 = 0, & \nabla_{e_2}e_2 = e_7, & \nabla_{e_2}e_3 = 0, & \nabla_{e_2}e_4 = 0, & \nabla_{e_2}e_5 = 0, & \nabla_{e_2}e_6 = 0, & \nabla_{e_2}e_7 = -e_2,\\
\nabla_{e_3}e_1 = 0, & \nabla_{e_3}e_2 = 0, & \nabla_{e_3}e_3 = e_7, & \nabla_{e_3}e_4 = 0, & \nabla_{e_3}e_5 = 0, & \nabla_{e_3}e_6 = 0, & \nabla_{e_3}e_7 = -e_3,\\
\nabla_{e_4}e_1 = 0, & \nabla_{e_4}e_2 = 0, & \nabla_{e_4}e_3 = 0, & \nabla_{e_4}e_4 = e_7, & \nabla_{e_4}e_5 = 0, & \nabla_{e_4}e_6 = 0, & \nabla_{e_4}e_7 = -e_4,\\
\nabla_{e_5}e_1 = 0, & \nabla_{e_5}e_2 = 0, & \nabla_{e_5}e_3 = 0, & \nabla_{e_5}e_4 = 0, & \nabla_{e_5}e_5 = e_7, & \nabla_{e_5}e_6 = 0, & \nabla_{e_5}e_7 = -e_5,\\
\nabla_{e_6}e_1 = 0, & \nabla_{e_6}e_2 = 0, & \nabla_{e_6}e_3 = 0, & \nabla_{e_6}e_4 = 0, & \nabla_{e_6}e_5 = 0, & \nabla_{e_6}e_6 = e_7, & \nabla_{e_6}e_7 = -e_6,\\
\nabla_{e_7}e_1 = 0, & \nabla_{e_7}e_2 = 0, & \nabla_{e_7}e_3 = 0, & \nabla_{e_7}e_4 = 0, & \nabla_{e_7}e_5 = 0, & \nabla_{e_7}e_6 = 0, & \nabla_{e_7}e_7 = 0.
\end{array} \tag{7.3}$$

From the above calculation it can be easily seen that in $M^7(\phi,\xi,\eta,g)$, $\eta(\xi) = -1$ and $\nabla_X \xi = \phi X$. Hence the manifold is an LP-Sasakian manifold.

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