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CERTAIN UNIFIED DOUBLE INTEGRALS ASSOCIATED WITH THE GENERALIZED LOMMEL–WRIGHT FUNCTION

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Abstract. The objective of this article is to establish a new class of unified double integrals associated with the generalized Lommel–Wright functions, which are represented in terms of Wright Hypergeometric function. Also, we establish integrals involving trigonometric, generalized Bessel and Struve functions as particular cases of the main results. Further, we obtain the reduction formulas for the Wright hypergeometric function.

1 Introduction

The Wright hypergeometric function defined by the series [19]:

$$ {}_p\psi_q \left[\begin{array}{c} (\alpha_1, A_1),\ ..,(\alpha_p, A_p); \\ \\ (\beta_1, B_1)...,(\beta_q, B_q) \end{array} z \right] = \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma(\alpha_j + A_j k) z^k}{\prod_{j=1}^{q} \Gamma(\beta_j + B_j k) k!}, \tag{1.1} $$

where the coefficients $A_1, ...A_p$ and $B_1, ...B_q$ are positive real numbers such that

$$ 1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \geq 0 \tag{1.2} $$

and slightly generalized form of (1.1) is given below.

$$ {}_p\psi_q \left[\begin{array}{c} (\alpha_1, 1),\ ..,(\alpha_p, 1); \\ \\ (\beta_1, 1)...,(\beta_q, 1) \end{array} z \right] = \frac{\prod_{j=1}^{p} \Gamma(\alpha_j)}{\prod_{j=1}^{q} \Gamma(\beta_j)} {}_pF_q \left[\begin{array}{c} \alpha_1, .., \alpha_p; \\ \\ \beta_1, ..., \beta_q \end{array} z \right], \tag{1.3} $$

where ${}_pF_q$ is the generalized hypergeometric function [17, 19] defined by

$$ {}_pF_q \left[\begin{array}{c} \alpha_1, .., \alpha_p; \\ \\ \beta_1, ..., \beta_q \end{array} z \right] = \sum_{k=0}^{\infty} \frac{(\alpha_1)_n, ..., (\alpha_p)_n z^n}{(\beta_1)_n, ..., (\beta_q)_n n!} = {}_pF_q(\alpha_1, ..., \alpha_p; \beta_1, ..., \beta_q; z), \tag{1.4} $$

where $(a)_n$ is the Pochhammer symbol [19].

The series representation of the generalized Lommel Wright function [7] is defined by

$$ J_{\nu,\lambda}^{\mu,m}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(k+1) (\frac{z}{2})^{2k+\nu+2\lambda}}{\Gamma(\lambda+k+1)^m \Gamma(\nu+k\mu+\lambda+1) k!}, \tag{1.5} $$

$$ (z \in \mathbb{C}/(-\infty, 0],\ m \in \mathbb{N},\ \nu, \lambda \in \mathbb{C}, \mu > 0) $$

Also, we have the following relations of generalized Lommel Wright functions with trigonometric functions and the generalized Bessel function and Struve function as follows:

$$J^{1,1}_{1/2,0}(z) = \sqrt{(\frac{2}{\pi z})}\sin(z). \tag{1.6}$$

$$J^{1,1}_{-1/2,0}(z) = \sqrt{(\frac{2}{\pi z})}\cos(z). \tag{1.7}$$

$$J^{\mu,1}_{\nu,\lambda}(z) = \mathbb{J}^{\mu}_{\nu,\lambda}(z). \tag{1.8}$$

$$J^{1,1}_{\nu,1/2}(z) = H_\nu(z). \tag{1.9}$$

For our present investigation, the following known result of Edward [6].

$$\int_0^1\int_0^1 t^\rho(1-y)^{\rho-1}(1-t)^{\sigma-1}(1-ty)^{1-\rho-\sigma}dtdy = \frac{\Gamma(\rho)\Gamma(\sigma)}{\Gamma(\rho+\sigma)}, \tag{1.10}$$

where $\Re(\rho) > 0$ and $\Re(\sigma) > 0$.

Integral formulas involving various special functions have been developed by many researchers (see, e.g., [1, 2, 4, 5, 9, 10, 15, 11, 12, 13, 14, 18]). In this sequel, here, we focus at establishing certain new generalized integral formula involving the generalized Lommel–Wright function $J^{\mu,m}_{\nu,\lambda}(z)$.

2 Main Results

The integral formulas involving Lommel–Wright function given in this section.

Theorem 2.1. *The following formula holds :*
For $\eta,\theta \in \mathbb{C}$ and $t > 0$ with $\Re(\eta+\nu+2\lambda) > 0, \Re(\theta+\nu+2\lambda) > 0$,

$$\int_0^1\int_0^1 t^\eta(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}J^{\mu,m}_{\nu,\lambda}\left[\frac{8t(1-t)(1-y)}{(1-ty)^2}\right]dydt = 2^{2\nu+4\lambda}$$
$$\times\, {}_3\psi_{m+2}\left[\begin{array}{c}(1,1),(\eta+\nu+2\lambda,2),(\theta+\nu+2\lambda,2);\\ (\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\eta+\theta+2\nu+4\lambda,4);\end{array} -16\right]. \tag{2.1}$$

Proof. On using (1.5) in the integrand of (2.1). Then interchanging the order of integration and summation which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^1\int_0^1 t^{\eta-1}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}J^{\mu,m}_{\nu,\lambda}\left[\frac{8t(1-t)(1-y)}{(1-ty)^2}\right]dydt$$
$$= 2^{2\nu+4\lambda}\sum_{k=0}^{\infty}\frac{\Gamma(k+1)(-16)^k}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!}$$
$$\times\int_0^1\int_0^1 t^{\eta+2k+2\nu+2\lambda}(1-y)^{(\eta+2k+\nu+2\lambda-1)}(1-t)^{\theta+2k+\nu+2\lambda-1}(1-ty)^{1-\eta-\theta-2(2k+\nu+2\lambda)}dtdy. \tag{2.2}$$

Now using (1.10) in the above equation we get

$$\int_0^1\int_0^1 t^\eta(1-y)^{2\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}J^{\mu,m}_{\nu,\lambda}\left[\frac{8t(1-t)(1-y)}{(1-ty)^2}\right]dtdy$$
$$= 2^{2\nu+4\lambda}\sum_{k=0}^{\infty}\frac{\Gamma(k+1)\Gamma(\eta+\nu+2\lambda+2k)\Gamma(\theta+\nu+2\lambda+2k)(-16)^k}{\Gamma(\lambda+k+1)^m\Gamma(\eta+\theta+2\nu+4\lambda+4k)\Gamma(\nu+k\mu+\lambda+1)k!}. \tag{2.3}$$

Finally, using (1.1) in the above equation, we get our assertion (2.1). This completes the proof of Theorem 2.1. □

Corollary 2.2. *On expanding the r.h.s of (2.3) in series form and then separating the resulting series into its even and odd terms, we get.*

$$\int_0^1\int_0^1 t^{\eta-1}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta} J_{\nu,\lambda}^{\mu,m}\left[\frac{8t(1-y)(1-t)}{(1-ty)^2}\right]dtdy$$
$$= \sqrt{\pi}\, 2^{2\nu+4\lambda}{}_3\psi_{m+3}\left[\begin{array}{c}(1,2),(\eta+\nu+2\lambda,4),(\theta+\nu+2\lambda+2,4);\\ \\ (\lambda+1,2),...,(\lambda+1,2),(\nu+\lambda+1,2\mu),(\eta+\theta+2\nu+4\lambda,8),(1/2,1);\end{array}\; 64\right] - (2)^{2\nu+4\lambda+5}\sqrt{\pi}$$
$$\times\, {}_3\Psi_{m+3}\left[\begin{array}{c}(2,2),(\eta+\nu+2\lambda+2,4),(\theta+\nu+2\lambda+2,4);\\ \\ (\lambda+2,2),...,(\lambda+2,2)(3/2,1),(\eta+\theta+2\nu+4\lambda+4,8),(\nu+\lambda+\mu+1,2\mu);\end{array}\; 64\right], \tag{2.4}$$

where $\Re(\eta+\nu+2\lambda)>0$ *and* $\Re(\theta+\nu+2\lambda)>0$.

Corollary 2.3. *The following formula holds:*
For $\eta,\theta\in C$ *and* $t>0$ *with* $\Re(\eta+\nu+2\lambda)>0, \Re(\theta+\nu+2\lambda)>0$,

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta} J_{\nu,\lambda}^{\mu,m}\left[\frac{8t(1-t)(1-y)}{(1-ty)^2}\right]dydt$$
$$= \frac{2^{2\nu+4\lambda}\Gamma(\eta+\nu+2\lambda)\Gamma(\theta+\nu+2\lambda)}{(\Gamma(\lambda+1))^m\Gamma(\nu+\lambda+1)\Gamma(\eta+\theta+2\nu+4\lambda)}$$
$$\times\, {}_5\mathbb{F}_{m+\mu+4}\left[\begin{array}{c}\Delta(2,\eta+\nu+2\lambda),\Delta(2,\theta+\nu+2\lambda),1;\\ (\lambda+1,1),...,(\lambda+1,1),\Delta(\mu,\nu+\lambda+1),\Delta(4,\eta+\theta+2\nu+4\lambda);\end{array}\; -\frac{1}{(\mu)^\mu}\right]. \tag{2.5}$$

where $(m;\lambda)$ abbreviates the array of m parameters $\lambda/m,(\lambda+1)/m,...,(\lambda+m-1)/m$, $m>0$.

3 Special Cases

Some integral formulas involving trigonometric function and generalized Lommel-Wright function will establish in this section.

Corollary 3.1. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=1/2$ *in (2.1) and then by using (1.6), we derive the following integral formula:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{(\eta-1/2)-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\sin\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$= 4\sqrt{\pi}\, {}_2\psi_2\left[\begin{array}{c}(\eta+1/2,2),(\theta+1/2,2);\\ \\ (\eta+\theta+1,4),(\frac{3}{2},1);\end{array}\; -16\right] \tag{3.1}$$

where $\Re(\eta+1/2)>0$ *and* $\Re(\theta+1/2)>0$.

Corollary 3.2. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=1/2$ *in (2.4) and then by using (1.6), we obtain:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{(\eta-1/2)-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\sin\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$= 4\sqrt{\pi}{}_2\psi_3\left[\begin{array}{c}(\eta+1/2,4),(\theta+1/2,4);\\ \\ (\eta+\theta+1,8),(\frac{3}{2},1),(1/2,1);\end{array}\; 64\right]$$
$$-(2)^7\sqrt{\pi}\, {}_2\psi_3\left[\begin{array}{c}(\eta+5/2,4),(\theta+5/2,4);\\ \\ (\eta+\theta+5,8),(3/2,1),(5/2,2);\end{array}\; 64\right], \tag{3.2}$$

where $\Re(\eta+1/2)>0$ *and* $\Re(\theta+1/2)>0$.

Corollary 3.3. *If we take* $m=1, \mu=1, \lambda=0$ *and* $\nu=1/2$ *in* (2.1) *and then by using* (1.6), *we get the following:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{2(\eta-\frac{1}{2})-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\sin\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$

$$=8\,\mathbb{B}(\eta+1/2,\theta+1/2)_4\mathbb{F}_5\left[\begin{array}{c}\Delta(2,\eta+1/2),\Delta(2,\theta+1/2);\\ \\ (3/2),\Delta(4,\eta+\theta+1);\end{array}\;-1\right], \tag{3.3}$$

where $\Re(\eta+1/2)>0$ *and* $\Re(\theta+1/2)>0$.

Corollary 3.4. *If we take* $m=1, \mu=1, \lambda=0$ *and* $\nu=-1/2$ *in* (2.1) *and then by using* (1.7), *we derive the following integral formula:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{(\eta-1/2)-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\cos\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$

$$=\sqrt{\pi}\;{}_2\psi_2\left[\begin{array}{c}(\eta-1/2,2),(\theta-1/2,2);\\ \\ (\eta+\theta-1,4),(\frac{1}{2},1);\end{array}\;-16\right] \tag{3.4}$$

where $\Re(\eta-1/2)>0$ *and* $\Re(\theta-1/2)>0$.

Corollary 3.5. *If we take* $m=1, \mu=1, \lambda=0$ *and* $\nu=-1/2$ *in* (2.4) *and then by using* (1.7), *we obtain:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{(\eta-1/2)-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\cos\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$

$$=\pi_2\psi_3\left[\begin{array}{c}(\eta-1/2,4),(\theta-1/2,4);\\ \\ (\eta+\theta-1,8),(\frac{1}{2},2),(1/2,1);\end{array}\;64\right]$$

$$-(2)^5\pi\;{}_2\psi_3\left[\begin{array}{c}(\eta+3/2,4),(\theta+3/2,4);\\ \\ (\eta+\theta+3,8),(3/2,1),(3/2,2);\end{array}\;64\right], \tag{3.5}$$

where $\Re(\eta-1/2)>0$ *and* $\Re(\theta=1/2)>0$.

Corollary 3.6. *If we take* $m=1, \mu=1, \lambda=0$ *and* $\nu=-1/2$ *in* (2.1) *and then by using* (1.7), *we get the following:*

$$\int_0^1\int_0^1 t^{\eta-1/2}(1-y)^{2(\eta-\frac{1}{2})-1}(1-t)^{(\theta-1/2)-1}(1-ty)^{2-\eta-\theta}\cos\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$

$$=\;\mathbb{B}(\eta-1/2,\theta-1/2)_4\mathbb{F}_5\left[\begin{array}{c}\Delta(2,\eta+1/2),\Delta(2,\theta+1/2);\\ \\ (3/2),\Delta(4,\eta+\theta+1);\end{array}\;-1\right], \tag{3.6}$$

where $\Re(\eta-1/2)>0$ *and* $\Re(\theta-1/2)>0$.

Corollary 3.7. *If we take* $m=1$ *in* (2.1) *and then by using* (1.8), *we derive the following integral formula:*

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{J}^{\mu}_{\nu,\lambda}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$

$$=2^{2\nu+4\lambda}\;{}_3\psi_3\left[\begin{array}{c}(1,1),(\eta+\nu+2\lambda,2),(\theta+\nu+2\lambda,2);\\ \\ (\lambda+1,1),(\nu+\lambda+1,\mu),(\eta+\theta+2\nu+4\lambda,4);\end{array}\;-16\right] \tag{3.7}$$

where $\Re(\eta+\nu+2\lambda)>0$ *and* $\Re(\theta+\nu+2\lambda)>0$.

Corollary 3.8. *If we take* $m=1$ *in* (2.4) *and then by using* (1.8)*, we obtain:*

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{J}^{\mu}_{\nu,\lambda}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$=2^{2\nu+4\lambda}\pi{}_3\psi_4\left[\begin{array}{c}(1,1),(\eta+\nu+2\lambda,4),(\theta+\nu+2\lambda,4);\\ \\(\lambda+1,1),(\nu+\lambda+1,2\mu),(\eta+\theta+2\nu+4\lambda,8),(1/2,1);\end{array}\ 64\right]$$
$$-(2)^{2\nu+4\lambda+5}\pi\,{}_3\psi_4\left[\begin{array}{c}(2,2),(\eta+\nu+2\lambda+2,4),(\theta+\nu+2\lambda+2,4);\\ \\(\lambda+2,2),(\nu+\lambda+\mu+1,2\mu),(3/2,1),(\eta+\theta+2\nu+4\lambda+4,8);\end{array}\ 64\right], \tag{3.8}$$

where $\Re(\eta+\nu+2\lambda)>0$ *and* $\Re(\theta+\nu+2\lambda)>0$.

Corollary 3.9. *If we take* $m=1$ *in* (2.1) *and then by using* (1.8)*, we get the following integral formula:*

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{J}^{\mu}_{\nu,\lambda}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$=\frac{2^{2\nu+4\lambda}}{\Gamma(\lambda+1)\Gamma(\nu+\lambda+1)}\mathbb{B}(\eta+\nu+2\lambda,\theta+\nu+2\lambda)$$
$${}_5\mathbb{F}_{\mu+5}\left[\begin{array}{c}\Delta(2,\eta+\nu+2\lambda),\Delta(2,\theta+\nu+2\lambda),1;\\ \\(\lambda+1),\Delta(4,\eta+\theta+2\nu+4\lambda),(\mu,\nu+\lambda+1);\end{array}\ -\frac{1}{\mu^{\mu}}\right], \tag{3.9}$$

where $\Re(\eta+\nu+2\lambda)>0$ *and* $\Re(\theta+\nu+2\lambda)>0$.

Corollary 3.10. *If we take* $m=1,\mu=1$ *and* $\lambda=1/2$ *in* (2.1) *and then by using* (1.9)*, we derive the following integral formula:*

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{H}_{\nu}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$=2^{2\nu+2}\,{}_3\psi_3\left[\begin{array}{c}(1,1),(\eta+\nu+1,2),(\theta+\nu+1,2);\\ \\(3/2,1),(\nu+3/2,1),(\eta+\theta+2\nu+2,4);\end{array}\ -16\right] \tag{3.10}$$

where $\Re(\eta+\nu+1)>0$ *and* $\Re(\theta+\nu+1)>0$.

Corollary 3.11. *If we take* $m=1,\mu=1$ *and* $\lambda=1/2$ *in* (2.4) *and then by using* (1.9)*, we obtain:*

$$\int_0^1\int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{H}_{\nu}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$=2^{2\nu+2}\sqrt{\pi}{}_3\psi_4\left[\begin{array}{c}(1,2),(\eta+\nu+1,4),(\theta+\nu+1,4);\\ \\(3/2,2),(\nu+3/2,2),(\eta+\theta+2\nu+2,8),(1/2,1);\end{array}\ 64\right]$$
$$-(2)^{2\nu+7}\sqrt{\pi}\,{}_3\psi_4\left[\begin{array}{c}(2,2),(\eta+\nu+3,4),(\theta+\nu+3,4);\\ \\(5/2,2),(\nu+5/2,2),(3/2,1),(\eta+\theta+2\nu+6,8);\end{array}\ 64\right], \tag{3.11}$$

where $\Re(\eta+\nu+1)>0$ *and* $\Re(\theta+\nu+1)>0$.

Corollary 3.12. *If we take $m = 1, \mu = 1$ and $\lambda = 1/2$ in (2.1) and then by using (1.9), we get the following integral formula:*

$$\int_0^1 \int_0^1 t^{\eta}(1-y)^{\eta-1}(1-t)^{\theta-1}(1-ty)^{1-\eta-\theta}\mathbb{H}_{\nu}\left(\frac{8y(1-t)(1-y)}{(1-ty)^2}\right)dtdy$$
$$= \frac{2^{2\nu+3}}{\sqrt{\pi}\Gamma(\nu+3/2)}\,\mathbb{B}(\eta+\nu+1,\theta+\nu+1)$$
$$\times\, {}_5\mathbb{F}_6\left[\begin{array}{c} \Delta(2,\eta+\nu+1),\Delta(2,\theta+\nu+1),1; \\ \\ (3/2),\Delta(4,\eta+\theta+2\nu+2),(1,\nu+3/2); \end{array} -\frac{1}{\mu^{\mu}}\right], \tag{3.12}$$

where $\Re(\eta+\nu+1) > 0$ *and* $\Re(\theta+\nu+1) > 0$.

4 Reducibility of the Wright Hypergeometic Function

A reduction formulas for the Wright hypergeometric function derived as follows:

$$ {}_3\psi_{m+2}\left[\begin{array}{c} (1,1)(\eta+\nu+2\lambda,2),(\theta+\nu+2\lambda,2); \\ (\lambda+1,1),...(\lambda+1,1)(\eta+\theta+2\nu+4\lambda,4)(\nu+\lambda+1,\mu); \end{array} -16\right] = \sqrt{\pi}$$
$$\times\, {}_3\psi_{m+3}\left[\begin{array}{c} (1,2),(\eta+\nu+2\lambda,4),(\theta+\nu+2\lambda,4); \\ (\lambda+1,2),...(\lambda+1,2)(\eta+\theta+2\nu+4\lambda,8)(1/2,1)(\nu+\lambda+1,2\mu); \end{array} 64\right] - \sqrt{\pi}(2)^5$$
$$\times\, {}_3\psi_{m+3}\left[\begin{array}{c} (2,2)(\eta+\nu+2\lambda+2,4),(\theta+\nu+2\lambda+2,4); \\ (\lambda+2,2),...(\lambda+2,2)(\eta+\theta+2\nu+4\lambda+4,8)(3/2,1)(\mu+\nu+\lambda+1,2\mu); \end{array} 64\right] \tag{4.1}$$

By comparing (2.1) and (2.4), results (4.1) can be established.

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