Palestine Journal of Mathematics

Vol. 9(1)(2020) , 456–463

INTEGRAL TRANSFORM OF EXTENDED MITTAG LEFFLER FUNCTION IN TERM OF FOX WRIGHT FUNCTION

M. Saif[1], A.H. Khan[1] and K.S. Nisar[2]

Communicated by Jose Luis Lopez-Bonilla

MSC 2010 Classifications: Primary 33C20; Secondary 33B15.

Keywords and phrases: Fox-Wright function, Integral transform, extended Mittag Leffler function, Bessel maitland function.

Abstract In this paper, we present some interesting integral transforms involving the extended Mittag-Leffler, which are given in term of Fox-Wright function. Based on the new result some integral formula with different special functions established as special cases of our main results for different values of parameters.

1 Introduction

The Mittag-Laffler function is the generalization of exponential function which occur naturally in the solution of fractional order and integral equation. Some application of Mittag-Laffler function is carried out in the study of Kinetic Equation, Study of Lorentz System, Random Walk, Complex system and also in applied problem such as Fluid flow, Electric network, Probability and Statistical distribution theory etc.

The Mittag-Leffler function was introduced by the Swedish Mathematician Gosta Mittag-Leffler [11] and its generalization introduced by Wiman [12] as:

$$E_{\alpha'}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha' k+1)} \quad\text{and}\quad E_{\alpha',\beta'}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha' k+\beta')}, \tag{1.1}$$

where $\alpha',\beta',z\in C$, $Re(\alpha')>0$, $Re(\beta')>0$ New generalization of $E_{\alpha',\beta'}$(z) introduced by Prabhakar [14] and further its generalization investigated by Shukla and Prajapati [15] in the following form as follows:

$$E^{\gamma}_{\alpha',\beta'}(z)=\sum_{k=0}^{\infty}\frac{(\gamma)_k\, z^k}{\Gamma(\alpha' k+\beta')k!} \quad\text{and}\quad E^{\gamma,q}_{\alpha',\beta'}(z)=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\, z^k}{\Gamma(\alpha' k+\beta')k!}, \tag{1.2}$$

where $\alpha',\beta',\gamma\in\mathbb{C}$, $Re(\alpha')>0$, $Re(\beta')>0$, $q\in(0,1)\cup\mathbb{N}$ and $(\gamma)_k$ is the Pochhammer symbol.

Recently, Özarslan and Yilmaz [13] have investigated following an extended Mittag-leffler, is defined by the series representation given as:

$$E^{\delta,c}_{\alpha',\beta'}(z,p)=\sum_{n=0}^{\infty}\frac{B_p(\delta+n,c-\delta)(c)_n z^n}{B(\delta,c-\delta)\Gamma(\alpha' n+\beta')n!}, \qquad (p\geq 0, Re(c)>0, Re(\delta)>0), \tag{1.3}$$

where $B_p(x,y)$ is the extension of Euler's beta function, which is defined as:

$$B_p(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}exp\left(\frac{-p}{t(1-t)}\right)dt, \qquad (Re(p)>0, Re(x)>0, Re(y)>0). \tag{1.4}$$

Further, generalization of extended Mittag-Leffler function which is defined by the series representation given as:

$$E^{\gamma,q,c}_{\alpha',\beta'}(z,p)=\sum_{n=0}^{\infty}\frac{B_p(\gamma+nq,c-\gamma)\,(c)_{nq}}{B(\gamma,c-\gamma)\,\Gamma(\alpha' n+\beta')}\frac{z^n}{n!}, \qquad (\,p\geq 0, Re(\gamma)>0, Re(c)>0), \tag{1.5}$$

where $(c)_{nq} = \frac{\Gamma(c+nq)}{\Gamma(\gamma)}$ denote the generalized Pochhammer symbol.

For put some particular values of this parameters we get following special cases given as:
If β' is replaced by $\beta' + 1$and z by $-z$, we get

$$E^{\gamma,q,c}_{\alpha',\beta'+1}(z) = J^{\lambda,\gamma,c}_{\alpha,q}(-z), \tag{1.6}$$

which is an extended Wright-Bessel function given by [10].
If we put p=0, q=1, c=1 in (1.5), we get

$$E^{\gamma,1,1}_{\alpha',\beta'}(z) = E^{\gamma,}_{\alpha',\beta'}(z). \tag{1.7}$$

The Fox-Wright function ([4], [16]) is defined by the series representation given as:

$$\begin{aligned} {}_q\Psi_s[z] &= {}_q\Psi_s \left[\begin{array}{c} (\alpha_1, A_1)............(\alpha_q, A_q) \\ (\beta_1, B_1)..........(\beta_s, B_s) \end{array} ; z \right] \\ &= \sum_{k=0}^{\infty} \frac{\Gamma(\alpha_1 + A_1 k)........\Gamma(\alpha_q + A_q k)}{\Gamma(\beta_1 + B_1 k).......\Gamma(\beta_s + B_s k)} \frac{z^k}{k!} \end{aligned} \tag{1.8}$$

$$\alpha_1, ...\alpha_q \ , \beta_1, ...\beta_s \in \mathbb{R}, \text{ such that } 1 + \sum_{i=1}^{s} B_i - \sum_{r=1}^{q} A_r > 0,$$

where $\Gamma(z)$ denotes the gamma function and q and s are non negative integer.

Integral transform (Fourier Transform, Hankel Transform, Hermite Transform, Melin Transform etc...) have been widely used in various problem of mathematical physics and applied mathematics. Integral transform involving a verity of special function have been established by many authors ([1], [3],[5], [6], [7]). The key motivation for pursuing theories for integral transform is that it gives a simple tool which is represented by an algebraic problem in the process of solving differential equation. In the most theories of integral transform , the kernel is doing the important role which transform one space to the other space in order to solve the solution. The main reason to transform is because it is not easy to solve the equation in the given space, or it is easy to find a characterestic for the special purpose.

- Edward established the following result [2]

$$\int_0^1 \int_0^1 u^\rho (1-v)^{\rho-1}(1-u)^{\sigma-1}(1-uv)^{1-\rho-\sigma} du dv = \frac{\Gamma(\rho)\Gamma(\sigma)}{\Gamma(\rho+\sigma)}. \tag{1.9}$$

  provided $\mathbb{R}(\rho) > 0$ and $\mathbb{R}(\sigma) > 0$

- We recall the following intresting and useful integral is given by MacRobert [8] for $Re(\alpha) > 0, Re(\beta) > 0, a$ and b are non zero constant

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au + b(1-u)]^{-\alpha-\beta} = \frac{\Gamma(\alpha)\Gamma(\beta)}{(a)^\alpha (b)^\beta \Gamma(\alpha+\beta)}. \tag{1.10}$$

2 Main results

In this section, we compute new integral formula involving extended Mittag Laffler function.These integral formula are expressed in term of Fox -Wright function as given in Theorem 2.1, Theorem 2.6 and Theorem 2.11.

Theorem 2.1. *For $Re(\alpha + \beta) > 0$, $Re(\gamma) > 0$, $\alpha', \beta', \gamma \in \mathbb{C}, p \geq 0$. a and b are non zero constant, then following integral formula holds true:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au + b(1-u)]^{-\alpha-\beta} E^{\gamma,q,c}_{\alpha',\beta'} \left(\frac{(1-u)}{[au + b(1-u)]}, p \right)$$

$$= \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\alpha)}\ {}_2\Psi_3\left[\begin{array}{l}(\gamma\,,1)\,,\ (c\,,q),\ \ (\gamma\,,1)\\(\alpha+\beta,1),\ \ (\beta',\alpha')\ (c,1)\end{array};(\frac{z}{b},p)\right]. \tag{2.1}$$

Proof. First we indicate the L.H.S of (2.1) by I and using (1.5), then interchanging the order of integration and summation, we get

$$I = \sum_{n=0}^{\infty}\frac{B_p(\gamma+nq,c-\gamma)\Gamma(c+nq)(z/b)^n}{\Gamma(c-\gamma)\Gamma(\alpha' n+\beta')\Gamma(\alpha+\beta+n)n!}$$

$$\times\int_0^1 u^{\alpha-1}(1-u)^{(\beta+n)-1}\,(au+b(1-u))^{-\alpha-(\beta+n)} \tag{2.2}$$

now we apply the integral formula (1.10) to the integral of (2.2) under the condition given in Theorem (2.1), we get

$$I = \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{B_p(\gamma+nq,c-\gamma)\Gamma(c+nq)(z/b)^n}{\Gamma(c-\gamma)\Gamma(\alpha' n+\beta')\Gamma(\alpha+\beta+n)n!} \tag{2.3}$$

after solving the above equation with the help of (1.8), we get the require result. This completes the proof. □

Remark 2.2. If we put $q=1$ in Theorem 2.1, we get following integral formula.

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}E^{\gamma,c}_{\alpha',\beta'}\left(\frac{(1-u)}{[au+b(1-u)]}\,,p\right)$$

$$= \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\alpha)}\ {}_1\Psi_2\left[\begin{array}{c}(\gamma\,,1)\\(\alpha+\beta,1),\ (\beta',\alpha')\end{array};(\frac{z}{b},p)\right]. \tag{2.4}$$

Corollary 2.3. *If we put $\beta=\beta'+1,\ z=-z$ in Theorem (2.1) and using (1.6) , then following integral formula holds true:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}J^{\gamma,q,c}_{\alpha',\beta'+1}\left(\frac{(1-u)}{[au+b(1-u)]},\ p\right)$$

$$= \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\alpha)}\ {}_2\Psi_3\left[\begin{array}{l}(\gamma\,,1)\,,\ (c,q)\\(\alpha+\beta,1),\ \ (\beta',\alpha'),\ (c,1)\end{array};(\frac{z}{b},p)\right]. \tag{2.5}$$

Corollary 2.4. *If we put $p=0,\ q=0$ in Theorem (2.1) , then following integral formula holds true:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}\,W_{\alpha',\beta'}\left(\frac{(1-u)z}{[au+b(1-u)]}\right)$$

$$= \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\alpha)}\ {}_0\Psi_2\left[\begin{array}{c}\\(\alpha+\beta;1),\ \ (\beta';\alpha')\end{array};z/b\right]. \tag{2.6}$$

Corollary 2.5. *If we put $p=0,q=c=1,\alpha',\beta'\in\mathbb{C}$ in Theorem(2.1), we get following integral formula holds good:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}E^{\gamma}_{\alpha',\beta'}\left(\frac{(1-u)z}{[au+b(1-u)]}\right)$$

$$= \frac{1}{(a)^{\alpha}(b)^{\beta}\Gamma(\alpha)}\ {}_1\Psi_2\left[\begin{array}{l}(\gamma,1)\\(\alpha+\beta,1),\ (\beta',\alpha')\end{array};\frac{z}{b}\right]. \tag{2.7}$$

Theorem 2.6. *For* $Re(\alpha) > 0, Re(\beta) > 0, Re(\alpha+\beta) > 0, \alpha', \beta' \in \mathbb{C}$ *such that and a and b are non negative integers then following integral formula holds good:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta} E^{\gamma,q,c}_{\alpha',\beta'}\left(\frac{u(1-u)}{[au+b(1-u)]^2},\ p\right)$$

$$= \frac{1}{(a)^\alpha (b)^\beta \Gamma(\alpha)}\ {}_2\Psi_2\left[\begin{array}{l}(\gamma,\ 1)\ ,\ (c,q)\\ (\beta',\alpha'),\ (c,1)\end{array};(\frac{z}{ab},p)\right]. \tag{2.8}$$

Proof. First we indicate the L.H.S of (2.8) by I and using (1.5) then interchanging the order of integration and summation, we get

$$I = \sum_{n=0}^{\infty} \frac{\beta_p(\gamma+nq; c-\gamma)(c)_{nq}}{\beta(\ \gamma;\ \ c-\gamma)\Gamma(\alpha' n+\beta')} \frac{(z)^n}{n!}$$

$$\times \int_0^1 u^{(\alpha+n)-1}(1-u)^{(\beta+n)-1}[au+b(1-u)]^{-(\alpha+n)-(\beta+n)} \tag{2.9}$$

now we apply the integral formula (1.10) to the integral of (2.9) under the condition given in Theorem(2.6), we get

$$I = \frac{1}{(a)^\alpha (b)^\beta \Gamma(\gamma)} \sum_{n=0}^{\infty} \frac{B_p(\gamma+nq, c-\gamma)\Gamma(c+nq)(z/ab)^n}{\Gamma(c-\gamma)\Gamma(\alpha' n+\beta')\Gamma(\alpha+\beta+n)n!} \tag{2.10}$$

after solving the above equation with the help of (1.8) we get the require result (2.8) of Theorem (2.1).

Remark 2.7. For $Re(\alpha) > 0,\ Re(\beta) > 0, \alpha', \beta', \gamma \in \mathbb{C}$ and $q = 1$ in Theorem (2.6), we get the following integral formula:

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta} E^{\gamma,c}_{\alpha',\beta'}\left(\frac{u(1-u)}{[au+b(1-u)]^2},\ p\right)$$

$$= \frac{\Gamma(\alpha)}{(a)^\alpha (b)^\beta}\ {}_1\Psi_1\left[\begin{array}{l}(\gamma,\ 1)\\ (\beta',\alpha')\end{array};(\frac{z}{ab},p)\right]. \tag{2.11}$$

Corollary 2.8. *If we put* $\beta = \beta'+1, z = -z$ *in Theorem* (2.6)*, then following integral formula holds true:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}\ J^{\gamma,q,c}_{\alpha',\beta'+1}\left(\frac{(1-u)}{[au+b(1-u)]},p\right)$$

$$= \frac{1}{(a)^\alpha (b)^\beta \Gamma(\alpha)}\ {}_2\Psi_3\left[\begin{array}{l}(\gamma\ ,1)\ ,\ (c,q)\\ (\alpha+\beta,2),\ (\beta'+1,\alpha'),\ (c,1)\end{array};(\frac{-z}{ab},p)\right]. \tag{2.12}$$

Corollary 2.9. *If we put* $p = 0, q = 0$ *in Theorem* (2.6) *, then following integral formula holds true:*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta}\ W_{\alpha',\beta'}\left(\frac{(1-u)}{[au+b(1-u)]},p\right)$$

$$= \frac{1}{(a)^\alpha (b)^\beta}\ {}_0\Psi_2\left[\begin{array}{l}\\ (\alpha+\beta,2),\ (\beta',\alpha')\end{array};\frac{z}{ab}\right]. \tag{2.13}$$

Corollary 2.10. *If we put* $p = 0,\ q = c = 1, \alpha', \beta', \gamma \in \mathbb{C},\ p \geq 0$ *in* (2.6)*, we get following result*

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}[au+b(1-u)]^{-\alpha-\beta} E^{\gamma}_{\alpha',\beta'}\left(\frac{u(1-u)z}{[au+b(1-u)]^2}\right)$$

$$= \frac{1}{(a)^\alpha (b)^\beta \Gamma(\alpha)}\ {}_1\Psi_2\left[\begin{array}{l}(\gamma,1)\\ (\alpha+\beta,2),\ (\beta',\alpha')\end{array};\frac{z}{ab}\right]. \tag{2.14}$$

Theorem 2.11. *For* $\alpha', \beta', \gamma \in \mathbb{C}, p \geq 0,\ Re(\alpha) > 0,\ Re(\beta > 0),\ Re(\alpha+\beta) > 0$, *then following integral formula holds good:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\ E_{\alpha',\beta'}^{\gamma,q,c}\left[ty(1-x)(1-xy),p\right] dxdy$$

$$= \frac{\Gamma(\beta)}{\Gamma(\gamma)}\ {}_3\Psi_3\left[\begin{array}{l} (\alpha\ ,1)\ ,\ (c,q),\ (\gamma,1) \\ (\alpha+\beta,1),\ (\beta',\alpha'),\ (c,\ 1) \end{array} ;t\right]. \tag{2.15}$$

Proof. First we denote the L.H.S of (2.15) by I and using (1.5) and interchanging the order of integration and summation, we get

$$I = \sum_{n=0}^{\infty} \frac{\beta_p(\gamma+nq; c-\gamma)}{\beta(\ \gamma,\ c-\gamma)}\ \frac{(c)_{nq}\ t^n}{\Gamma(\alpha' n+\beta')\ n!}$$

$$\times \int_0^1 \int_0^1 y^{(\alpha+n)}(1-x)^{(\alpha+n)-1}(1-y)^{\beta-1}(1-xy)^{1-(\alpha+n)-\beta} \tag{2.16}$$

now we apply the integral formula (1.10) to the integral of (2.16) under the condition given in Theorem (2.6), we get

$$I = \ \Gamma(\beta)\sum_{n=0}^{\infty} \frac{\beta_p(\gamma+nq; c-\gamma)\Gamma(c+nq)}{\Gamma(\gamma)\Gamma(c-\gamma)}\ \frac{\Gamma(\alpha+n)}{\Gamma(\gamma+\beta+n)}\ \frac{\Gamma(c+nq)}{\Gamma(\alpha' n+\beta')}\ \frac{t^n}{n!} \tag{2.17}$$

after solving the above equation with the help of (1.8) we get the required result. This completes the proof. □

Remark 2.12. For $\alpha', \beta', \gamma \in \mathbb{C},\ p\ \geq 0,\ Re(\alpha+\beta) > 0,\ Re(\gamma) > 0$ and $q = 1$ in Theorem (2.11), we get following integral formula:

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\ E_{\alpha',\beta'}^{\gamma,c}\left[ty(1-x)(1-xy),p\right] dxdy$$

$$= \frac{\Gamma(\beta)}{\Gamma(\gamma)}\ {}_2\Psi_2\left[\begin{array}{l} (\alpha\ ,1),\ (\gamma,1) \\ (\alpha+\beta\ ,1),\ (\beta',\alpha'), \end{array} ;t\right]. \tag{2.18}$$

Corollary 2.13. *If we put* $\beta' = \beta' + 1$, z *is replaced by -*z *in Theorem* (2.6)*, then following integral formula hold good:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\ E_{\alpha',\beta'}^{\gamma,q,c}\left[ty(1-x)(1-xy),p\right] dxdy$$

$$= \frac{\Gamma(\beta)}{\Gamma(\gamma)}\ {}_3\Psi_3\left[\begin{array}{l} (\alpha\ ,1)\ ,\ (c,q),\ \ (\gamma,1) \\ (\alpha+\beta,1),\ (\beta'+1,\alpha'),\ (c,1) \end{array} ;t\right]. \tag{2.19}$$

Corollary 2.14. *If we put* $p = 0, q = 0$ *in Theorem* (2.11)*, then following integral formula hold good:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)1-\alpha-\beta\ W_{\alpha',\beta'}\left[ty(1-x)(1-xy)\right] dxdy$$

$$= \Gamma(\beta)\ {}_1\Psi_2\left[\begin{array}{l} (\alpha,\ 1) \\ (\alpha+\beta,\ 1),\ \ (\beta',\ \alpha') \end{array} ;t\right]. \tag{2.20}$$

Corollary 2.15. *If we put $p = 0, q = c = 1$ in Theorem (2.11), then following integral formula holds good:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta} \quad E^{\gamma}_{\alpha',\beta'}\,[ty(1-x)(1-xy)]\,dxdy$$

$$= \frac{\Gamma(\beta)}{\Gamma(\gamma)}\,{}_2\Psi_2\left[\begin{array}{c} (\alpha\,,1)\ \ (\gamma,\ 1) \\ (\alpha+\beta,\ 1),\ (\beta',\ \alpha') \end{array};t\right]. \tag{2.21}$$

Theorem 2.16. *For $p \geq 0, Re(c) > 0, Re(\beta) > 0, Re(\gamma) > 0, Re(\alpha+\beta) > 0$, then following integral formula hold true:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\,E^{\gamma,q,c}_{\alpha',\beta'}\,[t(1-y)(1-xy)^{-1},p]\,dxdy$$

$$= \frac{\Gamma(\alpha)}{\Gamma(\gamma)\,\Gamma(c-\gamma)}\ {}_3\Psi_3\left[\begin{array}{c} (\beta\,,1)\,,\ \ (c,q),\ (\gamma,1) \\ (\alpha+\beta,1),\ \ (\beta',\alpha'),\ \ (c,1) \end{array};t\right]. \tag{2.22}$$

Proof. First we denote the L.H.S of (2.22) by I and using (1.5) then interchanging the order of integration and summation, we get

$$I = \sum_{n=0}^{\infty} \frac{\beta_p(\gamma+nq, c-\gamma)(c)_{nq}}{\beta(\ \gamma,\ \ c-\gamma)(\Gamma\alpha' n+\beta')}\frac{t^n}{n!}$$

$$\times \int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-(\beta+n)}\,dxdy \tag{2.23}$$

now we apply the integral formula (1.9) to the integral of (2.23) under the condition given in Theorem(2.16), we get

$$I =\ \Gamma(\alpha)\sum_{n=0}^{\infty}\frac{\beta_p(\gamma+nq;c-\gamma)}{\Gamma(\alpha+\beta+n)}\,\frac{\Gamma(\beta+n)}{\Gamma(\gamma+\beta+n)}\,\frac{\Gamma(c+nq)}{\Gamma(\alpha' n+\beta')}\,\frac{1}{\Gamma(c-\gamma)\Gamma(\gamma)}\,\frac{t^n}{n!} \tag{2.24}$$

after solving the above equation with the help of (1.8) we get the required result. This completes the proof. □

Remark 2.17. If we put q=1 in Theorem (2.16), we get the following integral formula

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\,E^{\gamma,c}_{\alpha',\beta'}\,[t(1-y)(1-xy)^{-1},p]\,dxdy$$

$$= \frac{\Gamma(\alpha)}{\Gamma(\gamma)\,\Gamma(c-\gamma)}\ {}_2\Psi_2\left[\begin{array}{c} (\beta\,,1)\,,\ (\gamma\,,1) \\ (\alpha+\beta\,,1),\ \ (\beta'\,,\alpha') \end{array};t\right]. \tag{2.25}$$

Corollary 2.18. *If we put $\beta' = \beta' + 1$, z is replaced by -z in Theorem (2.16), then following integral formula hold true:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta} \quad J^{\gamma,q,c}_{\alpha',\beta'+1}[t(1-y)(1-xy)^{-1},p]\,dxdy$$

$$= \frac{\Gamma(\alpha)}{\Gamma(\gamma)\,\Gamma(c-\gamma)}\ {}_3\Psi_3\left[\begin{array}{c} (\beta\ \ ;1)\,,\ \ (c;q),\ \ (\gamma;1) \\ (\alpha+\beta;1),\ \ (\beta';\alpha'),\ \ (c;1) \end{array};t\right]. \tag{2.26}$$

Corollary 2.19. *If we put $p = 0, q = 0$ in Theorem (2.16), we get the following integral formula*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta} \quad W_{\alpha',\beta'}\left(\frac{ty(1-x)}{(1-xy)}\right) dxdy$$

$$= \Gamma(\beta)\, {}_1\Psi_2 \left[\begin{array}{l} (\alpha\,,1) \\ (\alpha+\beta,1),\ (\beta'+1,\alpha') \end{array} ;t \right]. \tag{2.27}$$

Corollary 2.20. *If we put $p = 0, q = c = 1$ in Theorem (2.16), then following integral formula holds good:*

$$\int_0^1 \int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta} \quad E^{\gamma}_{\alpha',\beta'}\left(\frac{ty(1-x)}{(1-xy)}\right) dxdy$$

$$= \frac{\Gamma(\beta)}{\Gamma(\gamma)}\, {}_1\Psi_2 \left[\begin{array}{l} (\alpha\,,1)\ ,\ (\gamma,1) \\ (\alpha+\beta,1)\ ,\ (\beta',\alpha') \end{array} ;t \right]. \tag{2.28}$$

3 Concluding Remark

The outcome in the present paper is the computation of some integral transforms involving generalized extended Mittag-Leffler function in terms of Fox-Wright function. We observe that the generalized Bessel-Maitland function has a close relationship with some known special functions such as Mittag-Leffler function, Bessel function, etc. As a consequence, we have attempted to compute the integrals in form of different types of special functions by suitable replacement of parameters. Further, a lot of work can be put in the literature by writing the Fox-Wright function in terms of Fox H-function, Meijer G-function, etc. The results obtained here seems to be interesting and may potentially be useful in various applied problems.

References

[1] J. Choi, P. Agarwal, S. Mathur and S.D. Purohit, *Certain new integral formula involving the generalized Bessel function*, Bull. Korean. Math. Soc. **51** (4) (2014) 995-1003.

[2] J. Edward, *A treatise on the integral calculus*, Chelsea Publication Company, New York, **II** (1922).

[3] A. Erdelyi and W. Magnus, *Table of integral transform* , McGraw-Hill, New York, **I** (1954).

[4] A.A. Kilbas, M. Saigo, J.J. Trujillo, *On the generalized Wright function*, Fract. Calc. Appl. Anal. **5** (2002) 437-460.

[5] N.U. Khan, T. Kashmin, Electronic *Journal of Mathematical Analysis and Application*. Vol. **4** (2) (2016), 139-149.

[6] N.U. Khan, T. Kashmin, *Some integral for the generalized Bessel-Maitland function*, Elec. J. Math. Anal. Appl. **4** (2) (2016), 139-149.

[7] O. Khan, M. Kamarujjama and N.U. Khan, *Certain Integral Transforms Involving The Product of Galue Type Struve Function and Jacobi Polynomial*, Palestine J. Math (Accepted) (2018).

[8] T.M. MacRobert, *Beta functions formulas and integrals involving E-function*, Math.Annalen. **142** (1961) 450-452.

[9] E. Mittal, R.M. Panday and S. Joshi, *Application and Applied Mathematics*. **11** (1) (2016) 307-316.

[10] E.Mittal, R.M. Panday,and S. Joshi, *On extension of Mittag-Leffler function*, Appl. Appl. Math. **11** (2016), no. 1, 307-316.

[11] G.M. Mittag-Leffler,*Surla nouvelle function $E_\alpha(x)$*,C. R. Acad. Sci. Paris **137** (1903) 554-558.

[12] A. Wiman, *Uber den fundamental Satz in der Theorie der Funktionen $E_\alpha(x)$*, Acta. Math. **29** (1905) 191-201.

[13] M.A. Özarslan, B. Yilmaz, *The extended Mittaf-Leffler function and its properties*, Journal of Inequalities and Application, **85** (2014) 1-10.

[14] T.R. Prabhakar, *A singular integral equation with a generalized Mittafg-Leffler function in the kernel*, yokohama Math. J. **19** (1971) 7-15.

[15] A.K. Shukla, J.C. Prajapati, *On a generalization of Mittag-Laffler function and its properties*, J. Math. Anal. Appl. **336** (2007) 797-811.

[16] E.M. Wright, *The asymptotic expansion of the generalized hypergeometric function*, J. London. Math. Soc. **10** (1935) 287-293.

Author information

M. Saif[1], A.H. Khan[1] and K.S. Nisar[2], [1]Department of Applied Mathematics, Aligarh Muslim University, Aligarh-202002, India, [2]Department of Mathematics, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia,.
E-mail: `usmanisaf153@gmail.com`

Received: March 12, 2018.
Accepted: June 27, 2018.