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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS IN INTUITIONISTIC GENERALIZED FUZZY METRIC SPACES

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Abstract. In this paper, we obtain unique common fixed point theorems for six weakly compatible mappings in intuitionistic generalized fuzzy metric spaces.

1 Introduction

The theory of fuzzy sets has evolved in many directions after investigation of the notion of fuzzy sets by Zadeh [19]. Many authors have introduced the concept of a fuzzy metric space in different ways [5,8].George and Veeramani [4] modified the concept of a fuzzy metric space introduced by Kramosil and Michalek [8] and defined a Hausdroff topology on this fuzzy metric space. Alternatively, Mustafa and Sims [9] introduced a new notion of a generalized metric space called G-metric space. Rao et al. [15] proved two unique common coupled fixed point theorems for three mappings in symmetric G- fuzzy metric spaces. Sun and Yang [17] introduced the concept of G- fuzzy metric spaces and proved two common fixed point theorems for four mappings. Some interesting references on G- metric spaces are [9-12]. Park [14] introduced and discussed in a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [1], and the concept of a fuzzy metric space given George and Veeramani [5]. We have generalized the result of Rao [15]. Before giving our main results, we obtain unique common fixed point theorems for six weakly compatible mappings in intuitionistic generalized fuzzy metric spaces.

Definition 1.1 A 3-tuple (X, G, $*$) is called a G-Fuzzy Metric Space (Shortly GFMS) if X is an arbitrary nonempty set, $*$ is a continuous t-norm and G is a fuzzy set on the $X^3 \times (0, \infty)$ satisfying the following conditions: For each t, s > 0

(i) G(x, x, y, t) > 0 for all x, y$\in$ X with x$\neq$ y,

(ii) G(x, x, y, t) $\geq$ G(x, y, z, t) for all x, y, z $\in$ X with y $\neq$ z,

(iii) G(x, y, z, t) = 1 if and only if x = y = z,

(iv) G(x, y, z, t) = G(p(x, y, z), t) where p is a permutation function,

(v) G(x, y, z, t + s) $\geq$ G(a, y, z, t) * G(x, a, a, s) for all x,y, z, a $\in$ X,

(vi) G(x, y, z, .) : (0,∞) $\to$ [0,1] is continuous.

Definition 1.2 A 5 tuple (X, G, H, $*$, $\diamond$) is said to be an Intuitionistic Generalized Fuzzy Metric Space (Shortly GIFMS or IGFMS) if X is an arbitrary non-empty set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, G and H are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions: For every x, y, z, a $\in$ X and t, s > 0

(i) G(x, y, z, t) + H(x, y, z, t)$\leq$ 1,

(ii) G(x, x, y, t) > 0 for all x, y$\in$ X with x$\neq$ y,

(iii) G(x, x, y, t) $\geq$ G(x, y, z, t) for y $\neq$z,

(iv) G(x, y, z, t) $=1$ if and only if x = y = z,

(v) G(x, y, z, t) = G(p(x ,y, z), t), where p is a permutation function,

(vi) G(x, a, a, t) $*$ G(a, y, z, s) $\leq$ G(x, y, z, t + s),

(vii) G(x, y, z, .) :$(0, \infty) \rightarrow [0, 1]$ is continuous,

(viii) G is a non-decreasing of $\Re^+$, $\lim_{t\to\infty} G(x, y, z, t) = 1$,
$\lim_{t\to 0} G(x, y, z, t) = 0$ for all $x, y, z \in X, t > 0$,

(ix) H(x, x, y, t) < 1 for $x \neq y$,

(x) H(x, x, y, t) $\leq$ H(x, y, z, t) for y $\neq$ z,

(xi) H(x, y, z, t) = 0 if and only if x = y = z,

(xii) H(x, y, z, t) = H(p(x, y, z), t), where p is a permutation function,

(xiii) H(x, a, a, t) $\diamond$ H(a, y, z, s) $\geq H(x, y, z, t + s)$,

(xiv) H(x, y, z, .) : $(0, \infty) \rightarrow [0,1]$ is continuous,

(xv) H is a non-increasing function on $\Re^+$, $\lim_{t\to\infty} H(x, y, z, t) = 0$,
$\lim_{t\to 0}$ H(x, y, z, t) = 1 for all x, y, z $\in X, t > 0$.
In this case, the pair (G, H) is called an intuitionistic generalized fuzzy metric on X.

Lemma 1.3 Let (X, G, H, $*$, $\diamond$) be an intuitionistic generalized fuzzy metric space. Then G and H are continuous function on $X^3x(0,\infty)$. Now onwards, we assume the following condition :

$$\lim_{t\to\infty} G(x, y, z, t) = 1 \quad \text{and} \quad \lim_{t\to\infty} H(x, y, z, t) = 0 \text{ for all x,y, z} \in X$$

Lemma 1.4 Let (X, G, H, $*$, $\diamond$) be an intuitionistic generalized fuzzy metric space. If there exists k $\in$ (0,1) such that

$$\begin{aligned} min\{G(x, y, z, kt), G(u, v, w, kt)\} &\geq min\{G(x, y, z, t), G(u, v, w, t)\}. \\ max\{H(x, y, z, kt), H(u, v, w, kt)\} &\leq max\{H(x, y, z, t), H(u, v, w, t)\}. \end{aligned}$$

for all x, y, z, u, v, w $\in X$ and t $>$ 0, then x = y = z and u = v = w.

2 Main Result

Let ϕ denote the set of all continuous non decreasing functions ϕ, ψ : $[0,\infty) \rightarrow [0, \infty)$ such that $\phi^n(t) \rightarrow 0$ $as\ n \rightarrow \infty$ and $\psi^n(t) \rightarrow 1$ $as\ n \rightarrow \infty$ for all t $>$ 0. It is clear that $\phi(t) < t, \psi(t) > t$ for all t $>$ 0 and $\phi(0) = 0$ and $\psi(1) = 1$.
Theorem 2.1
$Let(X, G, H,*, \diamond$) be an intuitionistic generalized fuzzy metric space and
S, T, R, f, g, h : X $\rightarrow$ X be satisfying

(i) S(X) $\subseteq g(X), T(X) \subseteq$ h(X) and R(X) $\subseteq$ f(X),

(ii) One of the f(X), g(X) and h(X) is a complete subspace of X,

(iii) The pairs (S,f), (T,g) and (R,h) are weakly compatible and

$$G(Sx, Ty, Rz, t) \geq \phi\left(min\left\{ \begin{array}{r} G(fx, gy, hz, t), \frac{1}{3}[G(fx, Sx, Ty, t) \\ +G(gy, Ty, Rz, t) + G(hz, Rz, Sx, t)], \\ \frac{1}{4}[G(fx, Ty, hz, t) + G(Sx, gy, hz, t) \\ +G(fx, gy, Rz, t)] \end{array} \right\} \right) \quad (2.1.1)$$

$$H(Sx,Ty,Rz,t) \leq \psi\left(max\left\{\begin{array}{r} H(fx,gy,hz,t), \frac{1}{3}[H(fx,Sx,Ty,t) \\ +H(gy,Ty,Rz,t)+H(hz,Rz,Sx,t)], \\ \frac{1}{4}[H(fx,Ty,hz,t)+H(Sx,gy,hz,t) \\ +H(fx,gy,Rz,t)] \end{array}\right\}\right) \quad (2.1.2)$$

for all x, y, z $\in$ X,where $\phi, \psi \in \Phi$. Then either one of the pairs (S, f), (T, g) and (R,h) has a coincidence point or the maps S, T, R, f, g and h have a unique common fixed point in X.

Proof: Choose $x_0 \in X$. By (i), there exists $x_1, x_2, x_3 \in X$ such that $Sx_0 = gx_1 = y_0$, $Tx_1 = hx_2 = y_1$, $Rx_2 = fx_3 = y_2$. Inductively, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{3n} = Sx_{3n} = gx_{3n+1}$,$y_{3n+1} = Tx_{3n+1} = hx_{3n+2}$ and $y_{3n+2} = Rx_{3n+2} = fx_{3n+3}$, where n = 0, 1, 2... If $y_{3n} = y_{3n+1}$ then x_{3n+1} is a coincidence point of g and T. If $y_{3n+1} = y_{3n+2}$ then x_{3n+2} is a coincidence point of h and R. If $y_{3n+2} = y_{3n+3}$ then x_{3n+3} is a coincidence point of f ant S. Now assume that $y_n \neq y_{n+1}$ for all n. Denote $d_n = G(y_n, y_{n+1}, y_{n+2}, t)$ and $\rho_n = H(y_n, y_{n+1}, y_{n+2}, t)$. Putting $x = x_{3n}$, $y = x_{3n+1}$, $z = x_{3n+2}$ in (2.1.1), we get
$d_{3n} = G(y_{3n}, y_{3n+1}, y_{3n+2}, t) = G(Sx_{3n}, Tx_{3n+1}, Rx_{3n+2}, t)$

$$\geq \phi\left(min\left\{\begin{array}{r} G(fx_n, gx_{3n+1}, hx_{3n+2}, t), \frac{1}{3}[G(fx_{3n}, Sx_{3n}, Tx_{3n+1}, t) \\ +G(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, Sx_{3n}, t)], \\ \frac{1}{4}[G(fx_{3n}, Tx_{3n+1}, hx_{3n+2}, t) + G(Sx_{3n}, gx_{3n+1}, hx_{3n+2}, t) \\ +G(fx_{3n}, gx_{3n+1}, Rx_{3n+2}, t)] \end{array}\right\}\right)$$

$$\geq \phi\left(min\left\{\begin{array}{r} G(y_{3n-1}, y_{3n}, y_{3n+1}, t), \frac{1}{3}[G(y_{3n-1}, y_{3n}, y_{3n+1}, t) \\ +G(y_{3n}, y_{3n+1}, y_{3n+2}, t) + G(y_{3n+1}, y_{3n+2}, y_{3n}, t)], \\ \frac{1}{4}[G(y_{3n-1}, y_{3n+1}, y_{3n+1}, t) + G(y_{3n}, y_{3n}, y_{3n+1}, t) \\ +G(y_{3n-1}, y_{3n}, y_{3n+2}, t)] \end{array}\right\}\right)$$

$$\geq \phi\left(min\left\{\begin{array}{r} d_{3n-1}, \\ \frac{1}{3}[d_{3n-1} + d_{3n} + d_{3n}], \\ \frac{1}{4}[d_{3n-1} + d_{3n} + (d_{3n-1} + d_{3n})] \end{array}\right\}\right)$$

If $d_{3n} \leq d_{3n-1}$ then from lemma (1.4), we have $d_{3n} \geq \phi(d_{3n}) > d_{3n}$.
It is a contradiction. Hence $d_{3n} \geq d_{3n-1}$. Now from lemma (1.4), $d_{3n} \geq \phi(d_{3n-1}) > d_{3n}$.
$\rho_{3n} = H(y_{3n}, y_{3n+1}, y_{3n+2}, t) = H(Sx_{3n}, Tx_{3n+1}, Rx_{3n+2}, t)$

$$\leq \psi\left(max \begin{Bmatrix} H(fx_n, gx_{3n+1}, hx_{3n+2}, t), \frac{1}{3}[H(fx_{3n}, Sx_{3n}, Tx_{3n+1}, t) \\ +H(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + H(hx_{3n+2}, Rx_{3n+2}, Sx_{3n}, t)], \\ \frac{1}{4}[H(fx_{3n}, Tx_{3n+1}, hx_{3n+2}, t) + H(Sx_{3n}, gx_{3n+1}, hx_{3n+2}, t) \\ +H(fx_{3n}, gx_{3n+1}, Rx_{3n+2}, t)] \end{Bmatrix} \right)$$

$$\leq \psi\left(max \begin{Bmatrix} H(y_{3n-1}, y_{3n}, y_{3n+1}, t), \frac{1}{3}[H(y_{3n-1}, y_{3n}, y_{3n+1}, t) \\ +H(y_{3n}, y_{3n+1}, y_{3n+2}, t) + H(y_{3n+1}, y_{3n+2}, y_{3n}, t)], \\ \frac{1}{4}[H(y_{3n-1}, y_{3n+1}, y_{3n+1}, t) + H(y_{3n}, y_{3n}, y_{3n+1}, t) \\ +H(y_{3n-1}, y_{3n}, y_{3n+2}, t)] \end{Bmatrix} \right)$$

$$\leq \psi\left(max \begin{Bmatrix} \rho_{3n-1}, \\ \frac{1}{3}[\rho_{3n-1} + \rho_{3n} + \rho_{3n}], \\ \frac{1}{4}[\rho_{3n-1} + \rho_{3n} + (\rho_{3n-1} + \rho_{3n})], \end{Bmatrix} \right)$$

If $\rho_{3n} \geq \rho_{3n-1}$ then from lemma(1.4), we have $\rho_{3n} \leq \psi(\rho_{3n}) < \rho_{3n}$. It is a contradiction. Hence $\rho_{3n} \leq \rho_{3n-1}$. Now from lemma (1.4), $\rho_{3n} \leq \psi(\rho_{3n-1})$ Similarly, by putting $x = x_{3n+3}, y = x_{3n+1}, z = x_{3n+2}$ and $x = x_{3n+3}, y = x_{3n+4}, z = x_{3n+2}$ in (2.1.1) and (2.1.2) we get $d_{3n+1} \geq \phi(d_{3n})$, $d_{3n+2} \geq \phi(d_{3n+1})$ and
$\rho_{3n+1} \leq \psi(\rho_{3n})$, $\rho_{3n+2} \leq \psi(\rho_{3n+1})$
Thus, from lemma(1.4), Equations(2.1.1) and (2.1.2), we have

$$\begin{aligned} G(y_n, y_{n+1}, y_{n+2}, t) &\geq \phi(G(y_{n-1}, y_n, y_{n+1}, t)) \\ &\geq \phi^2(G(y_{n-2}, y_{n-1}, y_n, t)) \geq ... \geq \phi^n(G(y_0, y_1, y_2, t)) \end{aligned}$$

We have G(y$_n, y_n, y_{n+1}, t) \geq G(y_n, y_{n+1}, y_{n+2}, t) \geq \phi^n(G(y_0, y_1, y_2, t)$ and

$$\begin{aligned} H(y_n, y_{n+1}, y_{n+2}, t) &\leq \psi(H(y_{n-1}, y_n, y_{n+1}, t)) \\ &\leq \psi^2(H(y_{n-2}, y_{n-1}, y_n, t)) \leq ... \leq \psi^n(H(y_0, y_1, y_2, t)) \end{aligned}$$

We have H(y$_n, y_n, y_{n+1}, t) \leq H(y_n, y_{n+1}, y_{n+2}, t) \leq \psi^n(H(y_0, y_1, y_2, t)$.
Now for m $>$ n, we have

$$\begin{aligned} G(y_n, y_n, y_m, t) &\geq G(y_n, y_n, y_{n+1}, t) + G(y_{n+1}, y_{n+1}, y_{n+2}, t) + ... + G(y_{m-1}, y_{m-1}, y_m, t) \\ &\geq \phi^n(G(y_0, y_1, y_2, t)) + \phi^{n+1}(G(y_0, y_1, y_2, t)) + ... + \phi^{m-1}(G(y_0, y_1, y_2, t)) \\ &\rightarrow 1 \ \ as \ \ n \rightarrow \infty \end{aligned}$$

$$\begin{aligned} H(y_n, y_n, y_m, t) &\leq H(y_n, y_n, y_{n+1}, t) + H(y_{n+1}, y_{n+1}, y_{n+2}, t) + ... + H(y_{m-1}, y_{m-1}, y_m, t) \\ &\leq \psi^n(H(y_0, y_1, y_2, t)) + \psi^{n+1}(H(y_0, y_1, y_2, t)) + ... + \psi^{m-1}(H(y_0, y_1, y_2, t)) \\ &\rightarrow 0 \ \ as \ \ n \rightarrow \infty. \end{aligned}$$

Since $\phi^n(t) \rightarrow 1$ and $\psi^n(t) \rightarrow 0 \ \ as \ \ n \rightarrow \infty$ for t $> 0. Hence \{y_n\}$ is G- Cauchy.
Suppose f(X) is G- complete. Then there exists p,t $\in$ X such that $y_{3n+2} \rightarrow p$=ft . Since $\{y_n\}$ is G- Cauchy, it follows that $y_{3n} \rightarrow$ p and $y_{3n+1} \rightarrow p \ \ as \ \ n \ \rightarrow \infty$. and G(St, Tx$_{3n+1}, Rx_{3n+2}, t)$

$$\geq \phi\left(min\begin{Bmatrix} G(ft, gx_{3n}, hx_{3n+2}, t), \frac{1}{3}[G(ft, St, Tx_{3n+1}, t) \\ +G(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, St, t)], \\ \frac{1}{4}[G(ft, Tx_{3n+1}, hx_{3n+2}, t) + G(St, gx_{3n+1}, hx_{3n+2}, t) \\ +G(ft, gx_{3n+1}, Rx_{3n+2}, t)] \end{Bmatrix}\right)$$

Letting n $\to \infty$, we get

$$G(Sp, p, p, t) \geq \phi\left(min\begin{Bmatrix} 1, \frac{1}{3}[G(p, St, p, t) + 1 + G(p, p, St, t)], \\ \frac{1}{4}[1 + G(St, p, p, t) + 1)] \end{Bmatrix}\right)$$

G(St, p, p, t) $\geq \phi(G(St, p, p, t)$and
H(St, Tx$_{3n+1}, Rx_{3n+2}, t)$

$$\leq \psi\left(max\begin{Bmatrix} H(ft, gx_{3n}, hx_{3n+2}, t), \frac{1}{3}[H(ft, St, Tx_{3n+1}, t) \\ +H(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + H(hx_{3n+2}, Rx_{3n+2}, St, t)], \\ \frac{1}{4}[H(ft, Tx_{3n+1}, hx_{3n+2}, t) + H(St, gx_{3n+1}, hx_{3n+2}, t) \\ +H(ft, gx_{3n+1}, Rx_{3n+2}, t)] \end{Bmatrix}\right)$$

Letting n $\to \infty$, we get

$$H(Sp, p, p, t) \leq \psi\left(max\begin{Bmatrix} 0, \frac{1}{3}[H(p, St, p, t) + 0 + H(p, p, St, t)], \\ \frac{1}{4}[0 + H(St, p, p, t) + 0)] \end{Bmatrix}\right)$$

H(St, p,p, t) $\leq \psi(H(St, p, p, t), since\phi$ is non decreasing and ψ is non increasing.
Hence St = p. Thus p = ft = St. Since the pairs (S, f) is weakly compatible, we have fp = Sp.
Putting $x = p, y = x_{3n+1}, z = x_{3n+2}$ in (2.1), we get
G(Sp, Tx$_{3n+1}, Rx_{3n+2}, t)$

$$\geq \phi\left(min\begin{Bmatrix} G(fp, gx_{3n+1}, hx_{3n+2}, t), \frac{1}{3}[G(fp, Sp, Tx_{3n+1}, t) \\ +G(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, Sp, t)], \\ \frac{1}{4}[G(fp, Tx_{3n+1}, hx_{3n+2}, t) + G(Sp, gx_{3n+1}, hx_{3n+2}, t) \\ +G(fp, gx_{3n+1}, Rx_{3n+2}, t)] \end{Bmatrix}\right)$$

Letting n $\to \infty$, we have

$$G(Sp, p, p, t) \geq \phi\left(min\begin{Bmatrix} G(Sp, p, p, t), \frac{1}{3}[G(Sp, Sp, p, t) + 1 + G(p, p, Sp, t)], \\ \frac{1}{4}[G(Sp, p, p, t) + G(Sp, p, p, t) + G(Sp, p, p, t)] \end{Bmatrix}\right)$$

Since G(Sp, Sp, p, t) $\geq$ 2G(Sp, p, p, t), we have G(Sp, p, p, t) $\geq \phi(G(Sp, p, p, t))$ and
H(Sp, Tx$_{3n+1}, Rx_{3n+2}, t)$

$$\leq \psi\left(max \begin{Bmatrix} H(fp, gx_{3n+1}, hx_{3n+2}, t), \frac{1}{3}[H(fp, Sp, Tx_{3n+1}, t) \\ +H(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + H(hx_{3n+2}, Rx_{3n+2}, Sp, t)], \\ \frac{1}{4}[H(fp, Tx_{3n+1}, hx_{3n+2}, t) + H(Sp, gx_{3n+1}, hx_{3n+2}, t) \\ +H(fp, gx_{3n+1}, Rx_{3n+2}, t)] \end{Bmatrix} \right)$$

Letting n $\to \infty$, we have

$$H(Sp, p, p, t) \leq \psi\left(max \begin{Bmatrix} H(Sp, p, p, t), \frac{1}{3}[H(Sp, Sp, p, t) + 0 + H(p, p, Sp, t)], \\ \frac{1}{4}[H(Sp, p, p, t) + H(Sp, p, p, t) + H(Sp, p, p, t)] \end{Bmatrix} \right)$$

Since H(Sp, Sp, p, t) $\leq$ 2H(Sp, p, p, t), we have H(Sp, p, p, t) $\leq \psi(H(Sp, p, p, t))$.
Thus Sp = p. Hence fp = Sp = p. Since p = Sp $\in g(X), there exists v \in$ X such that p = gv.
Putting x = p, y = v, z = x_{n+2} in (2.1.1) we get

$$G(Sp, Tv, Rx_{3n+2}, t) \geq \phi\left(min \begin{Bmatrix} G(fp, gv, hx_{3n+2}, t), \frac{1}{3}[G(fp, Sp, Tv, t) \\ +G(gv, Tv, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, Sp, t)], \\ \frac{1}{4}[G(fp, Tv, hx_{3n+2}, t) + G(Sp, gv, hx_{3n+2}, t) \\ +G(fp, gv, Rx_{3n+2}, t)] \end{Bmatrix} \right)$$

Letting n $\to \infty$, we deduce that

$$G(p, Tv, p, t) \geq \phi\left(min \begin{Bmatrix} 1, \frac{1}{3}[G(p, p, Tv, t) + G(p, Tv, p, t) + 1], \\ \frac{1}{4}[G(p, Tv, p, t) + 1 + 1] \end{Bmatrix} \right) \geq \phi(G(p, Tv, p, t) \text{and}$$

$$H(Sp, Tv, Rx_{3n+2}, t) \leq \psi\left(max \begin{Bmatrix} H(fp, gv, hx_{3n+2}, t), \frac{1}{3}[H(fp, Sp, Tv, t) \\ +H(gv, Tv, Rx_{3n+2}, t) + H(hx_{3n+2}, Rx_{3n+2}, Sp, t)], \\ \frac{1}{4}[H(fp, Tv, hx_{3n+2}, t) + H(Sp, gv, hx_{3n+2}, t) \\ +H(fp, gv, Rx_{3n+2}, t)] \end{Bmatrix} \right)$$

Letting n $\to \infty$, we deduce that

$$H(p, Tv, p, t) \leq \psi\left(max \begin{Bmatrix} 0, \frac{1}{3}[H(p, p, Tv, t) + H(p, Tv, p, t) + 0], \\ \frac{1}{4}[H(p, Tv, p, t) + 0 + 0] \end{Bmatrix} \right) \leq \psi(H(p, Tv, p, t)$$

Since ϕ is non decreasing and ψ is non increasing. Thus Tv = p, so that p = Tv = gv.
Since the pair (T,g) is weakly compatible, we have Tp = gp.

$$G(Sp, Tp, Rx_{3n+2}, t) \geq \phi\left(min \begin{Bmatrix} G(fp, gp, hx_{3n+2}, t), \frac{1}{3}[G(fp, Sp, Tp, t) \\ +G(gp, Tp, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, Sp, t)], \\ \frac{1}{4}[G(fp, Tp, hx_{3n+2}, t) + G(Sp, gp, hx_{3n+2}, t) \\ +G(fp, gp, Rx_{3n+2}, t)] \end{Bmatrix} \right)$$

Letting n $\to \infty$, we have

$$G(p,Tp,p,t) \geq \phi\left(min\begin{Bmatrix} G(p,Tp,p,t), \frac{1}{3}[G(p,p,Tp,t)+G(Tp,Tp,p,t)+1], \\ \frac{1}{4}[G(p,Tp,p,t)+G(p,Tp,p,t)+G(p,Tp,p,t)] \end{Bmatrix}\right)$$

Since G(Tp, Tp, p, t)$\geq 2G(Tp,p,p,t)$,we have , G(p, Tp, p, t) $\geq \phi(G(p,Tp,p,t))$.

$$H(Sp,Tp,Rx_{3n+2},t) \leq \psi\left(max\begin{Bmatrix} H(fp,gp,hx_{3n+2},t), \frac{1}{3}[H(fp,Sp,Tp,t) \\ +H(gp,Tp,Rx_{3n+2},t)+H(hx_{3n+2},Rx_{3n+2},Sp,t)], \\ \frac{1}{4}[H(fp,Tp,hx_{3n+2},t)+H(Sp,gp,hx_{3n+2},t) \\ +H(fp,gp,Rx_{3n+2},t)] \end{Bmatrix}\right)$$

Letting n $\to \infty$, we have

$$H(p,Tp,p,t) \leq \psi\left(max\begin{Bmatrix} H(p,Tp,p,t), \frac{1}{3}[H(p,p,Tp,t)+H(p,Tp,p,t)+0], \\ \frac{1}{4}[H(p,Tp,p,t)+H(p,Tp,p,t)+H(p,Tp,p,t)] \end{Bmatrix}\right)$$

Since H(Tp, Tp, p, t)$\leq 2H(Tp,p,p,t), wehave, H(p,Tp,p,t) \leq \psi(H(p,Tp,p,t))$.
Thus Tp = p. Hence gp = Tp = p. Since p = Tp$\in$ h(X), there exists w $\in$ X such that p = hw.
Putting x = p, y = p, z = w in (2.1.1) we get

$$G(Sp,Tp,Rw,t) \geq \phi\left(min\begin{Bmatrix} G(fp,gp,hw,t), \frac{1}{3}[G(fp,Sp,Tp,t) \\ +G(gp,Tp,Rw,t)+G(hw,Rw,Sp,t)], \\ \frac{1}{4}[G(fp,Tp,hw,t)+G(Sp,gp,hw,t) \\ +G(fp,gp,Rw,t)] \end{Bmatrix}\right)$$

$$G(p,p,Rw,t) \geq \phi\left(min\begin{Bmatrix} 1, \frac{1}{3}[1+G(p,p,Rw,t)+G(p,Rw,p,t)], \\ \frac{1}{4}[1+1+G(p,p,Rw,t)] \end{Bmatrix}\right) \geq \phi(G(p,p,Rw,t))$$

$$H(Sp,Tp,Rw,t) \leq \psi\left(max\begin{Bmatrix} H(fp,gp,hw,t), \frac{1}{3}[H(fp,Sp,Tp,t) \\ +H(gp,Tp,Rw,t)+H(hw,Rw,Sp,t)], \\ \frac{1}{4}[H(fp,Tp,hw,t)+H(Sp,gp,hw,t) \\ +H(fp,gp,Rw,t)] \end{Bmatrix}\right)$$

$$H(p,p,Rw,t) \leq \psi\left(max\begin{Bmatrix} 0, \frac{1}{3}[0+H(p,p,Rw,t)+H(p,Rw,p,t)], \\ \frac{1}{4}[0+0+H(p,p,Rw,t)] \end{Bmatrix}\right) \leq \psi(H(p,p,Rw,t))$$

Since ϕ is non decreasing and ψ is non increasing. Thus Rw= p, so that p = hw = Rw.
Since the pair (R, h) is weakly compatible, we have Rp = hp.
Putting x = p, y = p, z = p in (2.1.1) we get,

$$G(p,p,Rp,t) = G(Sp,Tp,Rp,t) \geq \phi\left(min\left\{ \begin{array}{r} G(fp,gp,Rp,t), \frac{1}{3}[1 + G(p,p,Rp,t) \\ +G(Rp,Rp,p,t)], \frac{1}{4}[G(p,p,Rp,t) \\ +G(p,p,Rp,t) + G(p,p,Rp,t)] \end{array} \right\} \right)$$

$$H(p,p,Rp,t) = H(Sp,Tp,Rp,t) \leq \psi\left(max\left\{ \begin{array}{r} H(fp,gp,Rp,t), \frac{1}{3}[0 + H(p,p,Rp,t) \\ +H(Rp,Rp,p,t)], \frac{1}{4}[H(p,p,Rp,t) \\ +H(p,p,Rp,t) + H(p,p,Rp,t)] \end{array} \right\} \right)$$

Since G(Rp, Rp, p, t) $\geq$ 2G(p, p, Rp, t) and H(Rp, Rp, p, t) $\leq$ 2H(p, p, Rp, t).
We have G(p, p, Rp, t) $\geq \phi(G(p,p,Rp,t))$and H(p, p, Rp, t) $\leq \psi$ (H(p, p, Rp, t)).
Thus Rp = p, so that Rp = hp = p. It follows that p is a common fixed point of S, T, R, f, g and h. Uniqueness of common fixed point follows easily from (4) and (5). Similarly, we can prove the theorem when g(X) or h(X) is a complete subspace of X.

Corollary 2.2 Let (X, G, H, $*$, $\diamond$) be an intuitionistic generalized fuzzy metric space and S, T, R, f, g, h : X $\rightarrow$ X be satisfying

(i) S(X) $\subseteq g(X), T(X)$and R(X) $\subseteq$ f(X),

(ii) one of f(X), g(X) and h(X) is a complete subspace of X,

(iii) The pairs (S,f), (T,g) and (R,h) are weakly compatible and

(iv) G(Sx, Ty, Rz, t) $\geq \phi(G(fx,gy,hz,t)$and H(Sx, Ty, Rz, t) $\leq \psi$ (H(fx, gy, hz, t)) for all x, y, z $\in$ X, where $\phi \in \Phi, \psi \in \Psi$.

Then the maps S, T, R, f, g and h have a unique fixed point in X.

Corollary 2.3 Let (X,G,H,$*$,$\diamond$) be an intuitionistic generalized fuzzy metric space and S, T, R : X $\rightarrow$ X be satisfying G(Sx, Ty, Rz, t) $\geq \phi(G(x,y,z,t)$ and H(Sx, Ty, Rz, t) $\leq \psi$ (H(x, y, z, t)) for all x, y, z $\in$ X, where $\phi \in \Phi, \psi \in \Psi$. Then the maps S, T and R have a unique common fixed point in p $\in$ X and S, T, and R are G, H continuous at p.
Proof: There exists p$\in$ X such that p is the unique common fixed point of S, T and R as in Theorem 2.1 Let $\{y_n\}$ be any sequence in X which G, H converges to p. Then
G(Sy$_n, Sp, Sp, t) = G(Sy_n, Tp, Rp, t) \leq \phi$(G(y$_n, p, p, t)) \rightarrow 1$ $as\ n \rightarrow \infty\ and$
$H(Sy_n, Sp, Sp, t) = H(Sy_n, Tp, Rp, t) \geq \psi$ (H(y$_n, p, p, t)) \rightarrow 0$ $as\ n \rightarrow \infty$
Hence S is G, H- continuous at p. Similarly, we can show that T and R are also G, H- continuous at p.

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