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A note on Konhauser matrix polynomials

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Abstract. In this study, some new and known finite sums, operational formulas and generating matrix relations for Konhauser matrix polynomials of the second kind are given.

1 Introduction and Preliminaries

For various extensions of the properties of Konhauser matrix polynomials are motivated by researchers, the reader may be recalled in [1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Motivated by these contributions, the aim of this study is to obtain new and known interesting results of Konhauser matrix polynomials and also to prove the results involving an infinite series, finite sums, operational formulas and generating matrix relations of these matrix polynomials.

Throughout this paper, its spectrum $\sigma(A)$ symbolizes the set of all eigenvalues of a matrix A in $\mathbb{C}^{N\times N}$.

Definition 1.1. If P is a positive stable matrix in $\mathbb{C}^{N\times N}$, then Jódar and Cortés [2] defined the Gamma matrix function $\Gamma(P)$ as

$$\Gamma(P)=\int_0^\infty e^{-t}t^{P-I}dt;\quad t^{P-I}=\exp\Big((P-I)\ln t\Big),$$

where I symbolize the identity matrix in $\mathbb{C}^{N\times N}$.

Definition 1.2. For $A\in\mathbb{C}^{N\times N}$, Jódar and Cortés [3] defined the following Pochhammer symbol or shifted factorial as

$$(A)_n=A(A+I)(A+2I)\dots(A+(n-1)I)=\Gamma(A+nI)\Gamma^{-1}(A);\quad n\geq 1;\quad (A)_0=I$$

where $\Gamma(A)$ is an invertible matrix, its inverse coincides with $\Gamma^{-1}(A)$ such that $\sigma(A)\cap\mathbb{Z}_{\leq 0}=\emptyset$.

Recently, Varma et al. [12] defined the following Konhauser matrix polynomials of the second kind as:

$$Y_n^{(A,\lambda)}(x;k)=\frac{1}{n!}\sum_{r=0}^{n}\sum_{s=0}^{r}\frac{(-1)^s(\lambda x)^r}{s!(r-s)!}\left(\frac{1}{k}((s+1)I+A)\right)_n \tag{1.1}$$

where A is a matrix in $\mathbb{C}^{N\times N}$ such that the condition

$$Re(\mu)>-1,\quad \text{for all eigenvalues } \mu\in\sigma(A) \tag{1.2}$$

where λ is a complex number with $Re(\lambda) > 0$, $k \in \mathbb{N} = \mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ in [12]. Varma et al. [12] introduced the generating matrix function for Konhauser matrix polynomials of the second kind as

$$\sum_{n=0}^{\infty} Y_n^{(A,\lambda)}(x;k)t^n = (1-t)^{-\frac{1}{k}(A+I)} \exp\left[\lambda\, x[1-(1-t)^{-\frac{1}{k}}]\right]; |t| < 1. \tag{1.3}$$

Recently, Shehata [8] obtained the Rodrigues formula for Konhauser matrix polynomials $Y_n^{(A,\lambda)}(x;k)$ as

$$Y_n^{(A,\lambda)}(x;k) = \frac{1}{n!k^n} x^{-A-(kn+1)I} \exp\left(\lambda\, x\right)\left(x^{k+1}D\right)^n \left[x^{A+I} \exp\left(-\lambda\, x\right)\right]; D = \frac{d}{dx}. \tag{1.4}$$

2 Some new results of Konhauser matrix polynomials

Here, we apply the generating matrix function of Konhauser matrix polynomials to obtain an infinite series, finite sums, operational formulas and generating matrix relations for $Y_n^{(A,\lambda)}(x;k)$ with taking matrices the condition (1.2) is realized. The results to be obtained from our main findings are given in the following theorems.

Theorem 2.1. *Konhauser matrix polynomials $Y_n^{(A,\lambda)}(x;k)$ satisfy the summation formula*

$$Y_n^{(A,\lambda)}(x;k) = \sum_{r=0}^{n} \frac{1}{r!}\left(\frac{1}{k}(A-B)\right)_r Y_{n-r}^{(B+krI,\lambda)}(x;k), \tag{2.1}$$

where the matrices A and $B + krI$ for $k \in \mathbb{N}$ and $r = \mathbb{N} \cup 0 = \mathbb{N}_{\not{0}}$ in $\mathbb{C}^{N\times N}$ satisfying the condition (1.2).

Proof. Note that, we have the relation

$$\begin{aligned}
\left(1-t\right)^{-\frac{1}{k}(A+I)} &= \left(1-t\right)^{-\frac{1}{k}(A-B)}\left(1-t\right)^{-\frac{1}{k}(B+I)}\\
&= \left(1-t\right)^{-\frac{1}{k}(B+I)} \sum_{r=0}^{\infty} \frac{(-1)^r}{r!}\left(\frac{1}{k}(B-A)\right)_r \left(\frac{t}{1-t}\right)^r\\
&= \sum_{r=0}^{\infty} \frac{(-1)^r}{r!}\left(\frac{1}{k}(B-A)\right)_r \left(1-t\right)^{-\frac{1}{k}(B+(rk+1)I)} t^r.
\end{aligned}$$

From the generating matrix function (1.3) and the above relation, we observe

$$\begin{aligned}
&(1-t)^{-\frac{1}{k}(A+I)} \exp\left[\lambda\, x\left(1-(1-t)^{-\frac{1}{k}}\right)\right]\\
&= \sum_{n,r=0}^{\infty} \frac{(-1)^r}{r!}\left(\frac{1}{k}(B-A)\right)_r Y_n^{(B+rkI,\lambda)}(x;k)t^{n+r}\\
&= \sum_{n=0}^{\infty}\sum_{r=0}^{n} \frac{1}{r!}\left(\frac{1}{k}(A-B)\right)_r Y_{n-r}^{(B+rkI,\lambda)}(x;k)t^n
\end{aligned}$$

whence we obtain

$$\sum_{n=0}^{\infty} Y_n^{(A,\lambda)}(x;k)t^n = \sum_{n=0}^{\infty}\sum_{r=0}^{n} \frac{1}{r!}\left(\frac{1}{k}(A-B)\right)_r Y_{n-r}^{(B+rkI,\lambda)}(x;k)t^n.$$

Comparing the coefficients of t^n in the last equation, we obtain the desired relation (2.1). □

As the approach in the proof Theorem 2.1, one can easily see the next results.

Theorem 2.2. *The following are the finite sums for the Konhauser matrix polynomials :*

$$Y_n^{(A,\lambda)}(x;k)=\sum_{r=0}^{n-1}\frac{(-1)^r(n-1)!}{r!(n-r-1)!}Y_{n-r}^{(A+(kn-k)I,\lambda)}(x;k), \tag{2.2}$$

and

$$Y_n^{(A,\lambda)}(x;k)=\sum_{r=0}^{n-1}\frac{(n-1)!}{r!(n-r-1)!}Y_{n-r}^{(A+(k-kn+kr)I,\lambda)}(x;k). \tag{2.3}$$

Now, we give various generating matrix relations for the Konhauser matrix polynomials in the following theorem.

Theorem 2.3. *The following generating matrix relations for Konhauser matrix polynomials hold:*

$$\sum_{n=0}^{\infty}Y_n^{(A-knI,\lambda)}(x;k)=(1+t)^{\frac{1}{k}(A-(k-1)I)}\exp\left[\lambda\, x\left(1-(1+t)^{\frac{1}{k}}\right)\right], \tag{2.4}$$

and

$$\begin{aligned}&\sum_{n=0}^{\infty}\frac{(n+r)!}{n!r!}Y_{n+r}^{(A-knI,\lambda)}(x;k)\\&=(1+t)^{\frac{1}{k}(A-(k-1)I)}\exp\left[\lambda\, x\left(1-(1+t)^{\frac{1}{k}}\right)\right]Y_r^{(A,\lambda)}\left(x(1+t)^{\frac{1}{k}};k\right).\end{aligned} \tag{2.5}$$

Proof. From (1.4) and putting $u=x^{-k}$, we have

$$\begin{aligned}\sum_{n=0}^{\infty}Y_n^{(A-knI,\lambda)}(x;k)t^n&=\sum_{n=0}^{\infty}\frac{1}{n!k^n}x^{-A-I}\exp\left(\lambda\, x\right)\left(x^{k+1}D\right)^n\left[x^{A-(kn-1)I}\exp\left(-\lambda\, x\right)\right]t^n\\&=x^{-A-I}\exp\left(\lambda\, x\right)\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{d^n}{du^n}\left[u^{-\frac{1}{k}(A+I)+nI}\exp\left(-\lambda\, u^{-\frac{1}{k}}\right)\right].\end{aligned}$$

Using Lagrange theorem, we get

$$\begin{aligned}&\sum_{n=0}^{\infty}Y_n^{(A-knI,\lambda)}(x;k)t^n\\&=x^{-A-I}\exp\left(\lambda\, x\right)\left[(u(1+t)^{-1})^{-\frac{1}{k}(A+I)+nI}\exp\left(-\lambda\,(u(1+t)^{-1})^{-\frac{1}{k}}\right)\right](1+t)^{-1}\\&=(1+t)^{\frac{1}{k}(A-(k-1)I)}\exp\left[\lambda\, x\left(1-(1+t)^{\frac{1}{k}}\right)\right]\end{aligned}$$

which proves (2.4).

Again consider the left hand side of (2.5) and letting $u=x^{-k}$, we have

$$\begin{aligned}&\sum_{n=0}^{\infty}\frac{(n+r)!}{n!r!}Y_{n+r}^{(A-knI,\lambda)}(x;k)t^n\\&=\sum_{n=0}^{\infty}\frac{(n+r)!}{n!r!(n+r)!k^{n+r}}x^{-A-(kr+1)I}\exp\left(\lambda\, x\right)\left(x^{k+1}D\right)^{n+r}\left[x^{A-(kn-1)I}\exp\left(-\lambda\, x\right)\right]t^n\\&=\frac{1}{r!k^r}x^{-A-(kr+1)I}\exp\left(\lambda\, x\right)\left(x^{k+1}D\right)^r\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{d^n}{du^n}\left[u^{-\frac{1}{k}(A+I)+nI}\exp\left(-\lambda\, u^{-\frac{1}{k}}\right)\right].\end{aligned}$$

Using Lagrange theorem, we get

$$\sum_{n=0}^{\infty}\frac{(n+r)!}{n!r!}Y_{n+r}^{(A-knI,\lambda)}(x;k)t^{n}$$
$$=\frac{1}{r!k^{r}}x^{-A-(kr+1)I}\exp\left(\lambda\,x\right)\left(x^{k+1}D\right)^{r}\left[(u(1+t)^{-1})^{-\frac{1}{k}(A+I)}\exp\left(-\lambda\left(u(1+t)^{-1}\right)^{-\frac{1}{k}}\right)\right](1+t)^{-1}$$
$$=(1+t)^{\frac{1}{k}(A-(k-1)I)}\exp\left[\lambda\,x\left(1-(1+t)^{\frac{1}{k}}\right)\right]Y_{r}^{(A,\lambda)}\left(x(1+t)^{\frac{1}{k}};k\right)$$

which proves (2.5). □

Theorem 2.4. *The following result for Konhauser matrix polynomials holds true*

$$DY_{n}^{(A,\lambda)}(x;k)=\lambda Y_{n}^{(A,\lambda)}(x;k)-\lambda\sum_{m=0}^{n}\frac{1}{m!}\left(\frac{1}{k}\right)_{m}Y_{n-m}^{(A,\lambda)}(x;k). \tag{2.6}$$

Proof. Differentiating both sides in (1.3) with respect to x, we get

$$\sum_{n=0}^{\infty}\frac{\partial}{\partial x}Y_{n}^{(A,\lambda)}(x;k)t^{n}=\left[\lambda\left[1-(1-t)^{-\frac{1}{k}}\right]\right](1-t)^{-\frac{1}{k}(A+I)}\exp\left[\lambda\,x[1-(1-t)^{-\frac{1}{k}}]\right]$$
$$=\lambda\sum_{n=0}^{\infty}Y_{n}^{(A,\lambda)}(x;k)t^{n}-\lambda\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{1}{k}\right)_{m}Y_{n}^{(A,\lambda)}(x;k)t^{n+m}$$
$$=\lambda\sum_{n=0}^{\infty}Y_{n}^{(A,\lambda)}(x;k)t^{n}-\lambda\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{1}{m!}\left(\frac{1}{k}\right)_{m}Y_{n-m}^{(A,\lambda)}(x;k)t^{n}$$

which upon equating the coefficients of t^n, yields the desired result. □

Theorem 2.5. *The following formula for Konhauser matrix polynomials holds true*

$$(n+1)Y_{n+1}^{(A,\lambda)}(x;k)=\sum_{m=0}^{n}\frac{1}{k}\left[A+I-\frac{\lambda x}{m!}\left(\frac{1+k}{k}\right)_{m}I\right]Y_{n-m}^{(A,\lambda)}(x;k). \tag{2.7}$$

Proof. Differentiating both sides in (1.3) with respect to t, we get

$$\sum_{n=1}^{\infty}nY_{n}^{(A,\lambda)}(x;k)t^{n-1}=\frac{1}{k}(A+I)(1-t)^{-\frac{1}{k}(A+I)-I}\exp\left[\lambda\,x[1-(1-t)^{-\frac{1}{k}}]\right]$$
$$-\frac{\lambda x}{k}(1-t)^{-\frac{1+k}{k}}(1-t)^{-\frac{1}{k}(A+I)}\exp\left[\lambda\,x[1-(1-t)^{-\frac{1}{k}}]\right]$$
$$=\frac{1}{k}(A+I)\sum_{n=0}^{\infty}\sum_{m=0}^{n}Y_{n-m}^{(A,\lambda)}(x;k)-\frac{\lambda x}{k}\sum_{n=0}^{\infty}\sum_{p=0}^{n}\frac{1}{p!}\left(\frac{1+k}{k}\right)_{p}Y_{n-p}^{(A,\lambda)}(x;k)t^{n}$$
$$=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left[\frac{1}{k}(A+I)-\frac{\lambda x}{k}\frac{1}{m!}\left(\frac{1+k}{k}\right)_{m}I\right]Y_{n-m}^{(A,\lambda)}(x;k)t^{n}.$$

Comparing the coefficients of t^n, we arrive the desired result. □

Theorem 2.6. *The following interesting identities for Konhauser matrix polynomials hold true*

$$Y_{n}^{(A,\lambda)}(x;k)=\frac{1}{n!k^{n}}\prod_{m=0}^{n-1}\left(x(D-\lambda)I+A+(mk+1)I\right) \tag{2.8}$$

and

$$\sum_{m=0}^{n}\frac{n!}{m!}\Big(k^{n-m}x^{-mk}Y_{n-m}^{(A,\lambda)}(x;k)\Big)\Big(x^{k+1}D\Big)^{m}=\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big). \tag{2.9}$$

Proof. Since and repeating this process upto n times, we get

$$\begin{aligned}
\Big(x^{k+1}D\Big)^{n}\Big[\Big(x^{A+I}e^{-\lambda x}\Big)f\Big]&=\Big(x^{k+1}D\Big)^{n-1}\Big(x^{k+1}D\Big)\Big[\Big(x^{A+I}e^{-\lambda x}\Big)f\Big]\\
&=\Big(x^{k+1}D\Big)^{n-1}x^{k+1}\Big[(A+I)\Big(x^{A}e^{-\lambda x}\Big)f-\lambda\Big(x^{A+I}e^{-\lambda x}\Big)f+\Big(x^{A+I}e^{-\lambda x}\Big)Df\Big]\\
&=\Big(x^{k+1}D\Big)^{n-1}\Big[\Big(x^{A+(k+1)I}e^{-\lambda x}\Big)\Big(xD-\lambda xI+A+I\Big)f\Big]\\
&=\Big(x^{k+1}D\Big)^{n-2}x^{k+1}\Big[(A+(k+1)I)\Big(x^{A+kI}e^{-\lambda x}\Big)-\lambda\Big(x^{A+(k+1)I}e^{-\lambda x}\Big)\\
&+\Big(x^{A+(k+1)I}e^{-\lambda x}\Big)D\Big(xD-\lambda xI+A+I\Big)f\Big]
\end{aligned}$$

$$\begin{aligned}
&=\Big(x^{k+1}D\Big)^{n-2}\Big[\Big(x^{A+(2k+1)I}e^{-\lambda x}\Big)\Big(xDI-\lambda xI+A+(k+1)I\Big)\Big(xDI-\lambda xI+A+I\Big)f\Big]\\
&=\Big[\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\Big(xDI-\lambda xI+A+((n-1)k+1)I\Big)\ldots\Big(xDI-\lambda xI+A+(2k+1)I\Big)\\
&\Big(xDI-\lambda xI+A+(k+1)I\Big)\Big(xDI-\lambda xI+A+I\Big)f\Big].
\end{aligned}$$

Thus, we derive

$$\Big(x^{k+1}D\Big)^{n}\Big[\Big(x^{A+I}e^{-\lambda x}\Big)f\Big]=\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big)f. \tag{2.10}$$

When $f=1$, we have

$$\Big(x^{k+1}D\Big)^{n}\Big(x^{A+I}e^{-\lambda x}\Big)=\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big). \tag{2.11}$$

From (1.4) and (2.11), we obtain

$$n!k^{n}x^{A+(kn+1)I}e^{-\lambda x}Y_{n}^{(A,\lambda)}(x;k)=\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big).$$

Hence

$$Y_{n}^{(A,\lambda)}(x;k)=\frac{1}{n!k^{n}}\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big),$$

which proves (2.8)

Using Leibnitz rule of differentiation and (2.10), we have

$$\Big(x^{k+1}D\Big)^n\Big[\Big(x^{A+I}e^{-\lambda x}\Big)f\Big]=\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}\Big(x^{k+1}D\Big)^{n-m}\Big(x^{A+I}e^{-\lambda x}\Big)\Big(x^{k+1}D\Big)^m f$$
$$=\sum_{m=0}^{n}\frac{n!}{m!}\Big(k^{n-m}x^{A+(k(n-m)+1)I}\exp\big(-\lambda\, x\big)Y_{n-m}^{(A,\lambda)}(x;k)\Big)\Big(x^{k+1}D\Big)^m f. \tag{2.12}$$

From (2.10) and (2.12), we get the operational formula

$$\begin{aligned}&\sum_{m=0}^{n}\frac{n!}{m!}\Big(k^{n-m}x^{A+(k(n-m)+1)I}\exp\big(-\lambda\, x\big)Y_{n-m}^{(A,\lambda)}(x;k)\Big)\Big(x^{k+1}D\Big)^m f\\&=\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big)f.\end{aligned} \tag{2.13}$$

When $f=1$ in the above equation, we get

$$\begin{aligned}&\sum_{m=0}^{n}\frac{n!}{m!}\Big(k^{n-m}x^{A+(k(n-m)+1)I}\exp\big(-\lambda\, x\big)Y_{n-m}^{(A,\lambda)}(x;k)\Big)\Big(x^{k+1}D\Big)^m\\&=\Big(x^{A+(nk+1)I}e^{-\lambda x}\Big)\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big).\end{aligned}$$

Hence

$$\sum_{m=0}^{n}\frac{n!}{m!}\Big(k^{n-m}x^{-mk}Y_{n-m}^{(A,\lambda)}(x;k)\Big)\Big(x^{k+1}D\Big)^m=\prod_{m=0}^{n-1}\Big(x(D-\lambda)I+A+(mk+1)I\Big),$$

which proves (2.9). □

Theorem 2.7. *The following identity for Konhauser matrix polynomials holds true*

$$D^m\Big[\exp\big(-\lambda\, x\big)Y_n^{(A,\lambda)}(x;k)\Big]=(-\lambda)^m\exp\big(-\lambda\, x\big)Y_n^{(A+mI,\lambda)}(x;k). \tag{2.14}$$

Proof. Differentiating relation (1.3) with respect to x, we have

$$\sum_{n=0}^{\infty}DY_n^{(A,\lambda)}(x;k)t^n=\lambda\Big[1-(1-t)^{-\frac{1}{k}}\Big](1-t)^{-\frac{1}{k}(A+I)}\exp\Big[\lambda\, x[1-(1-t)^{-\frac{1}{k}}]\Big]$$
$$=\lambda\sum_{n=0}^{\infty}Y_n^{(A,\lambda)}(x;k)t^n-\lambda\sum_{n=0}^{\infty}Y_n^{(A+I,\lambda)}(x;k)t^n.$$

Equating the coefficients of t^n both sides, we get

$$DY_n^{(A,\lambda)}(x;k)t^n=\lambda Y_n^{(A,\lambda)}(x;k)-\lambda Y_n^{(A+I,\lambda)}(x;k)$$

or

$$\Big[D-\lambda\Big]Y_n^{(A,\lambda)}(x;k)=-\lambda Y_n^{(A+I,\lambda)}(x;k). \tag{2.15}$$

Using the formula (2.15), we get

$$D\Big[\exp\big(-\lambda\, x\big)Y_n^{(A,\lambda)}(x;k)\Big] = \exp\big(-\lambda\, x\big)DY_n^{(A,\lambda)}(x;k) - \lambda\exp\big(-\lambda\, x\big)Y_n^{(A,\lambda)}(x;k)$$

$$= \exp\big(-\lambda\, x\big)\Big[D-\lambda\Big]Y_n^{(A,\lambda)}(x;k)$$

or

$$D\Big[\exp\big(-\lambda\, x\big)Y_n^{(A,\lambda)}(x;k)\Big] = -\lambda\exp\big(-\lambda\, x\big)Y_n^{(A+I,\lambda)}(x;k). \tag{2.16}$$

Repeating operation of D, m times, we get

$$D^m\Big[\exp\big(-\lambda\, x\big)Y_n^{(A,\lambda)}(x;k)\Big] = (-\lambda)^m\exp\big(-\lambda\, x\big)Y_n^{(A+mI,\lambda)}(x;k).$$

This proves (2.14). □

We concluded this investigation by remarking that by obtaining the finite sums, operational formulas and generating matrix relations for Konhauser matrix polynomials of the second kind.

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