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Group $\{1, -1, i, -i\}$ Cordial Labeling of Special Graphs

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Abstract Let G be a (p, q) graph and A be a group. Let $f : V(G) \to A$ be a function. The order of $u \in A$ is the least positive integer n such that $u^n = e$. We denote the order of u by $o(u)$. For each edge uv assign the label 1 if $(o(f(u)), o(f(v))) = 1$ or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $n(n = 0, 1)$. A graph which admits a group A Cordial labeling is called a group A Cordial graph. In this paper we define group $\{1, -1, i, -i\}$ Cordial graphs and prove that the Jahangir graph $J_{3,n}(n \geq 3)$,the Jelly fish graphs $J(m, n)(m \leq n)$, the Dumbbell graph Db_n and the Flower graph Fl_n are all group $\{1, -1, i, -i\}$ Cordial for every n .

1 Introduction

Graphs considered here are finite, undirected and simple. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$. Cahit [3] introduced the concept of Cordial labeling. Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group $\{1, -1, i, -i\}$ cordial labeling and discussed that labeling for some standard graphs [1, 2] . In this paper we discuss the labeling for the Jahangir graph $J_{3,n}(n \geq 3)$, the Jelly fish graphs $J(m, n)(m \leq n)$, the Dumbbell graph Db_n and the Flower graph Fl_n . Terms not defined here are used in the sense of Harary[5] and Gallian [4].

The greatest common divisor of two integers m and n is denoted by (m, n) and m and n are said to be *relatively prime* if $(m, n) = 1$. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

A *path* is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n$, which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \leq i \leq n - 1$. A path on n vertices is denoted by P_n. A path $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n, e_n, v_1$ is called a cycle and a cycle on n vertices is denoted by C_n.

Given two graphs G and H, $G + H$ is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv/u \in V(G), v \in V(H)\}$. A Wheel W_n is defined as $C_n + K_1$ and the graph obtained by subdividing the edges on the cycle of a wheel exactly once is called the Gear graph. The Helm H_n is the graph obtained from a wheel W_n by attaching a pendent edge at each vertex of the $n-$ cycle.

2 Group $\{1, -1, i, -i\}$ Cordial graphs

Definition 2.1. Let G be a (p, q) graph and consider the group $A = \{1, -1, i, -i\}$ with multiplication. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if $(o(f(u)), o(f(v))) =$ 1or 0 otherwise. f is called a group $\{1, -1, i, -i\}$ Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $n(n = 0, 1)$. A graph which admits a group $\{1, -1, i, -i\}$ Cordial labeling is called a group $\{1, -1, i, -i\}$ Cordial graph.

Example 2.2. A simple example of a group $\{1, -1, i, -i\}$ Cordial graph is given in Fig. 2.1.

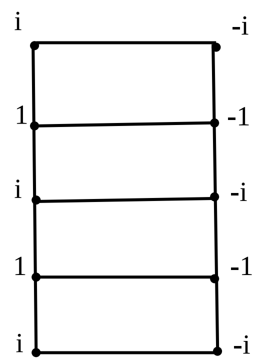

Fig. 2.1

We now investigate the group $\{1, -1, i, -i\}$ Cordial labeling of some (p, q) graphs.
The Jahangir graph $J_{m,n}(n \geq 3)$ was introduced by Surahmar and Tomescu[6] in 2006.

Definition 2.3. The Jahangir graph $J_{m,n}(n \geq 3)$ is a graph with $mn + 1$ vertices, consisting of a cycle C_{mn} with one additional vertex which is adjacent to n vertices of C_{mn} at distance m to each other on C_{mn}.

Remark 2.4. The Jahangir graph $J_{1,n}$ is the Wheel and $J_{2,n}$ is the gear graph.

Theorem 2.5. *The Jahangir graph* $J_{3,n}$ $(n \geq 3)$ *is group* $\{1, -1, i, -i\}$ *cordial for all* n.

Proof. Let the vertices on the cycle be labeled as $u_1, u_2, ..., u_{3n}$ and let the central vertex be labeled as w. Assume that w is adjacent to $u_i(i \equiv 1(mod\, 3))$. Number of vertices = $3n + 1$ and number of edges = $4n$. Group $\{1, -1, i, -i\}$ cordial labelings for $n = 3$ and $n = 4$ are given in Table 1. Suppose $n \geq 5$.

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	w
3	-1	1	1	-1	1	-1	i	i	$-i$				$-i$
4	-1	1	1	-1	1	1	-1	i	i	i	$-i$	$-i$	$-i$

Table 1

Case(i): $3n + 1 \equiv 0(mod\, 4)$.
Let $3n + 1 = 4k(k \in \mathbb{Z})$. Each vertex label should appear k times and each edge label should appear $\frac{8k-2}{3}$ times in a group $\{1, -1, i, -i\}$ cordial labeling. Note that $k = 3r+1(r \in \mathbb{Z}, r \geq 1)$. So the vertices on the cycle are $u_i(1 \leq i \leq 12r + 3)$ where $u_i(i \equiv 1(mod\, 3), 1 \leq i \leq 12r + 3)$ are of degree 3 and others are of degree 2. Label the vertices $u_i(1 \leq i \leq 6r - 2\, , i \equiv 1(mod\, 3))$ with 1. Also choose $r + 1$ vertices among $u_i(6r \leq i \leq 12r + 3, i \neq 1(mod\, 3))$ and give them label 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = 3 \times 2r + (r + 1)2 = \frac{8k-2}{3}$.

Case(ii): $3n + 1 \equiv 1(mod\, 4)$.
Let $3n + 1 = 4k + 1(k \in \mathbb{Z})$. Three vertex labels should appear k times and one vertex label should appear $k + 1$ times. Each edge label should appear $\frac{8k}{3}$ times in a group $\{1, -1, i, -i\}$ cordial labeling. In this case $k = 3r(r \in \mathbb{Z}, r \geq 2)$. Now the vertices on the cycle are

$u_i(1 \leq i \leq 12r)$ where $u_i(i \equiv 1(mod\ 3))$ are of degree 3 and others are of degree 2. Label the vertices $u_i(1 \leq i \leq 6r-8\ ,\ i \equiv 1(mod\ 3))$ with 1. Also choose $r+3$ vertices $u_i(6r-6 \leq l \leq 12r, i \neq 1(mod\ 3))$ and give them label 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = 3 \times (2r-2) + (r+3)2 = 8r$.

Case(iii): $3n+1 \equiv 2(mod\ 4)$
Let $3n+1 = 4k+2(k \in \mathbb{Z})$. Two vertex labels should appear k times and 2 vertex labels should appear $k+1$ times. Each edge label should appear $2n = \frac{8k+2}{3}$ times. Now $k = 3r+2(r \geq 1, r \in \mathbb{Z})$. The vertices on the cycle are $u_i(1 \leq i \leq 12r+9)$. Label the vertices $u_i(1 \leq i \leq 6r-2, i \equiv 1(mod\ 3))$ with 1. Also choose $r+3$ vertices among $u_i(6r \leq i \leq 12r+9, l \neq 1(mod\ 3))$ and give them label 1. Label the remaining vertices arbitrarily so that $k+1$ vertices get label -1, k vertices get label i and k vertices get label $-i$. Number of edges with label $1 = 2r \times 3 + 2(r+3) = 8r+6$.

Case(iv): $3n+1 \equiv 3(mod\ 4)$
Let $3n+1 = 4k+3(k \in \mathbb{Z})$. Three vertex labels should appear $k+1$ times and 1 vertex label should appear k times. Each edge label should appear $\frac{8k+4}{3}$ times. Now $k = 3r+1(r \geq 1, r \in \mathbb{Z})$. The vertices on the cycle are $u_i(1 \leq i \leq 12r+6)$. Label the vertices $u_i(1 \leq i \leq 6r-2, l \equiv (mod\ 3))$ with 1. Also choose $r+2$ vertices among $u_i(6r \leq i \leq 12r+6, l \neq i(mod\ 3))$ and give them label 1. Label the remaining vertices arbitrarily so that $k+1$ vertices get label -1 , $k+1$ vertices get label i and k vertices get label $-i$. Number of edges with label $1 = 2r \times 3 + 2(r+2) = 8r+4$. Table 2 shows that in all cases, the given labeling is group $\{1,-1,i,-i\}$ cordial. □

$3n+1$	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
$4k$	k	k	k	k	$\frac{8k-2}{3}$	$\frac{8k-2}{3}$
$4k+1$	$k+1$	k	k	k	$\frac{8k}{3}$	$\frac{8k}{3}$
$4k+2$	$k+1$	$k+1$	k	k	$\frac{8k+2}{3}$	$\frac{8k+2}{3}$
$4k+3$	$k+1$	$k+1$	$k+1$	k	$\frac{8k+4}{3}$	$\frac{8k+4}{3}$

Table 2

An illustration of the labeling for $J_{3,5}$ is given in Fig. 2.2.

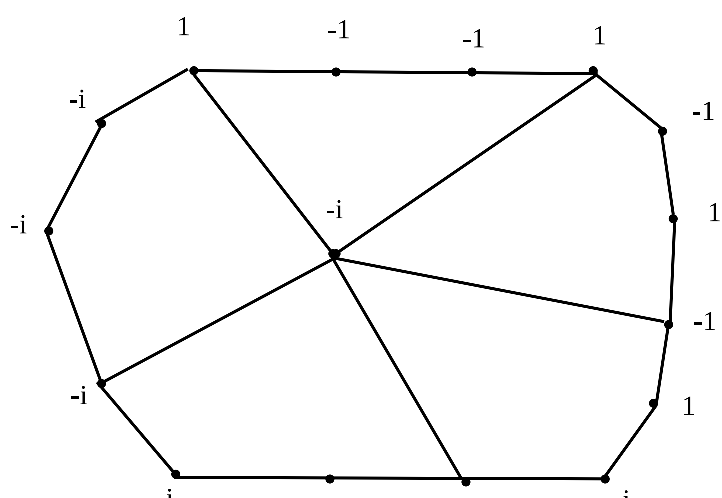

Fig. 2.2

Definition 2.6. Jelly fish graphs $J(m,n)(m \leq n)$ are obtained from a cycle $C_4 : uxvyu$ by joining x and y with an edge and appending m pendent edges to u and n pendent edges to v.

Theorem 2.7. *Jelly fish graphs $J(m,n)(m \le n)$ are group $\{1,-1,i,-i\}$ cordial if and only if either $m+n \le 10$ or $3m-6 \le n \le 3m+6$.*

Proof. Let the m pendent vertices adjacent to u be labeled as $u_1, u_2, ..., u_m$ and the n pendent vertices adjacent to v be labeled as $v_1, v_2, ..., v_n$. Number of vertices in $J(m,n)$ is $m+n+4$ and number of edges is $m+n+5$.

Case(i): $m+n \equiv 0(mod\,4)$.

Let $m+n = 4k, k \ge 1, k \in \mathbb{Z}$. Each vertex label should appear $k+1$ times. One edge label should appear $2k+2$ times and another should appear $2k+3$ times.

Subcase(1): $f(u) \ne 1$ and $f(v) \ne 1$.

If $f(x) = 1$ and $f(y) \ne 1$,then every other vertex with label 1 will yield only one edge with label 1. So $k = 2k-1$ or $k = 2k$ so that $k = 1$ or $k = 0$. If both $f(x) = 1$ and $f(y) = 1$, then $k-1 = 2k-3$ or $k-1 = 2k-2$ and so $k = 2$ or $k = 1$. If $k = 1, m+n = 4$ and if $k = 2, m+n = 8$. If $k = 1$, label x and v_1 with 1 and remaining vertices arbitrarily so that 2 vertices get label -1, 2 vertices get label i and 2 vertices get label $-i$. If $k = 2$, label x, y and v_1 with 1 and remaining vertices arbitrarily so that each vertex label appears on 3 vertices.

Subcase(2): $f(u) = 1$ and $f(v) = 1$.

This induces label 1 to $m+n+4$ edges and so this case is impossible.

Subcase(3): $f(u) = 1$ and $f(v) \ne 1$.

If both $f(x) \ne 1$ and $f(y) \ne 1$, then either $k = 2k-m$ or $k = 2k-m+1$. So either $k = m$ or $k = m-1$ and so $n = 3m$ or $n = 3m-4$. In both the cases, label the vertices $v_1, v_2, ..., v_k$ with 1 and the remaining vertices arbitrarily so that each vertex label appears on exactly $k+1$ vertices. Suppose either $f(x) = 1$ or $f(y) = 1$. Without loss of generality, let $f(x) = 1$. Then as above, $k = m+1$ or $k = m$. In both the cases, label the vertices $v_1, v_2, ..., v_{k-1}$ with 1 and the remaining vertices arbitrarily so that each vertex label appears on exactly $k+1$ vertices. When $k = m-1, m$ or $m+1$, we have $n = 3m-4, 3m, 3m+4$ accordingly.

Subcase(4): $f(u) \ne 1$ and $f(v) = 1$.

As in Subcase(3), by symmetry, we have $k = n, n-1$ or $n+1$. But , by assumption $m \le n$ and so in this case $n \le 2$.

Case(ii): $m+n \equiv 1(mod\,4)$.

Let $m+n = 4k+1, k \ge 0, k \in \mathbb{Z}$. Three vertex labels should appear $k+1$ times and one vertex label should appear $k+2$ times. Each edge label should appear $2k+3$ times.

Subcase(1):$f(u) \ne 1$ and $f(v) \ne 1$.

If $f(x) = 1$ and $f(y) \ne 1$, then either $k = 2k$ or $k+1 = 2k$ so that $k = 0$ or 1. If $k = 0$,take $f(y) = -1, f(u) = f(v) = i$ and $f(v_1) = -i$. If both $f(x) = 1$ and $f(y) = 1$, then either $k-1 = 2k-2$ or $k = 2k-2$ so that $k = 1$ or $k = 2$. If $k = 1, f(x) = 1, f(v_1) = 1$ and $f(v_2) = 1$. Label the remaining vertices arbitrarily so that each vertex label appears on 2 vertices. If $k = 2$, let $f(x) = 1, f(y) = 1, f(v_1) = 1$ and $f(v_2) = 1$. Label the remaining vertices arbitrarily so that each vertex label appears on 3 vertices. As $k = 0, 1$ or 2, we have $m+n = 1, 5$ or 9.

Subcase(2):$f(u) = 1$ and $f(v) = 1$.

As in Subcase(2) of Case(i), this is impossible.

Subcase(3):$f(u) = 1$ and $f(v) \ne 1$.

If both $f(x) \ne 1$ and $f(y) \ne 1$, then either $k = 2k-m+1$ or $k+1 = 2k-m+1$ so that $k = m-1$ or $k = m$. In the former case,label $v_1, v_2, ..., v_k$ with 1 and the remaining vertices arbirarily so that $k+1$ vertices get label -1, $k+1$ vertices get label i and $k+2$ vertices get label $-i$. In the latter case, label $v_1, v_2, ..., v_{k+1}$ with 1 and the remaining vertices arbitrarily so that each of the vertex labels $-1, i$ and $-i$ appear on $k+1$ vertices. Suppose $f(x) = 1$ and $f(y) \ne 1$. Then as above $k = m$ or $k = m+1$. If $k = m+1$, label the vertices $v_1, v_2, ..., v_k$ with 1 and the remaining vertices arbitrarily so that each of the vertex labels $-1, i$ and $-i$ appear on $k+1$ vertices. As $k = m-1, m, m+1$, we have $n = 3m-3, 3m+1$ or $3m+5$.

Subcase(4): $f(u) \ne 1$ and $f(v) = 1$.

As in Subcase(3), we get $k = n-1, n$ or $n+1$. As $m \le n$, in these cases, $n \le 2$.

Case(iii): $m+n \equiv 2(mod\,4)$.

Let $m+n = 4k+2, k \ge 0, k \in \mathbb{Z}$. Two vertex labels should appear $k+1$ times and two other vertex labels should appear $k+2$ times. One edge label should appear $2k+3$ times and another should appear $2k+4$ times.

Subcase(1): $f(u) \neq 1$ and $f(v) \neq 1$.
If $f(x) = 1$ and $f(y) \neq 1$, then there are are four possibilities; $k = 2k, k = 2k + 1, k + 1 = 2k$ and $k+1 = 2k+1$. Hence $k = 0$ or 1. If $k = 0, f(x) = 1, f(y) = -1, f(u) = f(v) = i, f(u_1) = f(v_1) = -i$. If $k = 1, f(x) = 1, f(v_1) = f(v_2) = 1$. Label remaining vertices arbitrarily so that 3 vertices get label $-1, 2$ vertices get label i and 2 vertices get label $-i$. If both $f(x) = 1$ and $f(y) = 1$, then $k = 0, 1$ or 2. If $k = 2, f(x) = f(y) = f(v_1) = f(v_2) = 1$. Label the remaining vertices arbitrarily so that 4 vertices get label $-1, 3$ vertices get label i and 3 vertices get label $-i$. As $k = 0, 1, 2, m + n = 2, 6$ or 10.
Subcase(2): $f(u) = 1$ and $f(v) = 1$.
As in previous cases, this is not possible.
Subcase(3): $f(u) = 1$ and $f(v) \neq 1$.
If both $f(x) \neq 1$ and $f(y) \neq 1$ then $k = m - 1, m - 2$ or m. If $k = m - 1$ or $m - 2$, label $v_1, v_2, ..., v_k$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ vertices get label $-1, k + 2$ vertices get label i and $k + 2$ vertices get label $-i$. If $k = m$, label $v_1, v_2, ..., v_{k+1}$ with 1. Label the remaining vertices arbitrarily so that $k + 2$ vertices get label $-1, k + 1$ vertices get label i and $k + 1$ vertices get label $-i$. Suppose $f(x) = 1$ and $f(y) \neq 1$. As above, $k = m - 1, m$or $m + 1$. If $k = m$, label $v_1, v_2, ..., v_{k-1}$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ vertices get label $-1, k + 2$ vertices get i and $k + 2$ vertices get label $-i$. As $k = m - 1, m - 2, m, m + 1$, we have $n = 3m - 6, 3m - 2, 3m + 2$ or $3m + 6$.
Subcase(4): $f(u) \neq 1$ and $f(v) = 1$.
As in Subcase(3), we get $k = n - 2, n - 1, n$ or $n + 1$. As $m \leq n$, we have $n \leq 3$.
Case(iv): $m + n \equiv 3(mod\, 4)$
Let $m + n = 4k + 3, k \geq 0, k \in \mathbb{Z}$. Three vertex labels should appear $k + 2$ times and one vertex label should appear $k + 1$ vertices. Each edge label should appear $2k + 4$ times.
Subcase(1): $f(u) \neq 1$ and $f(v) \neq 1$.
If $f(x) = 1$ and $f(y) \neq 1$, then either $k+1 = 2k+1$ or $k = 2k+1$ so that $k = 0$ or -1; If $k = 0$, $f(v_1) = 1$; Label the remaining vertices arbitrarily so that 2 vertices get label $-1, 2$ vertices get label i and 1 vertex get label $-i$. If both $f(x) = 1$ and $f(y) = 1$, then either $k = 2k - 1$ or $k - 1 = 2k - 1$ so that $k = 1$ or $k = 0$. If $k = 1$, $f(v_1) = 1$. Label the remaining vertices arbitrarily so that 3 vertices get label $-1, 3$ vertices get label i and 2 vertices get label $-i$. As $k = 0, 1, m + n = 3$ or $m + n = 7$.
Subcase(2): $f(u) = 1$ and $f(v) = 1$.
As in previous cases, this is impossible.
Subcase(3): $f(u) = 1$ and $f(v) \neq 1$.
If both $f(x) \neq 1$ and $f(y) \neq 1$,then $k = m - 1$ or $k = m - 2$. If $k = m - 1$, label $v_1, v_2, ..., v_{k+1}$ with 1 and remaining vertices arbitrarily so that $k+2$ vertices get label $-i, k+2$ vertices get label i and $k + 1$ vertices get label $-i$. If $k = m - 2$, label $v_1, v_2, ..., v_k$ with 1 and remaining vertices arbitrarily so that $k + 2$ vertices get label -1, $k + 2$ vertices get label i and $k + 2$ vertices get label $-i$. Suppose $f(x) = 1$ and $f(y) \neq 1$. Then $k = 2k - m$ or $k - 1 = 2k - m$ so that $k = m$ or $k = m - 1$. If $k = m$, label $v_1, v_2, ..., v_k$ with 1. Label the remaining vertices arbitrarily so that $k + 2$ vertices get label -1, $k + 2$ vertices get label i and $k + 1$ vertices get label $-i$. As $k = m - 2, m - 1$ or m, we have $n = 3m - 5, 3m - 1$ or $3m + 3$.
Subcase(4): $f(u) \neq 1$ and $f(v) = 1$.
As in Subcase(3), we get $k = n - 2, n - 1$ or n. As $m \leq n$, we have $n \leq 2$. □

An illustration of the labeling is given for $J(3, 5)$ in Fig. 2.3.

Definition 2.8. The graph obtained by joining two disjoint cycles $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ with an edge u_1v_1 is called dumbbell graph Db_n.

Theorem 2.9. *The Dumbbell graph Db_n is group $\{1, -1, i, -i\}$ cordial for every n.*

Proof. Number of vertices in Db_n is $2n$ and number of edges is $2n + 1$.
Case (i): n is even.
Let $n = 2k, k \geq 2, k \in \mathbb{Z}$.

In a group $\{1, -1, i, -i\}$ cordial labeling, each vertex label should appear k times. One edge label should appear $2k$ times and another $2k+1$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label 1 is $3 + 2(k-1) = 2k+1$.

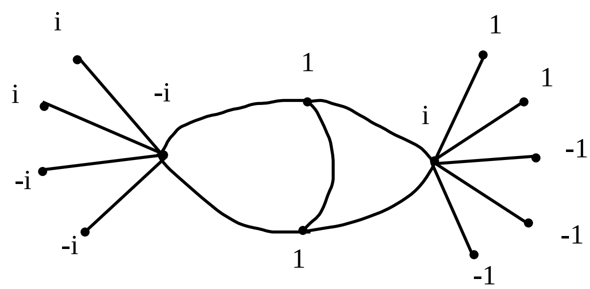

Fig. 2.3

Case (ii): n is odd.
Let $n = 2k+1, k \geq 1, k \in \mathbb{Z}$.
In a group $\{1, -1, i, -i\}$ cordial labeling, two vertex labels should appear k times and two vertex labels should appear $k+1$ times . One edge label should appear $2k+1$ times and another $2k+2$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that k of them get label -1, $k+1$ of them get label i and $k+1$ of them get label $-i$. Number of edges with label 1 is $3 + 2(k-1) = 2k+1$.
Table 3 shows that in all cases, the given labeling is group $\{1, -1, i, -i\}$ cordial. □

n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
$2k$	k	k	k	k	$2k$	$2k+1$
$2k+1$	k	k	$k+1$	$k+1$	$2k+2$	$2k+1$

Table 3

An illustration of the labeling is given for Db_6 in Fig. 2.4.

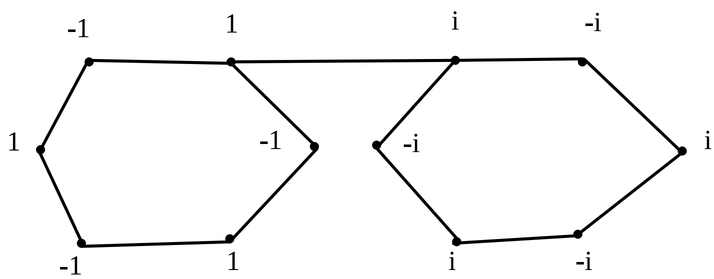

Fig. 2.4

Definition 2.10. A flower graph Fl_n is the graph obtained from a Helm by joining each pendent vertex to the central vertex of the Helm.

Theorem 2.11. *The Flower graph Fl_n is group $\{1, -1, i, -i\}$ cordial for every n.*

Proof. Let u be the center of the Wheel W_n. Let $u_1, u_2, ..., u_n$ be the vertices on the cycle of W_n and $v_1, v_2, ..., v_n$ be the pendent vertices of the helm such that v_i is adjacent to u_i for $1 \leq i \leq n$. Number of vertices in Fl_n is $2n+1$ and number of edges is $4n$.

Case (i): $n \equiv 0(mod\, 4)$.
Let $n = 4k, k \geq 1, k \in \mathbb{Z}$. In a group $\{1,-1,i,-i\}$ cordial labeling, three of the vertex labels should appear $2k$ times and one vertex label $2k+1$ times. Each edge label should appear $8k$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that $2k$ of them get label -1, $2k$ of them get label i and $2k+1$ of them get label $-i$. Number of edges with label 1 is $4(2k) = 8k$.
Case (ii): $n \equiv 1(mod\, 4)$.
Let $n = 4k+1, k \geq 1, k \in \mathbb{Z}$. In a group $\{1,-1,i,-i\}$ cordial labeling, one vertex label should appear $2k$ times and three other vertex labels $2k+1$ times. Each edge label should appear $8k+2$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-2}, v_2$ with 1. Label the remaining vertices arbitrarily so that $2k+1$ of them get label -1, $2k+1$ of them get label i and $2k$ of them get label $-i$. Number of edges with label 1 is $4(2k) + 2 = 8k+2$.
Case (iii): $n \equiv 2(mod\, 4)$.
Let $n = 4k+2, k \geq 1, k \in \mathbb{Z}$. In a group $\{1,-1,i,-i\}$ cordial labeling, three of the vertex labels should appear $2k+1$ times and one vertex label $2k+2$ times. Each edge label should appear $8k = 4$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that $2k+1$ of them get label -1, $2k+1$ of them get label i and $2k+2$ of them get label $-i$. Number of edges with label 1 is $4(2k+1) = 8k+4$.
Case (iv): $n \equiv 3(mod\, 4)$.
Let $n = 4k+3, k \geq 0, k \in \mathbb{Z}$. In a group $\{1,-1,i,-i\}$ cordial labeling, three vertex labels should appear $2k+2$ times and one vertex label should appear $2k+1$ times. Each edge label should appear $8k+6$ times. Define a labeling f as follows:
Label $u_1, u_3, u_5, ..., u_{n-2}, v_2$ with 1. Label the remaining vertices arbitrarily so that $2k+2$ of them get label -1, $2k+2$ of them get label i and $2k+1$ of them get label $-i$. Number of edges with label 1 is $4(2k+1) + 2 = 8k+6$.
Table 4 shows that in all cases, the given labeling is group $\{1,-1,i,-i\}$ cordial. □

n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
$4k, k \geq 1, k \in \mathbb{Z}$	$2k$	$2k$	$2k$	$2k+1$	$8k$	$8k$
$4k+1, k \geq 1, k \in \mathbb{Z}$	$2k+1$	$2k+1$	$2k+1$	$2k$	$8k+2$	$8k+2$
$4k+2, k \geq 1, k \in \mathbb{Z}$	$2k+1$	$2k+1$	$2k+1$	$2k+2$	$8k+4$	$8k+4$
$4k+3, k \geq 0, k \in \mathbb{Z}$	$2k+2$	$2k+2$	$2k+2$	$2k+1$	$8k+6$	$8k+6$

Table 4

An illustration of the labeling is given for Fl_6 in Fig. 2.5.

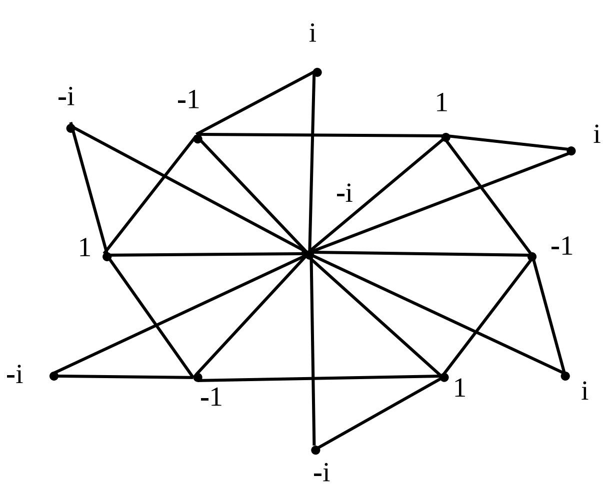

Fig 2.5

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