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A FEW THOUGHTS ON THE AESTHETICS OF MATHEMATICS IN RESEARCH AND TEACHING

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Abstract Mathematical aesthetic, having a variety of forms, is commonly experienced by mathematicians as a genuine reality and is available at every level of study. This short essay—in hopefully moving beyond standardised hermeneutic critique—attests to its authenticity through the words of some mathematical proponents, and reminds us that it should be promoted to children and students as a phenomenon that is as accessible as it is incontestable.

To view mathematics as devoid of aesthetic is to picture something else completely.

1 Introduction

The German mathematician, theoretical physicist and philosopher Hermann Weyl wrote the following (referring to a prominent Danish intellectual) in 1951 as he reviewed a half-century of mathematics:

> "[He] once said religion deals with what concerns man unconditionally. In contrast (but with equal exaggeration) one may say that mathematics talks about the things which are of no concern at all to man." [14, p. 523].

Even if this were once true, albeit loosely, it is no longer the case. Religion (that is, contemplative and non-aggressive religion) is nowadays too often hijacked across the globe to suit political and idealogical aims, while the rise of applied mathematics—quickened by the onset of World War II—has secured its well documented growth and relevance to society along with a small number of branches in pure mathematics. Of equal importance—but unknown to the masses—mathematics is the owner of an aesthetic essence able to bewitch and enrapture those who tread its soothing waters, garnishing it with extra currency when compared with other fields; here we examine this a little, first with reference to the rarefied air of academic research and then in the context of education more widely.

2 Aesthetics in Research

Bertrand Russell was an eclectic Welsh mathematician, philosopher, historian and social/political critic who wrote of mathematics that it is, when "rightly viewed", in possession of

> "not only truth, but supreme beauty ... austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, ..." [8, p. 60].

This oft used quote has been echoed by others, including Weyl himself who said that mathematics has "the inhuman quality of starlight, brilliant and sharp, but cold". There are a few grains of truth here, yes, though the discipline is so very much more. In aligning the "stern perfection" of mathematics with similar aspects of "the greatest art", Russell was no doubt biased by his experience as a logician first and foremost, from which mindset his imagery was produced (a first rate one he may have been, but his expertise lay in just one small area of a large operational canvass, and this skewed his vision which was evidently centred on the grand edifice of pure

reason); Weyl, on the other hand, gave himself a more expansive mathematical brief, so it is of note to see the same linguistic tone adopted.

Neither of these perspectives gets to grips with the emotions that bubble up from connecting with a subject whose grace and allure lies at its core and awakens the spirits. Henri Poincaré's areas of interest matched broadly those of Weyl, and the French academic thought that mathematics had a triple end, one of which was that its innate aesthetic properties—in addition to stimulating enquiry within nature and philosophy—should touch practitioners in ways that painting and music do. The Proceedings of the August 1897 International Congress of Mathematics (held in Zurich) record that he opined thus:

> "Peu de privilégiés sont appelés à la goûter pleinement, cela est vrai, mais n'est-ce pas ce qui arrive pour les arts les plus nobles?" ["Only the privileged few are called to enjoy it fully, this is true, but is this not what happens for all the noblest arts?"] [7, p. 82].

The congress was the first of its kind (the next, in Paris three years later, saw David Hilbert announce his famous list of 23 then unsolved problems), as the mathematical community—recognising the benefits to be reaped through co-operation and collegiality fostered by physical interaction between academics—moved to trigger new links across national borders and spread not only technical results but higher-minded hypotheses and critical thinking. These involved aesthetic elements to both mathematical study and research, on which deliberations began to crystallise throughout Europe and beyond as prevailing instruments of work (such as geometric intuition) started to wane and people began to think more deeply about such things as proof, rigour, axiomatisation, inferential argument and in time metatheories, all within the framework of human participation as mathematics began to move itself away from the sciences towards relative independence and self-sufficiency; questions concerning what it actually means to be a mathematician—and how the paradigm of aesthetic should be interpreted—began to take hold among the rank and file of party members, so to speak, and they have prompted discussion among us ever since. Real mathematicians revel in the quiet satisfaction to be taken from just sitting and writing out mathematics—the space in which we work, the implements of our choice (our mathematical *mise en scène*, as it were), the solitude and the delineations we make, all combine to generate feelings that fall within aesthetic boundaries, overarched by temperament, outlook and disposition of the individual. While some fields prosper as others become exhausted, mathematics—largely evolutionary, occasionally revolutionary, but overwhelmingly to most just a constant source of gratification—sustains and rewards through aesthetic gifts accorded to those fortunate enough to be in a position to receive them, and always has; the human mind finds in abundance the peace, solice and cheer it craves as it taps into a timeless occupation with aesthetic at its very heart.

What makes aesthetic so difficult to model and study is its ambiguity in acting as a marker of experiential events (in a metaphysical sense) and also as a descriptive attribute associated with objects (real or otherwise). These overspill into mathematical territory where some things—constructs, notation, and so forth—are universally regarded as pleasing, but where, too, the internalisations of aesthetic are mainly personal affairs, triggered in research by subjective entities such as *depth, rarity, usefulness, stylishness, powerfulness, ingenuity, refinement* and *importance*, to name a few, along with other traits appropriated for hegemonic purpose. One sees this in the context of creativity from the passages of prominent mathematicians such as Jacques S. Hadamard, Peter D. Lax, David E. Smith, John von Neumann, Godfrey H. Hardy and Serge Lang found in [1], endorsing the premise that aesthetic at the upper end of intellectual endeavour has many manifestations and means different things to people in the way it shapes their approach to work. It is not synthetic—involving hightened sensations, cerebral epiphanies, or other gentler and longer lasting effects—but both literary devices and language struggle to characterise it for mathematicians in ways that tally normatively with the lives of laymen.

Some commendable attempts have been made in this regard, though, one or two very recently. As an example, consider the American-based Russian mathematician Edward Frenkel who has captured the textures of his life as a top mathematician in a deep relationship with the discipline. To paraphrase from his award-winning 2013 text [2, p. 233], mathematical formulas and equations convey information—sometimes abstruse, sometimes related to our physical surroundings—offering outposts of freedom to be creative within a passionate pursuit involving

struggles with the mathematical unknown before our very eyes and also with ourselves. They elicit strong inner stirrings that link with intuition, feelings and intellect in ways that may be acute and profound. His dedicated search for mathematical knowledge and truth is there for all to see, and the way this is expressed does a great service in allowing the reader a glimpse of what makes mathematicians love what they do; being in tune with the aesthetics of mathematics plays a central role for him day-to-day, as it does for thousands of others, and has culminated artistically in the production of the short allegorical 2010 film *Rites of Love and Math*—a fictional piece symbolising the constant quest for meaning and certitudes hidden in the realities around us (a mathematician creates a definitive formula of love and so, realising that it can be used for evil as well as for good while beset by the powers that be, to save it from falling into the wrong hands he tattoos it on the body of the woman he loves before taking his own life). Distinguished American mathematician Michael Harris points out in his 2015 *tour de force* book [4] that entering the mathematical life is mostly a matter of seeking an orientation among, or identifying, worthwhile problems, and often obliges one to have a running conversation with the history of the subject. Relations with past mathematics can impact upon a contemporary mathematician, for an understanding of contextual aesthetic unites us across eras with our predecessors and provides an acknowledgement of the social/cultural progression of mathematics. He also hints at what I refer to as 'spiritual' aesthetic, noting that while mathematics is not a religion the way we talk about it draws heavily at times on religious dialogue that conveys the devotion and commitment to subject shared by many of us.

Polish born Stanisław Ulam extolled in his biography [13] the virtue of aesthetic as a major influence throughout mathematics, observing that few non-mathematicians are able to apprehend or grasp the full implications of its worth which for professionals is undeniable in their daily contacts with each other and with mathematics itself. Stating that value judgements in mathematics are far from easy, he asserted that aesthetic scrutiny is alone not enough to allow the relative assessment and grading of one's results. He believed that degrees of mathematical beauty would be formalised at some point so as to provide more comprehension (leading to the idea of 'super beauty' at loftier ground), noting at the time that

> "So far when anyone has tried to analyze the aesthetic criteria of mathematics too precisely, whatever was proposed has seemed too narrow. It has to appeal to connections with other theories of the external world or to the history of the development of the human brain, or else it is purely aesthetic and very subjective ..." [13, p. 276].

He went on to propound that there must be something objective, if not actually defined, about the appraisal of beauty offered up by mathematics, remarking that it is dependent for some on utility within the field or other sciences, while for others (G.H. Hardy being the best known) it is quite the opposite. To me, lack of concensus running through our profession still exists with little surprise—one personality reaps much from applying his or her work in the resolution of a physical or practical problem, but another might retreat for aesthetic comfort to isolated analytical purity away from the pragmatic spoilers. Harris reports of experiments which—tracing responses via activation of certain brain regions—claim to have shown there is a faculty of (auditory and visual) beauty that is divorced from the modality through which it is conveyed, intimating that we might be hardwired to embrace and exploit mathematical beauty in the same physical way. When mathematicians speak of beauty he suspects there are good reasons to think they mean pleasure, in some sense, of definite and quite singular type. If this is deployed to communicate a sort of approval then we are entitled to ask what kind it is and how it might differ in reference to things deemed to be 'true' or 'good'. Wider aesthetic judgement in mathematics is, he says, hampered by its meagre lexicon, writing

> "*Pretty, appealing, attractive*, and the like, carry less weight than the all-purpose *beautiful* and the perennial favorite *elegant*, and in practice they overlap with words like *clever* or *ingenious* that reflect a different set of concerns." [4, p. 307].

According to Sinclair and Pimm [10, pp. 13-14], the French engineer and writer François Le Lionnais thought that our ties with the aesthetic depend on our orientation as either a 'classical' or 'romantic' mathematician, harbouring desires for equilibrium, harmony and order, or else tendencies towards form obliteration, imbalance and pathology, a distinction made by the English-born American theoretical physicist and mathematician Freeman Dyson who distin-

guished between what he termed 'unifiers' and 'diversifiers'. The counter view has also been taken, however, based on the belief that a mathematician's judgements are non-subjective and instead depend solely on the mathematics itself, making it possible to formulate decisive aesthetic criteria—this has led, for one thing, to the principles of 'minimal completeness' and 'maximal applicability' as viable contenders, along with other factors and formulaic estimates.

Mathematical aesthetic remains something of an enigma, beyond academia and even within—indisputably vital to our subject, yet still resistant to categorisation and dilution to simplified terms that would cause it to be weakened or altered. Rather than it be impossible to remain passive about, or indifferent to, mathematical aesthetic is something to which most research mathematicians give little time nor are much troubled by. It is not, however—as was suggested some sixty years ago (and since disputed, too)—subservient to achievement and thus robbed of any epistemic interest [10, p. 11], but complementary to it. Harris condenses the merit of mathematics as a trilogy of goodness, truthfulness and beauty, writing

> "Pure research in mathematics as in other fields is *good* because it often leads to useful practical consequences (. . . the [so called] *Golden Goose* argument); it is *true* because it offers . . . access to certain truths; it is *beautiful*, an art form." [4, pp. ix–x].

It exists, he says, as a second (virtual) life of shape, number, order and rule that we are able to conjure (remote from our first and authentic life), where everything is not only as it should be but we are happy that we know why. Aesthetic inhabits such a life, unquestionably, but Harris, rather interestingly, recognises that it is one that is not fully controllable and is itself punctured by pathos, a point on which Pimm and Sinclair have written with reference to the state of melancholic contemplation useful in readiness for, and during, those periods when creative energy works with aesthetic arousal to assist the mathematical mind. [6, p. 233].

In concluding this section we should also note that mathematical writing of research articles—while very much proceduralised, with emphasis on the production of self-contained works that adhere to established patterns and syntactic models replete with subject-specific references and terminologies—itself offers an arena where self-expression is able to induce an element of the aesthetic for both creator and reader. This can be brought about by the compositional structure to a paper, partnered with tenor, transparency, modulation, pace and deictic choices employed. Viewed as something of a short story, licence to personalise some discourse integrants can elevate written technical mathematics still further. It has been said that mathematics sits between rhetoric and poetry, so there is no reason why this should not be reflected in what we generate within our community as outputs, striking a balance between the author's wish to give a compelling account of a mathematical passage that fulfils its morphological commitments with an eagerness to display authority, readability, stylistic non-neutrality and aesthetic strength.

3 Aesthetics in Education

Axiology is a longstanding branch of philosophy, being a collective term for the study of ethics and aesthetics that each tackle notions of worth in our evaluation of the nature of values and how these are arrived at; the latter—which over the last several decades has been looked at more so from the newer stance of mind-body inseparability in scholarly work drawn from cognitive science, anthropology, philosophy and neuroscience—has particular resonance and flavour in the realm of mathematics, pure or applied. American mathematician and Fields Medalist William Thurston considered aesthetic and utilitarian sides to mathematics as being in fact quite close, writing

> "Our aesthetic instincts draw us to mathematics of a certain depth and connectivity. The very depth and beauty of the patterns makes them likely to be manifested, in unexpected ways, in other parts of mathematics, science, and the world.
>
> To share in the delight and the intellectual experience of mathematics—to fly where before we walked—that is the goal of a mathematical education." [12, p. 848].

It is not unreasonable that those who ply their trade professionally will (through a power dynamic entrenched within default enculturation) see mathematics as divulging a special sort of aesthetic which needs no reification; there is an absence of external critics—widespread in some of the arts

and humanities—to affirm what is experienced as agencies of articulation, and it can be viewed as a preserve of the elite who accept their classified status without dwelling on it a great deal, if at all. The posture taken on exclusive enlightenment as an inherited endowment of indulgence—shared by Hardy, Poincaré, Russel and a fair proportion of others past and present—is fuelled by the belief that like the mathematics itself, its aesthetic nuances are in a class of their own and satisfyingly *sui generis*, within reach only to those who are guardians of mathematical sophistication. This is not the whole story, however. Students of HE sector mathematics often meet both the surprising and inevitable in our subject, encountering shades of aesthetic from such things as the following: working through an enlivening instance of algebraic simplification; successfully repeating a 'trick' in different contexts; mastering a technique/strategy; gaining insight into a solution path; moving from the particular to the general in extending a result, idea or concept; following through an inventive proof, or subtle line of assertions; translating a successful mental mode for use in a fresh mathematical setting; making a helpful association between things outwardly disparate; effecting simulation or visualisation through computer code; finding simplicity and organisation lurking in apparent complexity or chaos. I, for one, sometimes highlight any satisfying characteristic of the mathematics I present to students so as to share my own sentiments. When teaching calculus I tell them, for example, that I never tire of working through a constructive proof of the Product/Quotient and Chain Rules of differentiation taken for granted when younger. I mean it when I say this to them, and hope my enthusiasm for aesthetic delicacies therein is clear. I also ensure they are familiar with the great Paul Erdős' reference to any exceptionally succinct or deft proof as being "from The Book!" (God's fictitious tome containing all the theorems of mathematics with their most 'beautiful' proofs), a phrase sanctioned by him for praise of something he considered consummate.

Within the crucible of research, us adepts—with added richness to our work—will feel them more intensely as they are allied with the joy of originality in the problems we formulate and subsequently interrogate. The senses are vitalised, wherein a perception of the aesthetic might act as a latent guide or imperative in the prosecution of research—even if one is not conscious of it—or more overtly as an unmistakable motivator of intent. Components of mathematical aesthetic should ideally extend unashamedly into the school classroom, where it has long been felt that there is something about the way the subject of mathematics is taught that causes children to function in narrow domains, to rely on routine procedures and selective algorithms, and to think in predominantly convergent ways about mathematical problems. Opportunities and encouragement to break from the stereotype, to overcome fixations and rigidity in approaches to problem solving, and to think flexibly and divergently, seem to have almost totally disappeared throughout secondary level education, where a dearth of mathematical aesthetic—and suppression of the sensitivities to it—has, as noted by Griffiths [3], been brought about by schooling that is driven predominantly by test scoring and metric imposed by government; there is no warrant to persist with this kind of delivery, whose visible footprint has stamped unceremoniously on mathematical *jouissance* and felicity to which every child has the right to be exposed if they are to discover the precious bounty on offer. Mathematics—as a compulsory subject with which to grapple—can be intimidating enough as it is, but when only lip service at best is paid to its aesthetic strands at the expense of obsessive assessment directives and restricting measures of attainment, it is little wonder that the enchanting and seductive sides of mathematics are lost along the way and forever remain unknown to whole swathes of the population. Students miss out without even knowing it, and teachers are not well placed to pass these things on nowadays as their concentration lies elsewhere; those inadequately trained or educated in the field (of which we hear there are too many in the U.K.) fair even worse. To address this is a challenge indeed—since mathematical aesthetic may be ephemeral or vague at one moment, and then tangible or more palpable the next, with inbuilt mathematical hierarchies revealing it in both the simple and the not so simple—but we should try.

Pertinent to mention is a purposeful and very comprehensive disquisition by Sinclair [9] who describes current narratives and more traditional beliefs associated with the topic of aesthetic. Previous works have suggested aesthetic reactions in mathematics demand well grounded subject knowledge and learning skills, only acquiring momentum and becoming consolidated in more advanced study—the conclusion, therefore, is that school staff focus initial energies on teaching basic tools before addressing the aesthetic maturity of their pupils. She, however, argues for an acqaintance with mathematical aesthetic to be at once a captivating, liberating and unifying force

across all of lower tier instruction, and I agree. Its democratisation—by increased awareness and promotion throughout this layer of the education spectrum—could only be a movement for good as it would equip the next generation with the means to respect this organic segment of mathematics, to nurture their sensibilities appropriately and to cherish them for what they are; they might then represent the subject more faithfully, with fidelity and conviction, wherever they go and whatever they do in life.

This call has been heard elsewhere. The American mathematician turned educationalist Paul Lockhart—in his visceral commentary on today's western mathematical education systems—has bemoaned the preparation of teachers as the root cause in perpetuating a lack of appreciation for mathematics as, first and foremost, an art form. Whatever useful place it has should, he says, be subordinate to its aesthetic and creative facets around which curricula should be designed, and he queries why modern day practices exhibit no mindfulness of its history, philosophy, status and aesthetic content. Lockhart is of the view that, as a bare minimum, teachers must be mathematical evangelists, be both knowledgable and competent, and visibly thrive in what it can give, warning that

> "If teaching is reduced to mere data transmission, if there is no sharing of excitement and wonder, if teachers themselves are passive recipients of information and not creators of new ideas, what hope is there for their students?" [5, p. 46].

He declares that when mathematics is, to a teacher, little more than a series of seemingly arbitrary rules, formulas, identities, methods and procedures to apply (rather than something arising from creative/exploratory processes coupled with aesthetic engagement through, and inspired by, choices, opinions, tastes and wants) then they will be absorbed by the audience; the discipline then becomes mapped out and given structure along precisely these paths of shallow thinking, and the cycle of what he calls "pseudo-mathematics" continues—a soulless and sullen distortion of the real thing which is done a huge disservice by the blatant perversion served up to each yearly cohort and carried into adulthood. Ultimately, we would do well to remember the following as a working mantra that informs, in part at least, the delivery of mathematics:

> "If there is anything like a unifying aesthetic principle in mathematics, it is this: *simple is beautiful*. Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are *imaginary*." [5, p. 24].

He urges us to repress and remove some of the more dismal features of the way the subject is packaged for the classroom, feeling that a communal declaration to bring it to life is the least that mathematics deserves—there is, after all, an almost boundless latitude and flexibility to evoke aesthetic cognisance and engender a familiarity with it given sufficient appetite to realise them. We do now have some excellent facilitating technological resouces at our disposal, remember, so this doesn't seem like an overly ambitious ask.

Mathematicians carry a unique attitude and ethos that pulls them towards aesthetic with an almost relentless inexorability. How this is inculcated into the young—given Sinclair's accurate belief that the aesthetic dimension of mathematics thinking/learning has received only intermittent attention in academic literature, remaining decidedly third wheel to the behemoths of cognition and affect—is an issue to take seriously, as it has ramifications for our subject in ways which most of us have probably not properly contemplated. Between one person and the next, those lines demarcating aesthetic mathematical predilections are a mix of the flexible and the unyielding, defining provinces and smaller districts—sometimes clear, sometimes opaque—of both mutual separation and commonality that shift in line with circumstance, environment, nationality, training, and more; at any one point in time the sum total of this patchwork across the whole community forms a principal and sincere statement about the distinctive hallmark of mathematics whose *de facto* exclusivity should be celebrated while simultaneously opened up for everyone. Artists who weave mathematical threads into their works, to enhance or underpin them, have a useful voice in education as they will normally disclose a keen interest in the mathematical aesthetic and provide a window into an alternative universe of explanation and explication. As someone interested in the semiotic analysis of mathematical imagery, Sarah Stengle—who tries to question the veil of assumptions through which mathematics is regarded as impassive and whose textual expressivity is thought of as highly regulated and dry—left us with some pertinent comments in a paper from 2000. Positing that an entirely correct, but bor-

ing, mathematical proof is second rate (I would disagree here, as it only *might* be), just as an artistically dull portrait can be representatively excellent, she had in mind the vocabulary used to decipher the relative working *modus operandi* of mathematicians and fine artists; in particular, Stengle wanted to encapsulate instinct as an usher to aesthetic in each group, writing

> "Often the outcome is described in terms of discovery, meaning that [it] was not known beforehand but seemed to exist a priori. Both often have only a sense of the outcome, rather than knowledge of it, and follow their intuition to their goals, which they recognize only when they get there. "There" is [where] things "feel" resolved and complete. The mathematical discovery has to withstand the rigid demands of the discipline, while [its counterpart] is subject to constant reinterpretation and debate." [11, p. 165],

in which "Success ... feels suprapersonal and enduring." (p. 161). Of course poetry, literature and music—through workshops, seminars, summer schools, *etc.*, where heavy mathematical prerequisites may be avoided—are also used to arouse and make more meaningful the aesthetics of mathematics in those who do not naturally lean favourably towards them.

We end by pondering Sinclair and Pimm's slightly more optimistic impression [10, p. 3] that there has been something of a *rapprochement* among mathematics, science and the arts, in tandem with a rising popularity of mathematics in mainstream culture (media-led—and compromised somewhat as a result—but discernible and long overdue). Both are put forward as manifestations of a re-emergent affinity between things mathematical and aesthetic that recalls their closeness in the days of the ancient Greeks who—celebrating them as integral to their beliefs about knowledge, understanding, cultivation, edification, and so on—saw them acting as a bridge between the raw world of sense and experience, and the divine world of perfection and beauty. Perhaps, then, all is not lost if mathematical aesthetic is truly at the start of a *renaissance* whereby people agree that it is a prime asset and *sine qua non* of mathematics, concede that it mirrors something bigger in us as a species, and embrace it resolutely in society; we shall see.

4 Summary

It has long been sensed by people with an interest in such matters that processes embedded in mathematical research and the psychology attached to them can be matched to a greater or lesser extent with those in other pursuits, and we can be thankful for individuals articulate enough to write or speak on this with rank and substance as they link our aspirations, ventures and artifacts with different spheres of enterprise. It is only possible to think in these terms because mathematics is creative in a very fundamental way—self-contained, uncontaminated and wholesome—wherein its aesthetics live and breathe, waiting to show themselves as we pass through the sacred doors of the temple in which they reside. Russell—pondering activities that add to human existence beyond the practical mechanics of life—spoke of Plato's observation that there was something of "divine necessity" in mathematics that "cannot be set aside" [8, p. 60]; this, surely, is why its aesthetics are so treasured and so satisfying when we stumble across them, as they embody much of what is glorious about the subject and keep alive our individual mathematical propinquities which are constantly nourished and renewed.

I finish by proffering the following thought as an immutable fact despite the ever changing faces of teaching and research. *The terrain occupied by mathematics is an inhospitable one in many ways, but the riches bestowed upon those able to settle there and prosper more than justify the investement in time and effort required—aesthetic wellbeing is but one of them, and quite possibly the most significant of all.*

La créativité implicitement requise par la recherche mathématiques sera éternellement enlacée à celle du génie du'n peintre ou poète ou écrivain ou musicien.

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