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CERTAIN INTEGRAL TRANSFORMS OF K-BESSEL FUNCTION.

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Abstract The Euler, Laplace, Whittaker and fractional Fourier transforms of $k-$Bessel function is established in this paper. The results obtained here are expressed in terms of generalized Wright function.

1 Introduction

The k-Pochhammer symbol $(\lambda)_{n,k}$ is defined (for $\lambda,\ \nu \in \mathbb{C};\ \ k \in \mathbb{R}$) by

$$\begin{aligned}(\lambda)_{\nu,k} :=& \frac{\Gamma_k(\lambda+\nu k)}{\Gamma_k(\lambda)} \quad (\lambda \in \mathbb{C}\setminus\{0\}) \\ =& \begin{cases} 1 & (\nu=0) \\ \lambda(\lambda+k)\cdots(\lambda+(n-1)k) & (\nu=n\in\mathbb{N}); \end{cases}\end{aligned} \tag{1.1}$$

and the k-gamma function has the relation

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\,\Gamma\left(\frac{z}{k}\right). \tag{1.2}$$

is such that $\Gamma_k(z) \to \Gamma(z)$ if $k \to 1$ [1]. The Bessel function of fist kind has the power series representation of the form [3]

$$J_\upsilon(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{2}\right)^{2k+\upsilon}}{\Gamma(k+\upsilon+1)\,k!}, \tag{1.3}$$

The $k-$Bessel function defined in [2] (also see [4]) and represented as follows:

$$J^k_{\pm\vartheta}(z) := \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{z}{2}\right)^{2r\pm\frac{\vartheta}{k}}}{\Gamma_k(rk\pm\vartheta+k)\,r!}, k\in\mathbb{R}^+, \vartheta\in\mathbb{I}, \vartheta>-k. \tag{1.4}$$

The generalized hypergeometric function represented as follows [5]:

$${}_pF_q\left[\begin{array}{l}(\alpha_p);\\(\beta_q);\end{array} z\right] = \sum_{n=0}^{\infty} \frac{\Pi_{j=1}^{p}(\alpha_j)_n}{\Pi_{j=1}^{q}(\beta_j)_n}\frac{z^n}{n!}, \tag{1.5}$$

provided $p \le q; p = q+1$ and $|z| < 1$ and $(\alpha)_n$ is well known Pochhammer symbol, $\alpha \in \mathbb{C}$ (see [5])

The generalized Wright function ${}_p\Psi_q(z)$ is given by the series

$$\begin{aligned}{}_p\Psi_q(z) &= {}_p\Psi_q\left[\begin{array}{l}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle| z\right] \\ &= \frac{\prod_{j=1}^{q}\Gamma(\beta_j)}{\prod_{i=1}^{p}\Gamma(\alpha_i)}\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i k)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j k)}\frac{z^k}{k!},\end{aligned} \tag{1.6}$$

where $a_i, b_j \in \mathbb{C}$, and $\alpha_i, \beta_j \in \mathbb{R}$ $(i = 1, 2, \ldots, p; j = 1, 2, \ldots, q)$. Asymptotic behavior of this function for large values of argument of $z \in \mathbb{C}$ were studied in [6] and under the condition

$$\sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i > -1 \tag{1.7}$$

was found in the work [7, 8].

The generalized $k-$Wright function introduced in [9] as:
For $k \in \mathbb{R}^+; z \in \mathbb{C}, \alpha_i, \beta_j \in \mathbb{R}$ $(\alpha_i, \beta_j \neq 0; i = 1, 2, ..., p; j = 1, 2, ..., q)$ and $(a_i + \alpha_i n), (b_j + \beta_j n) \in \mathbb{C} \backslash k\mathbb{Z}^-$

$$\begin{aligned} {}_p\Psi_q^k(z) &= {}_p\Psi_q^k\left[\left(\begin{array}{cc} (a_i, \alpha_i)_{1,p} & |z \\ (b_j, \beta_j)_{1,q} & \end{array}\right)\right] \\ &= \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(a_i + \alpha_i n)}{\prod_{j=}^{q} \Gamma_k(b_j + \beta_j n)} \frac{z^n}{n!}. \end{aligned} \tag{1.8}$$

Now, we recall the following definitions,

Definition 1.1. Euler Transform:

Let $\alpha, \beta \in \mathbb{C}$ and $\Re(\alpha), \Re(\beta) > 0$,then the Euler transform of the function $f(z)$ is defined by

$$B\{f(z); \alpha, \beta\} = \int_0^1 z^{\alpha-1}(1-z)^{\beta-1} f(z)\, dz. \tag{1.9}$$

Definition 1.2. Laplace Transform:

The Laplace transform of the function $f(t)$ is defined as

$$F(s) = L\{f(t); s\} = \int_0^{\infty} e^{-st} f(t)\, dt, \Re(s) > 0. \tag{1.10}$$

Definition 1.3. Fourier transform
The following integral gives the Fourier transform

$$\widehat{u} = \Im[u](w) = \int_R u(t)\, e^{iwt} dt. \tag{1.11}$$

where $u = u(t)$ be a function of the space $S(R)$ Shwartzian space of function that decay rapidly at ∞ together with all derivatives.

Definition 1.4. The Fractional Fourier Transform (FFT)
Let u be the function belonging to $\phi(\mathbb{R})$, the Lizorkin space of functions,where

$$\phi(\mathbb{R}) = \{\phi \in S(\mathbb{R})\} : \Im[\phi] \in V(\mathbb{R})$$

and $V(\mathbb{R})$ is the set of functions defined by

$$V(\mathbb{R}) = \{v \in S(\mathbb{R})\} : V_{(0)}^{(u)} = 0, n = 0, 1, 2, ..$$

then FFT of order α, $0 \leq \alpha \leq 1$ is given by

$$\begin{aligned} U_\alpha(w) &= \Im_\alpha[u](w) \\ &= \int_R e^{iw^{1/\alpha} t} u(t)\, dt \end{aligned} \tag{1.12}$$

particularly, if $\alpha = 1$ (1.12) reduces to FT and for $w > 0$ (1.12) reduces to FFT given by Luchko et al [10].

The aim of this paper is to derive the Euler,Laplace, Whittaker and Fractional Fourier transforms of $k-$ Bessel function given in (1.4).

2 Main Results

In this section, we give some theorems and corollaries as main results.

Theorem 2.1. *If* $k \in \mathbb{R}^+; a, b \in \mathbb{C},\ \vartheta \in \mathbb{I}$ *and* $\vartheta > -k$, *then*

$$\int_0^1 z^{a-1}(1-z)^{b-1} J_v^k(xz^\sigma)\,dz$$
$$= \Gamma(b)\left(\frac{x}{2k}\right)^{\frac{\vartheta}{k}} {}_1\Psi_2\left[\begin{array}{c}(a+\frac{\vartheta\sigma}{k}, 2\sigma)\\ (\frac{\vartheta}{k}+1, 1), (a+b+\frac{\vartheta\sigma}{k}, 2\sigma)\end{array}\Bigg| -\frac{x^2}{4k}\right].$$

Proof. The left-hand side of theorem 1 denoted by by $\mathcal{I}_1$.Using the definition of $k-$Bessel function and (1.9) , we have

$$\begin{aligned}\mathcal{I}_1 &= \int_0^1 z^{a-1}(1-z)^{b-1} J_v^k(xz^\sigma)\,dz\\ &= \int_0^1 z^{a-1}(1-z)^{b-1}\sum_{r=0}^{\infty}\frac{(-1)^r\left(\frac{xz^\sigma}{2}\right)^{2r+\frac{\vartheta}{k}}}{\Gamma_k(rk+\vartheta+k)\,r!}dz,\end{aligned}$$

Interchanging the integration and summation under the given conditions, we get

$$\begin{aligned}\mathcal{I}_1 &= \sum_{r=0}^{\infty}\frac{(-1)^r x^{2r+\frac{\vartheta}{k}}}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\int_0^1 z^{a+2r\sigma+\frac{\vartheta}{k}\sigma-1}(1-z)^{b-1}\,dz,\\ &= \left(\frac{x}{2}\right)^{\frac{\vartheta}{k}}\sum_{n=0}^{\infty}\frac{(-1)^r x^{2r}}{2^{2r}\Gamma_k(rk+\vartheta+k)\,r!}\frac{\Gamma\left(a+2\sigma r+\frac{\vartheta\sigma}{k}\right)\Gamma(b)}{\Gamma\left(a+b+2\sigma r+\frac{\vartheta\sigma}{k}\right)}.\end{aligned}$$

Now using (1.1), we get

$$\mathcal{I}_1 = \Gamma(b)\left(\frac{x}{2k}\right)^{\frac{\vartheta}{k}}\sum_{n=0}^{\infty}\frac{(-1)^r x^{2r}}{2^{2r}k^r\Gamma\left(\frac{rk+\vartheta+k}{k}\right)r!}\frac{\Gamma\left(a+2\sigma r+\frac{\vartheta\sigma}{k}\right)}{\Gamma\left(a+b+2\sigma r+\frac{\vartheta\sigma}{k}\right)}.$$

In view of the definition of (1.6) , we arrived the desired result. □

Corollary 2.2. *If we take* $k=1$, *in theorem* 2.1, *then we have the following integral representations for Bessel function of first kind* $J_v(z)$ *:*

$$\int_0^1 z^{a-1}(1-z)^{b-1} J_v(xz^\sigma)\,dz$$
$$= \Gamma(b)\left(\frac{x}{2}\right)^{\vartheta} {}_1\Psi_2\left[\begin{array}{c}(a+\vartheta\sigma, 2\sigma)\\ (\vartheta+1, 1), (a+b+\vartheta\sigma, 2\sigma)\end{array}\Bigg| -\frac{x^2}{4}\right].$$

Theorem 2.3. *If* $k \in \mathbb{R}^+, a \in \mathbb{C}, \vartheta \in \mathbb{I}, \vartheta > -k$, $\Re(s) > 0$ *and* $\left|\frac{x}{s^\sigma}\right| < 1$ *then*

$$\int_0^\infty z^{a-1}e^{-sz} J_v^k(xz^\sigma)\,dz$$
$$= \left(\frac{x}{2ks^\sigma}\right)^{\frac{\vartheta}{k}} s^{-a}\,{}_1\Psi_1\left[\begin{array}{c}(a+\frac{\sigma\vartheta}{k}, 2\sigma)\\ (\frac{\vartheta}{k}+1, 1)\end{array}\Bigg| -\frac{x^2}{4ks^{2\sigma}}\right].$$

Proof. Let $\mathcal{I}_2$ denoted by the left hand side of Theorem 2.3 and applying 1.4, we get

$$\begin{aligned}\mathcal{I}_2 &= \int_0^\infty z^{a-1}e^{-sz} J_v^k(xz^\sigma)\,dz\\ &= \int_0^\infty z^{a-1}e^{-sz}\sum_{r=0}^{\infty}\frac{(-1)^r\left(\frac{xz^\sigma}{2}\right)^{2r+\frac{\vartheta}{k}}}{\Gamma_k(rk+\vartheta+k)\,r!}dz.\end{aligned}$$

Interchanging the integration and summation allow us to write,

$$\mathcal{I}_2=\sum_{r=0}^{\infty}\frac{(-1)^r x^{2r+\frac{\vartheta}{k}}}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\int_0^1 z^{a+2r\sigma+\frac{\vartheta}{k}\sigma-1}e^{-sz}dz,$$

In view of the definition of Laplace transform and (1.1), we get

$$\begin{aligned}\mathcal{I}_2 &= \sum_{r=0}^{\infty}\frac{(-1)^r x^{2r+\frac{\vartheta}{k}}}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\frac{\Gamma\left(\frac{\sigma\vartheta}{k}+2\sigma r+a\right)}{s^{\frac{\sigma\vartheta}{k}+2\sigma r+a}}\\ &= \left(\frac{x}{2ks^{\sigma}}\right)^{\frac{\vartheta}{k}}s^{-a}\sum_{r=0}^{\infty}\frac{(-1)^r x^{2r}}{2^{2r}k^r\Gamma\left(\frac{rk+\vartheta+k}{k}\right)r!}\frac{\Gamma\left(\frac{\sigma\vartheta}{k}+2\sigma r+a\right)}{s^{2\sigma r}}.\end{aligned}$$

In view of definition (1.6), we get the desired result. □

Corollary 2.4. *If we set $k=1$ in theorem 2.3, we have*

$$\begin{aligned}&\int_0^{\infty}z^{a-1}e^{-sz}J_v(xz^{\sigma})\,dz\\ &= \left(\frac{x}{2s^{\sigma}}\right)^{\vartheta}s^{-a}\,{}_1\Psi_1\left[\begin{array}{c}(a+\sigma\vartheta,2\sigma)\\(\vartheta+1,1)\end{array}\Bigg|-\frac{x^2}{4s^{2\sigma}}\right].\end{aligned}$$

In the following theorem, we derive the Whittaker transform of generalized $k-$Bessel function. Here, we recall the following results:

$$\int_0^{\infty}t^{v-1}e^{-\frac{t}{2}}W_{\lambda,\mu}(t)\,dt=\frac{\Gamma\left(\frac{1}{2}+\mu+v\right)\Gamma\left(\frac{1}{2}-\mu+v\right)}{\Gamma(1-\lambda+v)},\Re(v\pm\mu)>-1/2, \tag{2.1}$$

where the Whittaker function $W_{\lambda,\mu}(t)$ is given in [11] (also see [12]).

$$W_{\lambda,\mu}(t)=\frac{\Gamma(-2\mu)}{\Gamma\left(\frac{1}{2}-\mu-\lambda\right)}M_{\lambda,\mu}(t)+\frac{\Gamma(2\mu)}{\Gamma\left(\frac{1}{2}+\mu-\lambda\right)}M_{\lambda,-\mu}(t)$$

where $M_{\lambda,\mu}(t)$ is defined as

$$M_{\lambda,\mu}(t)=z^{\frac{1}{2}+\mu}e^{-\frac{1}{2}t}\,{}_1F_1\left(\frac{1}{2}+\mu+v;2\mu+1;t\right).$$

Theorem 2.5. *If $k\in\mathbb{R}^+,\delta,\rho,a,b\in\mathbb{C},\vartheta\in\mathbb{I},\vartheta>-k$ and $\Re\left(\rho\pm2\delta r+\frac{\delta v}{k}+\mu\right)>-\frac{1}{2}$ then*

$$\begin{aligned}&\int_0^{\infty}t^{\rho-1}e^{-\frac{pt}{2}}W_{\lambda,\mu}(pt)\,J_{\vartheta}^{k}\left(wt^{\delta}\right)dt\\ &= p^{1-\rho-\frac{\delta\vartheta}{k}}\left(\frac{1}{2k}\right)^{\frac{\vartheta}{k}}{}_2\Psi_2\left[\begin{array}{c}\left(\frac{1}{2}+\mu+\rho+\frac{\delta\vartheta}{k},2\delta\right),\left(\frac{1}{2}-\mu+\rho+\frac{\delta\vartheta}{k},2\delta\right)\\ \left(\frac{\vartheta}{k}+1,1\right),\left(1-\lambda+\rho+\frac{\delta\vartheta}{k},2\delta\right)\end{array}\Bigg|\frac{-w}{4kp^{2\delta}}\right].\end{aligned}$$

Proof. Let $pt=v$ then

$$\begin{aligned}&\int_0^{\infty}t^{\rho-1}e^{-\frac{pt}{2}}W_{\lambda,\mu}(pt)\,J_{\vartheta}^{k}\left(wt^{\delta}\right)dt\\ &= \int_0^{\infty}e^{-\frac{v}{2}}\left(\frac{v}{p}\right)^{\rho-1}W_{\lambda,\mu}(v)\sum_{n=0}^{\infty}\frac{(-1)^r w^r}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\left[\left(\frac{v}{p}\right)^{\delta}\right]^{2r+\frac{\vartheta}{k}}\frac{1}{p}dv.\end{aligned}$$

Interchanging the integration and summation allow us to write

$$=2^{-\frac{\vartheta}{k}}p^{-\rho-\frac{\delta\vartheta}{k}}\sum_{n=0}^{\infty}\frac{(-1)^r w^r}{2^{2r}p^{2\delta r}\Gamma_k(rk+\vartheta+k)\,r!}\int_0^{\infty}e^{-\frac{v}{2}}v^{\rho+2\delta r+\frac{\delta\vartheta}{k}-1}W_{\lambda,\mu}(v)\,dv,$$

Using the formula for Whittaker transform (2.1), we get

$$
\begin{aligned}
&= 2^{-\frac{\vartheta}{k}}p^{-\rho-\frac{\delta\vartheta}{k}}\sum_{n=0}^{\infty}\frac{1}{\Gamma_k(rk+\vartheta+k)\,r!}\left(-\frac{w}{4p^{2\delta}}\right)^r \\
&\quad\times\frac{\Gamma\left(\frac{1}{2}+\mu+2\delta r+\rho+\frac{\delta\vartheta}{k}\right)\Gamma\left(\frac{1}{2}-\mu+2\delta r+\rho+\frac{\delta\vartheta}{k}\right)}{\Gamma\left(1-\lambda+\rho+2\delta r+\frac{\delta\vartheta}{k}\right)}.
\end{aligned}
$$

In view of (1.1), (1.2) and (1.6), we get the desired result. □

Corollary 2.6. *If we take $k=1$ in Theorem 2.5 then we get*

$$
\begin{aligned}
&\int_0^{\infty}t^{\rho-1}e^{-\frac{pt}{2}}W_{\lambda,\mu}(pt)\,J_{\vartheta}\left(wt^{\delta}\right)dt \\
&= p^{-\rho-\delta\vartheta}2^{-\vartheta}\,{}_2\Psi_2\left[\begin{array}{c}\left(\frac{1}{2}+\mu+\rho+\delta\vartheta,2\delta\right),\left(\frac{1}{2}-\mu+\rho+\delta\vartheta,2\delta\right)\\ (\vartheta+1,1),(1-\lambda+\rho+\vartheta,2\delta)\end{array}\middle|\frac{-w}{4p^{2\delta}}\right].
\end{aligned}
$$

Theorem 2.7. *If $k\in\mathbb{R}^+$,$\vartheta\in\mathbb{I}$,$\vartheta>-k$ and $0<\alpha\leq 1$,then*

$$
\begin{aligned}
&\Im_{\alpha}\left[J_{\vartheta}^{k}(t)\right](w) \\
&= (2wk)^{-\frac{\vartheta}{k}}\sum_{r=0}^{\infty}\frac{\Gamma\left(\frac{\vartheta}{k}+1+2r\right)w^{-\frac{1}{\alpha}}(i)^{-2r-1}}{\Gamma\left(r+\frac{\vartheta}{k}+1\right)r!}\frac{w^{-\frac{2r}{\alpha}}}{(4k)^r}.
\end{aligned}
$$

Proof. Using the definition of 1.4 and FFT, interchanging the integration and summation gives

$$
\begin{aligned}
\Im_{\alpha}\left[J_{\vartheta}^{k}(t)\right](w) &= \int_R \exp\left(iw^{\frac{1}{\alpha}}t\right)\sum_{r=0}^{\infty}\frac{(-1)^r\left(\frac{t}{2}\right)^{2r+\frac{\vartheta}{k}}}{\Gamma_k(rk+\vartheta+k)\,r!} \\
&= \sum_{r=0}^{\infty}\frac{(-1)^r}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\int_R \exp\left(iw^{\frac{1}{\alpha}}t\right)t^{2r+\frac{\vartheta}{k}}dt,
\end{aligned}
$$

Let $iw^{\frac{1}{\alpha}}t=-\xi$ then

$$
\begin{aligned}
&= \sum_{r=0}^{\infty}\frac{(-1)^r}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,r!}\int_{-\infty}^{0}\exp(-\xi)\left(\frac{-\xi}{iw^{\frac{1}{\alpha}}}\right)^{2r+\frac{\vartheta}{k}}\left(\frac{-d\xi}{iw^{\frac{1}{\alpha}}}\right) \\
&= \sum_{r=0}^{\infty}\frac{(-1)^r}{2^{2r+\frac{\vartheta}{k}}\Gamma_k(rk+\vartheta+k)\,(i)^{2r+1}w^{\frac{2r+1}{\alpha}+\frac{\vartheta}{k}}r!}\int_0^{\infty}e^{-\xi}\xi^{2r+\frac{\vartheta}{k}}d\xi.
\end{aligned}
$$

Now using the formula (1.1) and definition of gamma function, we get

$$
=(2)^{-\frac{\vartheta}{k}}k^{-\frac{\vartheta}{k}}\sum_{r=0}^{\infty}\frac{\Gamma\left(\frac{\vartheta}{k}+1+2r\right)(i)^{-2r-1}}{w^{\frac{\vartheta}{k}}\Gamma\left(r+\frac{\vartheta}{k}+1\right)r!}\frac{w^{-\frac{2r+1}{\alpha}}}{(4k)^r}.
$$

which gives the required result. □

Corollary 2.8. *If we take $k=1$ in Theorem 2.7, we get,*

$$
\begin{aligned}
&\Im_{\alpha}\left[J_{\vartheta}^{1}(t)\right](w) \\
&= (2w)^{-\vartheta}\sum_{r=0}^{\infty}\frac{\Gamma(\vartheta+1+2r)\,w^{-\frac{1}{\alpha}}(i)^{-2r-1}}{\Gamma(r+\vartheta+1)\,r!}\frac{w^{-\frac{2r}{\alpha}}}{4^r}.
\end{aligned}
$$

In conclusion, by taking $k=1$ in the main results, we obtained the integral transforms of Bessel function of first kind.

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