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A Review of the Integral Transforms-Based Decomposition Methods and their Applications in Solving Nonlinear PDEs

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Abstract The integral transform based decomposition method comprises of coupling a particular integral transform defined in a time domain and the well-known Adomian decomposition method (ADM) established by G. Adomian. Though, integral transforms are as old as at least two hundred years, but of recent, many researchers seem to turn their attention to the development of this an old area in the last two decades, and more especially this current one. However, with the emergence of these newly initiated integral transforms and the decomposition method on the other hand; the decomposition methods based on these integral transforms and the just introduced transforms that are yet to be coupled with the ADM will be reviewed and coupled respectively. Further, the decomposition method obtained in each mixture (i.e. based on each integral transform) will be utilized to treat nonlinear partial differential equations (PDEs), as the methods are firmly believed to need no linearization, no discretization and no perturbation among others in comparison with other methods for solving nonlinear differential equations.

1 Introduction

Mathematical models encountered in applied mathematics, mathematical physics, and engineering science mostly tend to be nonlinear partial differential equations or even coupled system of nonlinear partial differential equations. These nonlinear equations cannot generally be solved directly via utilizing only the known integral transforms due to the nonlinearity present. However, on coupling these integral transforms with the decomposition method introduced by G. Adomian [18] in particular, a remarkable approximate exact solutions are attained. Besides, integral transforms are tools used in solving linear ordinary/partial differential equations and integral equations among other applications as they are also used in control engineering applications. To talk a little about these integral transforms defined in the time domain; the oldest integral transform and also the most commonly used is the Laplace transform by P.S. Laplace in (1780s) [45]. Others include Stieltjes transform (1894) [56], Mellin transform (1896) [30], Hankel transform [17], Hilbert transform (1912) [11], Radon transform (1917) [34], Laguerre transform (1960) [38] and wavelet transform (1982) [2] among others. Furthermore, of recent, Watugula introduced the Sumudu transform in the year (1993) [22], Khan and Khan (2008) [63] initiated the Natural transform. The Elzaki transform was introduced by [55] in the year (2011) and the Aboodh transform was devised in (2013) by [37]. Other recent integral transforms include the new integral transform developed in (2013) by Kashuri and Fundo [3]. The new integral transform or $\mathbb{M}$-transform [29] was by Srivastava $et\ al$ in (2015) . The ZZ transform was initiated by Zafar [64] in (2016) and finally the Ramadan Group (RG) transform was by Ramadan $et\ al$ in (2016) [49]. On the other hand, the classical Adomian decomposition method (ADM) was established by G. Adomian in 1980s that generates its solution in form of a convergent series whose terms are determined recursively [18]. The convergence and analysis aspect of ADM are discussed in [1, 35-36, 59-61] among others, and it is also regarded as a reliable method for treating both differential/integral and partial differential equation after the successes of many researches such as in [12, 23,44] and a quick review of the method can be seen in [20-21, 32, 39].

To give an overview of the method, consider the more general nonlinear partial differential equation written in an operator form:

$$Lu + Ru + Nu = g, \tag{1.1}$$

where Nu represent the nonlinear terms. Solving for Lu,

$$Lu = g - Ru - Nu. \tag{1.2}$$

Where L is invertible, an equivalent ie expressed as

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu. \tag{1.3}$$

If this corresponds to an initial-value problem, the integral operator L^{-1} may be regarded as definite integrations from 0 to t. As a particular case, if L is a second-order operator, L^{-1} is a two-fold integration operator and

$$L^{-1}Lu = u - u(0) - tu_t(0).$$

Thus, Eq.(1.3) can be written as

$$u = u(0) + tu_t(0) + L^{-1}g - L^{-1}Ru - L^{-1}Nu. \tag{1.4}$$

Now, replacing the unknown function u by an infinite series of u_m's, i.e.,

$$u = \sum_{m=0}^{\infty} u_m, \tag{1.5}$$

and the nonlinear terms Nu by an infinite series of the Adomian polynomials A_m's given by

$$Nu = \sum_{m=0}^{\infty} A_m(u_0, u_1, u_2, ...), \quad m = 0, 1, 2, ..., \tag{1.6}$$

where,

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\Big[N\Big(\sum_{i=0}^{\infty}\lambda^i u_i\Big)\Big]_{\lambda=0}, \quad m = 0, 1, 2, ... \tag{1.7}$$

with few terms as follows:

$$\begin{aligned}
A_0 &= N(u_0),\\
A_1 &= u_1 N'(u_0),\\
A_2 &= u_2 N'(u_0) + \frac{1}{2!}u_1^2 N''(u_0),\\
A_3 &= u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!}u_1^3 N'''(u_0),\\
A_4 &= u_4 N'(u_0) + \left(\frac{1}{2!}u_2^2 + u_1 u_3\right) N''(u_0) + \frac{1}{2!}u_1^2 u_2 N'''(u_0) + \frac{1}{4!}u_1^4 N^{(iv)}(u_0),\\
&\vdots
\end{aligned} \tag{1.8}$$

Using Eq.'s (1.5) and (1.6) into Eq.(1.4) after identifying $u(0) + tu_t(0) + L^{-1}g$ with u_0, we get

$$\sum_{m=0}^{\infty} u_m = u_0 - L^{-1}R\sum_{m=0}^{\infty} u_m - L^{-1}\sum_{m=0}^{\infty} A_m. \tag{1.9}$$

We thus obtain the solution recursively as

$$\begin{aligned}
u_1 &= -L^{-1}Ru_0 - L^{-1}A_0,\\
u_2 &= -L^{-1}Ru_1 - L^{-1}A_1,\\
u_3 &= -L^{-1}Ru_2 - L^{-1}A_2,\\
&\vdots\\
u_{n+1} &= -L^{-1}Ru_n - L^{-1}A_n.
\end{aligned} \tag{1.10}$$

The general solution is written in compact form as

$$\begin{cases} u_0 = u(0) + tu_t(0) + L^{-1}g & , \quad n = 0 \\ u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n & , \quad n \geqslant 0. \end{cases} \tag{1.11}$$

Furthermore, the Adomian decomposition method has undergone several modifications by many researchers in an attempt to improve the efficiency of ADM. Adomian and Rach [19] introduced modified Adomian polynomials which converge slightly faster than the original polynomials and are also convenient for computer computations. Adomian also introduced accelerated Adomian polynomials [20-21].

Wazwaz [7] split the initial iteration in Eq. (1.11) into two, that is

$$u_0 = f_0 + f_1,$$

thereby expressing the general solution in Eq.(1.11) as

$$\begin{cases} u_0 = f_0 & , \quad n = 0 \\ u_1 = f_1 - L^{-1}Ru_0 - L^{-1}A_0 & , \quad n = 1 \\ u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n & , \quad n \geqslant 1. \end{cases} \tag{1.12}$$

Further, Wazwaz and El-Sayed [8] used the Taylor series expansion to expand the initial iteration into an infinite series in what they called the new modification of the Adomian decomposition method for linear and nonlinear operators, that is,

$$u_0 = \sum_{n=0}^{\infty} f_m$$

$$\begin{cases} u_0 = f_0 & , \quad n = 0 \\ u_{n+1} = f_{n+1} - L^{-1}Ru_n - L^{-1}A_n & , \quad n \geqslant 0. \end{cases} \tag{1.13}$$

New modification called the two-step ADM was by Luo [58], the restarted Adomian method [65], to conclude, some modifications of ADM can also be seen in [5-6, 9-10, 28, 33, 41, 50-51, 62]. It is also good however to note that several researchers are of the opinion that the classical ADM is handy and advantageous due to its convenient algorithm that is easily remembered despite all the available modifications obtainable in the literature.

Now, having confirmed and proven the effectiveness and efficiency of ADM through the works of many researchers, this and more are what makes ADM a unique method and further led to coupling it with some available integral transforms defined in the time domain to further devise more methods to solve mostly initial value problems of differential and partial differential equations forms among others. Firstly, on using Laplace transform [45], Khuri [52] used the Laplace transform coupled with ADM and named it Laplace decomposition method (LDM) as a byproduct of ADM to solve some nonlinear differential equations. Khuri [66] applied LDM to Bratu's problem, Islam *et al* [53] and Khan and Austin [57] used LDM respectively in solving various problems. Hussain and Khan [41] and Eltayeb *et al* [26] applied the modified LDM to solve nonlinear and couple nonlinear PDEs and system of Emden-Fowler type equations respectively, and Yin *et al* [67] applied it on Lane-Emden type differential equations. Gadain [24] further used double LDM to treat coupled singular and nonsingular thermoelastic system. For Sumudu transform [16, 22], the Sumudu decomposition method (SDM) [27] was applied by Eltayeb and Kilicman to solve nonlinear system of nonlinear PDEs, Ramadan and Al-Luhaibi [40] used the SDM to solve nonlinear wave-like equations with variable coefficients, while Eltayeb *et al* [25] applied modified SDM on Lane-Emden-type differential equations, see also [13] for SLM. For Natural transform [63], Loonker and Banerji solved the fractional differential equations using Natural decomposition method (NDM) [14], Rawashdeh and Maitama solved coupled systems of nonlinear PDEs using the Natural decomposition method [43] while Suleiman *et al* [54] applied the Natural decomposition method on telegraph equations. Next is the Elzaki transform

[55], the Elzaki decomposition method EDM was applied by Ziane and Hamdi-Cherif [15] to resolve some nonlinear partial differential equations. Khalid *et al* [42] applied EDM to a class of nonlinear differential equations, while Nuruddeen [46] used it in solving linear and nonlinear Schrodinger equations. Further, the Aboodh transform was coupled with ADM by Nuruddeen and Nass [47-48] to solve some heat and wave-like equations, while the fractional diffusion equation by solved Nuruddeen and Aboodh [68] using the same method. Furthermore, the new integral transform developed by Kashuri and Fundo [3] is not found in the literature to be coupled with ADM in this regards; but will be coupled accordingly in this paper. Moreover, the Ramadan Group (RG) transform was by Ramadan *et al* [49] has the same properties with Natural decomposition.

However, in this paper, an attempt shall be made to revisit and review this newly introduced integral transforms coupled with the Adomian decomposition method and utilized simultaneously to tackle some nonlinear partial differential equations arising in real-life applications such as heat conduction equations, wave propagation equations, Burger's equations, telegraph equations and other nonlinear nonhomogeneous partial differential equations among others.

2 Recent Integral Transforms

We present here the some integral transforms starting with the oldest and most used one, the Laplace transform, followed by the some recent transforms such as Sumudu transform, Natural transform, Elzaki transform, Aboodh transform, the new integral transform by Kashuri & Fundo, the new integral transform by Srivastava *et al*, the ZZ transform by Zain Ul Abadin and lastly the Ramadan Group (RG) transform.

2.1 Laplace Transform

The classical Laplace transform of $u(t)$ is defined (in the usual manner) by ([38], [45])

$$\mathfrak{L}[u(t)] = \int_0^\infty u(t)e^{-st}dt = U(s),\ Re(s) > 0. \tag{2.1}$$

The n^{th} order derivative of $u(t)$ with respect to t using Laplace transform is given by

$$\mathfrak{L}\{u^n(t)\} = s^n U(s) - \sum_{k=0}^{n-1} s^{n-k-1}u^k(0). \tag{2.2}$$

2.2 Sumudu Transform

The Sumudu transform over the set A of functions given by

$$A = \{u : |u(t)| < Me^{|t|/k_j}\ \ t \in (-1)^j \times [0,\infty);\ (M, k_1, k_2 > 0\ )\}$$

is defined by ([22])

$$\mathbb{S}[u(t)] = \int_0^\infty u(vt)e^{-t}dt = U(v),\ v \in (-k_1, k_2). \tag{2.3}$$

The n^{th} order derivative of $u(t)$ with respect to t using Sumudu transform is given by

$$\mathbb{S}\{u^n(t)\} = \frac{U(v)}{v^n} - \sum_{k=0}^{n-1} \frac{u^k(0)}{v^{n-k}}. \tag{2.4}$$

2.3 Natural Transform

The Natural transform over the set A of functions given by

$$A = \{u : |u(t)| < Me^{|t|/k_j}\ \ t \in (-1)^j \times [0,\infty);\ (M, k_1, k_2 > 0\ )\}$$

is defined by ([63])

$$\mathbb{N}^{+}[u(t)] = \int_{0}^{\infty} u(vt)e^{-st}dt = U(v,s),\ v,s \in (-k_1,k_2). \tag{2.5}$$

And the n^{th} order derivative of $u(t)$ with respect to t using Natural transform is given by

$$\mathbb{N}^{+}\{u^{n}(t)\} = \frac{s^{n}}{v^{n}}U(v,s) - \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{v^{n-k}} u^{k}(0). \tag{2.6}$$

2.4 Elzaki Transform

The Elzaki transform over the set A of functions given by

$$A = \{u : |u(t)| < Me^{|t|/k_j}\ \ t \in (-1)^{j} \times [0,\infty);\ (M,k_1,k_2 > 0\ )\}$$

is defined by ([55])

$$E[u(t)] = v\int_{0}^{\infty} u(t)e^{-\frac{t}{v}}dt = U(v),\ v \in (k_1,k_2). \tag{2.7}$$

The n^{th} order derivative of $u(t)$ with respect to t using Elzaki transform is given by

$$E\{u^{n}(t)\} = \frac{U(v)}{v^{n}} - \sum_{k=0}^{n-1} v^{2-n+k}u^{k}(0). \tag{2.8}$$

2.5 Aboodh Transform

The Aboodh transform over the set A of functions given by

$$A = \{u : |u(t)| < Me^{-vt}\ \ t \in (-1)^{j} \times [0,\infty);\ (M,k_1,k_2 > 0\ )\}$$

is defined by ([37])

$$A[u(t)] = \frac{1}{v}\int_{0}^{\infty} u(t)e^{-vt}dt = U(v),\ v \in (k_1,k_2). \tag{2.9}$$

The n^{th} order derivative of $u(t)$ with respect to t using Aboodh transform is given by

$$A\{u^{n}(t)\} = v^{n}U(v) - \sum_{k=0}^{n-1} \frac{u^{k}(0)}{v^{2-n+k}}. \tag{2.10}$$

2.6 New Integral Transform by Kashuri & Fundo

The new integral transform over the set A of functions given by

$$A = \{u : |u(t)| \leq Me^{\frac{|t|}{k_j{}^2}}\ \ t \in (-1)^{j} \times [0,\infty);\ (M,k_1,k_2 > 0\ )\}$$

is defined by ([3])

$$K[u(t)] = \frac{1}{v}\int_{0}^{\infty} u(t)e^{-\frac{t}{v^2}}dt = U(v),\ v \in (-k_1,k_2), \tag{2.11}$$

and the n^{th} order derivative of $u(t)$ with respect to t using Kashuri & Fundo transform is given by

$$K\{u^{n}(t)\} = \frac{1}{v^{2n}}U(v) - \sum_{k=0}^{n-1} \frac{u^{k}(0)}{v^{2(n-k)-1}}. \tag{2.12}$$

2.7 New Integral Transform by Srivastava *et al*

If a function $u(t)$ is continuous or piecewise continuous in $[0,\infty)$ satisfying the property that, for given $K>0, T>0$ and $\beta>0$,

$$|u(t)| \leq Kt^{\Re(\rho)e^{\frac{t}{\beta}}} \;\; for\; all\; t>T,$$

then the $\mathbb{M}$-transform of $u(t)$ given by ([29])

$$\mathbb{M}_{\rho,m}[u(t)](s,v) = \int_0^\infty \frac{u(vt)e^{-st}}{(t^m+v^m)^\rho}dt = U(s,v), \tag{2.13}$$

exists for all $v \in (0,\mu)$ and u such that $\Re(u) > \frac{\mu}{\beta}$.

2.8 ZZ Transform

The ZZ transform for any exponential order function $u(t)$ and $t \geq 0$ is defined by ([64]) as

$$H\{u(t)\} = s\int_0^\infty u(vt)e^{-st}dt = U(s,v), \tag{2.14}$$

and the n^{th} order derivative of $u(t)$ with respect to t using ZZ transform is given by

$$H\{u^n(t)\} = \frac{s^n}{v^n}U(s,v) - \sum_{k=0}^{n-1}\frac{s^{n-k}}{v^{n-k}}u^k(0). \tag{2.15}$$

2.9 Ramadan Group (RG) Transform

The RG transform over the set A of functions given by

$$A = \{u : |u(t)| \leq Me^{\frac{|t|}{k_j}} \;\; t \in (-1)^j \times [0,\infty);\; (M, k_1, k_2 > 0\;)\}$$

is defined by ([49])

$$RG[u(t)] = \int_0^\infty u(vt)e^{-st}dt = U(v,s),\; v \in (k_1,k_2), \tag{2.16}$$

and the n^{th} order derivative of $u(t)$ with respect to t using RG transform is given by

$$RG\{u^n(t)\} = \frac{s^n}{v^n}U(v) - \sum_{k=0}^{n-1}\frac{s^{n-k-1}}{v^{n-k}}u^k(0). \tag{2.17}$$

3 Integral Transform Based Decomposition Method

We consider the more general form of nonhomogeneous two-dimensional nonlinear partial differential equation

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = h(x,t), \tag{3.1}$$

with the initial conditions

$$u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad x \in [a,b], \tag{3.2}$$

where $L = \frac{\partial^2}{\partial^2 t}$ is the second order linear differential operator, R is the remaining linear operator of order less than L and $Nu(x,t)$ represents the general nonlinear differential operator with $h(x,t)$ as a nonhomogeneous term.

Furthermore, since we are going to use varieties of the integral transforms stated above coupled with the ADM [18], we now present the integral transform based decomposition method

with an arbitrary integral transform say $\mathbb{P}$.
Now, on applying this arbitrary transform $\mathbb{P}$ in t on both sides of Eq.(3.1) we get

$$\mathbb{P}\{Lu(x,t)\} + \mathbb{P}\{Ru(x,t)\} + \mathbb{P}\{Nu(x,t)\} = \mathbb{P}\{h(x,t)\}. \tag{3.3}$$

Then, on using the differentiation property in each of the various transforms as the case maybe, we get

$$\mathbb{P}\{au(x,t)\} - bu(x,t) - cu_t(x,t) + \mathbb{P}\{Ru(x,t)\} + \mathbb{P}\{Nu(x,t)\} = \mathbb{P}\{h(x,t)\}, \tag{3.4}$$

where $a \neq 0,\ b$ and c are functions of one or two variables coming from the respective integrals as transform parameters. We also assume that $\mathbb{P}^{-1}\{\frac{b}{a}\} = 1$ and $\mathbb{P}^{-1}\{\frac{c}{a}\} = t$.
We thereafter as the next step represent the solution $u(x,t)$ by an infinite series

$$u(x,t) = \sum_{m=0}^{\infty} u_m(x,t), \tag{3.5}$$

and the nonlinear operator $Nu(x,t)$ by

$$Nu(x,t) = \sum_{m=0}^{\infty} A_m, \tag{3.6}$$

where A_m's are the Adomian polynomial given in Eq.(1.7). Thus, on substituting Eq.(3.5) and Eq.(3.6) into Eq.(3.4) we obtain

$$\begin{aligned} \mathbb{P}\{L\sum_{m=0}^{\infty} u_m(x,t)\} - \frac{b}{a}u(x,0) - \frac{c}{a}u_t(x,0) + \frac{1}{a}\mathbb{P}\{R\sum_{m=0}^{\infty} u_m(x,t)\}+ \\ \frac{1}{a}\mathbb{P}\{\sum_{m=0}^{\infty} A_m\} = \frac{1}{a}\mathbb{P}\{h(x,t)\}. \end{aligned} \tag{3.7}$$

Thus, on comparing the both sides of Eq.(3.7) and thereafter taking the inverse transform, after using the initial conditions given in Eq.(3.2) we then obtain the general solution recursively as

$$\begin{cases} u_0(x,t) = f(x) + tg(x) + \mathbb{P}^{-1}\{\frac{1}{a}\mathbb{P}\{h(x,t)\}\} & , \quad n = 0 \\ u_{n+1}(x,t) = -\mathbb{P}^{-1}\{\frac{1}{a}\mathbb{P}\{Ru_n(x,t) + A_n\}\} & , \quad n \geqslant 0. \end{cases} \tag{3.8}$$

4 Application of the Methods

To demonstrate the application of the aforesaid various integral transforms based decompositions, we consider the following initial value problems modelled in nonlinear partial differential equations like nonlinear wave equation, nonlinear heat, Burger's equation and other various nonlinear nonhomogeneous PDEs. Each problem will however be solved using a specific coupling accordingly.

4.1 Example One

Consider the nonlinear PDE [Kashuri *at al* [4]]given by

$$u_t = u_x^2 + uu_{xx}, \tag{4.1}$$

with the initial condition

$$u(x,0) = x^2. \tag{4.2}$$

Here, we couple the **Laplace transform with ADM** on Eq.(4.1), we get the general solution recursively as

$$\begin{cases} u_0(x,t) = u(x,0) & , \quad n = 0 \\ u_{n+1}(x,t) = \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{A_n\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{B_n\}\} & , \quad n \geq 0, \end{cases} \tag{4.3}$$

where A_n's and B_n's are the Adomian polynomials with few terms expressed from Eq.(1.7) as follows

$$A = u_x^2, \tag{4.4}$$

$$\begin{aligned} A_0 &= u_{0_x}^2, \\ A_1 &= 2u_{0x}u_{1_x}, \\ A_2 &= 2u_{0_x}u_{2_x} + u_{1_x}^2, \end{aligned} \tag{4.5}$$

and so on.

$$B = uu_{xx}, \tag{4.6}$$

$$\begin{aligned} B_0 &= u_0 u_{o_{xx}}, \\ B_1 &= u_0 u_{1_{xx}} + u_{0_{xx}} u_1, \\ B_2 &= u_0 u_{2_{xx}} + u_1 u_{1_{xx}} + u_2 u_{0_{xx}}, \end{aligned} \tag{4.7}$$

and so on.
So we get few iterations as follows

$$u_0(x,t) = x^2, \tag{4.8}$$

$$\begin{aligned} u_1(x,t) = &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{A_0\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{B_0\}\}, \\ &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{4x^2\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{2x^2\}\}, \\ &= -\mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{6x^2\}\}, \\ &= 6x^2 t, \end{aligned} \tag{4.9}$$

$$\begin{aligned} u_2(x,t) = &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{A_1\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{B_1\}\}, \\ &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{48x^2 t\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{24x^2 t\}\}, \\ &= -\mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{72x^2 t\}\}, \\ &= 36x^2 t^2, \end{aligned} \tag{4.10}$$

$$\begin{aligned} u_3(x,t) = &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{A_2\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{B_2\}\}, \\ &= \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{432x^2 t^2\}\} + \mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{216x^2 t^2\}\}, \\ &= -\mathfrak{L}^{-1}\{\frac{1}{s}\mathfrak{L}\{648x^2 t^2\}\}, \\ &= 216x^2 t^3, \end{aligned} \tag{4.11}$$

and so on. Thus, summing the above iterations we obtain

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = x^2\Big(1 + (6t) + (6t)^2 + (6t)^3 + \dots\Big), \tag{4.12}$$

which is leading to the exact solution

$$u(x,t) = \frac{x^2}{1-6t}. \tag{4.13}$$

4.2 Example Two

Consider the nonlinear PDE [Biazar [31]]

$$u_t + uu_x = u_{xx}, \tag{4.14}$$

with the initial condition

$$u(x,0) = 2x, t > 0. \tag{4.15}$$

Applying the **Sumudu transform coupled with ADM**, we get the solution of Eq.(4.14) as

$$\begin{cases} u_0(x,t) = u(x,0) \quad , \quad n = 0 \\ u_{n+1}(x,t) = \mathbb{S}^{-1}\{v\mathbb{S}\{u_{n_{xx}}\}\} - \mathbb{S}^{-1}\{v\mathbb{S}\{A_n\}\} \quad , \quad n \geq 0, \end{cases} \tag{4.16}$$

where A_n's are the Adomian polynomials with few terms expressed from Eq.(1.7) as follows

$$A = uu_x, \tag{4.17}$$

$$\begin{aligned} A_0 &= u_0 u_{o_x}, \\ A_1 &= u_0 u_{1_x} + u_{0_x} u_1, \\ A_2 &= u_0 u_{2_x} + u_1 u_{1_x} + u_2 u_{0_x}, \end{aligned} \tag{4.18}$$

and so on. So we get few iterations as follows

$$u_0(x,t) = 2x, \tag{4.19}$$

$$\begin{aligned} u_1(x,t) &= \mathbb{S}^{-1}\{v\mathbb{S}\{u_{0_{xx}}\}\} - \mathbb{S}^{-1}\{v\mathbb{S}\{A_0\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{u_{0_{xx}}\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{4x\}\}, \\ &= -4xt, \end{aligned} \tag{4.20}$$

$$\begin{aligned} u_2(x,t) &= \mathbb{S}^{-1}\{v\mathbb{S}\{u_{1_{xx}}\}\} - \mathbb{S}^{-1}\{v\mathbb{S}\{A_1\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{u_{1_{xx}}\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{-16xt\}\}, \\ &= 8xt^2, \end{aligned} \tag{4.21}$$

$$\begin{aligned} u_3(x,t) &= \mathbb{S}^{-1}\{v\mathbb{S}\{u_{2_{xx}}\}\} - \mathbb{S}^{-1}\{v\mathbb{S}\{A_2\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{u_{2_{xx}}\}\}, \\ &= -\mathbb{S}^{-1}\{v\mathbb{S}\{48xt^2\}\}, \\ &= -16xt^3, \end{aligned} \tag{4.22}$$

and so on. Thus, summing the above iterations we obtain

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = 2x\Big(1 - (2t) + (2t)^2 - (2t)^3 + ...\Big), \tag{4.23}$$

which is leading to the exact solution

$$u(x,t) = \frac{2x}{1+2t}. \tag{4.24}$$

4.3 Example Three

Consider nonlinear nonlinear PDE [Kashuri *at al* [4]]

$$u_{tt} + uu_x = -sin(t), \tag{4.25}$$

with the initial conditions

$$u(x,0) = 0,\ and\ u_t(x,0) = 1. \tag{4.26}$$

Applying the **Natural transform coupled with ADM** to Eq.(4.25), we get

$$\begin{cases} u_0(x,t) = u(x,0) + tu_t(x,0) - \mathbb{N}^-\{\frac{v^2}{s^2}\mathbb{N}^+\{\frac{v}{s^2+v^2}\}\} & , \quad n = 0 \\ u_{n+1}(x,t) = -\mathbb{N}^-\{\frac{v^2}{s^2}\mathbb{N}^+\{A_n\}\} & , \quad n \geq 0, \end{cases} \tag{4.27}$$

where A_n's are the Adomian polynomials with few terms expressed from Eq.(1.7) as follows

$$A = uu_x \tag{4.28}$$

$$\begin{aligned} A_0 &= u_0 u_{o_x}, \\ A_1 &= u_0 u_{1_x} + u_{0_x} u_1, \\ A_2 &= u_0 u_{2_x} + u_1 u_{1_x} + u_2 u_{0_x}, \end{aligned} \tag{4.29}$$

and so on. So we get few iterations as follows

$$\begin{aligned} u_0(x,t) &= t - \mathbb{N}^-\{\frac{v^2}{s^2}\{\frac{v}{s^2+v^2}\}\}, \\ &= t - \{t - sin(t)\}, \\ &= sin(t), \end{aligned} \tag{4.30}$$

$$\begin{aligned} u_1(x,t) &= -\mathbb{N}^-\{\frac{v^2}{s^2}\mathbb{N}^+\{A_0\}\}, \\ &= 0, \end{aligned} \tag{4.31}$$

$$\begin{aligned} u_2(x,t) &= -\mathbb{N}^-\{\frac{v^2}{s^2}\mathbb{N}^+\{A_1\}\}, \\ &= 0, \end{aligned} \tag{4.32}$$

$$\begin{aligned} u_3(x,t) &= -\mathbb{N}^-\{\frac{v^2}{s^2}\mathbb{N}^+\{A_2\}\}, \\ &= 0, \end{aligned} \tag{4.33}$$

and so on. Thus, summing the above iterations we obtain

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = sin(t) + 0 + 0 + 0..., \tag{4.34}$$

which is leading to the exact solution

$$u(x,t) = sin(t). \tag{4.35}$$

Note that, we used the convolution theorem of Natural transform into Eq.(4.25) expressed as

$$\mathbb{N}^+\{f * g\} = vF(v)G(v),$$

where, $F(v) = \mathbb{N}^+\{f\}$ and $G(v) = \mathbb{N}^+\{g\}$. Note also that, the **Ramadan Group (RG) transform** [37] has similar properties with the **Natural transform** [27]; so we use only the Natural transform being the older transform among the two.

4.4 Example Four

Consider nonlinear nonlinear Schroedinger differential equation [Wazwaz [7]]

$$iu_t + u_{xx} + 2|u|^2 u, \tag{4.36}$$

with the initial condition

$$u(x,0) = e^{ix}. \tag{4.37}$$

Applying the **Elzaki transform coupled with ADM** to Eq.(4.36), we obtain the general solution recursively given by

$$\begin{cases} u_0(x,t) = u(x,0) \quad , \quad n = 0 \\ u_{n+1}(x,t) = iE^{-1}\{vE\{u_{n_{xx}}\}\} + 2iE^{-1}\{vE\{A_n\}\} \quad , \quad n \geq 0. \end{cases} \tag{4.38}$$

Where A_n's are the Adomian polynomials to be determined from the nonlinear term

$$Nu = |u|^2 u = u^2 \overline{u}, \tag{4.39}$$

and $\overline{u}$ is the conjugate of u, with few terms using the formula in Eq.(1.7) expressed as:

$$\begin{aligned} A_0 &= u_0^2 \overline{u}_0, \\ A_1 &= 2u_0 u_1 \overline{u}_0 + u_0^2 \overline{u}_1, \\ A_2 &= 2u_0 u_2 \overline{u}_0 + u_1^2 \overline{u}_0 + 2u_0 u_1 \overline{u}_1 + u_0^2 \overline{u}_2, \end{aligned} \tag{4.40}$$

and so on.
We now express few components as follows:

$$u_0(x,t) = e^{ix}, \tag{4.41}$$

$$\begin{aligned} u_1(x,t) &= iE^{-1}\{vE\{u_{0_{xx}}\}\} + 2iE^{-1}\{vE\{A_0\}\} \\ &= iE^{-1}\{i^2 v^3 e^{ix}\} + 2iE^{-1}\{v^3 e^{ix}\}, \\ &= ite^{ix}, \end{aligned} \tag{4.42}$$

$$\begin{aligned} u_2(x,t) &= iE^{-1}\{vE\{u_{1_{xx}}\}\} + 2iE^{-1}\{vE\{A_1\}\} \\ &= iE^{-1}\{i^3 v^4 e^{ix}\} + 2iE^{-1}\{iv^4 e^{ix}\}, \\ &= \frac{-t^2 e^{ix}}{2!}, \end{aligned} \tag{4.43}$$

$$\begin{aligned} u_3(x,t) &= iE^{-1}\{vE\{u_{2_{xx}}\}\} + 2iE^{-1}\{vE\{A_2\}\} \\ &= iE^{-1}\{v^5 e^{ix}\} + 2iE^{-1}\{-v^5 e^{ix}\}, \\ &= \frac{-it^3 e^{ix}}{3!}, \end{aligned} \tag{4.44}$$

and so on. Thus, summing the above iterations we obtain

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = e^{ix}\Big(1 + (it) + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \ldots\Big), \tag{4.45}$$

which is leading to the exact solution

$$u(x,t) = e^{i(x+t)}. \tag{4.46}$$

4.5 Example Five

Consider the nonlinear wave equation [Kaya [12]] given by

$$u_t + u_x^2 = 0, \tag{4.47}$$

with the initial condition

$$u(x,0) = -x^2. \tag{4.48}$$

Applying the **Aboodh transform coupled with ADM** to Eq.(4.47), we get general solution

$$\begin{cases} u_0(x,t) = u(x,0), & n = 0 \\ u_{n+1}(x,t) = -A^{-1}\{\frac{1}{v}A\{A_n\}\}, & n \geq 0. \end{cases} \tag{4.49}$$

where A_n's are the Adomian polynomials to be obtained from the nonlinear term

$$Nu = u_x^2, \tag{4.50}$$

given some few terms as

$$\begin{aligned} A_0 &= u_{o_x}^2, \\ A_1 &= 2u_{0_x}u_{1_x}, \\ A_2 &= 2u_{0_x}u_{2_x} + u_{1_x}^2, \end{aligned} \tag{4.51}$$

and so on. Now, few terms of the solution are as follows

$$u_0(x,t) = -x^2, \tag{4.52}$$

$$\begin{aligned} u_1(x,t) &= -A^{-1}\{\frac{1}{v}A\{A_0\}\}, \\ &= -A^{-1}\{\frac{4x^2}{v^3}\}, \\ &= -4x^2t, \end{aligned} \tag{4.53}$$

$$\begin{aligned} u_2(x,t) &= -A^{-1}\{\frac{1}{v}A\{A_1\}\}, \\ &= -A^{-1}\{\frac{32x^2}{v^4}\}, \\ &= -16x^2t^2, \end{aligned} \tag{4.54}$$

$$\begin{aligned} u_3(x,t) &= -A^{-1}\{\frac{1}{v}A\{A_2\}\}, \\ &= -A^{-1}\{\frac{1}{v}A\{\frac{384x^2}{v^5}\}, \\ &= -64x^2t^3, \end{aligned} \tag{4.55}$$

and so on. Summing the above iterations, we get the solution as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = -x^2\left(1 + 4t + 16t^2 + 64t^3 + ...\right), \tag{4.56}$$

which is leading to the exact solution

$$u(x,t) = \frac{x^2}{4t+1}. \tag{4.57}$$

4.6 Example Six

Consider the nonhomogeneous nonlinear PDE [Kashuri *et al* [4]]

$$u_t - uu_x = 0, \tag{4.58}$$

with the initial condition

$$u(x,0) = x. \tag{4.59}$$

Applying the **Kashuri and Fundo transform coupled with ADM** to Eq.(4.58), we get

$$\begin{cases} u_0(x,t) = u(x,0), & n = 0 \\ u_{n+1}(x,t) = K^{-1}\{v^2 K\{A_n\}\}, & n \geq 0. \end{cases} \tag{4.60}$$

where A_n's are the Adomian polynomials with few terms expressed from Eq.(1.7) as follows

$$A = uu_x, \tag{4.61}$$

$$\begin{aligned} A_0 &= u_0 u_{o_x}, \\ A_1 &= u_0 u_{1_x} + u_{0_x} u_1, \\ A_2 &= u_0 u_{2_x} + u_1 u_{1_x} + u_2 u_{0_x}, \end{aligned} \tag{4.62}$$

and so on.
So we get few iterations as follows

$$u_0(x,t) = x, \tag{4.63}$$

$$\begin{aligned} u_1(x,t) &= K^{-1}\{v^2 K\{A_0\}\}, \\ &= K^{-1}\{v^2 K\{x\}\}, \\ &= K^{-1}\{xv^3\}, \\ &= xt, \end{aligned} \tag{4.64}$$

$$\begin{aligned} u_2(x,t) &= K^{-1}\{v^2 K\{A_1\}\}, \\ &= K^{-1}\{v^2 K\{2xt\}\}, \\ &= K^{-1}\{2xv^5\}, \\ &= xt^2, \end{aligned} \tag{4.65}$$

$$\begin{aligned} u_3(x,t) &= K^{-1}\{v^2 K\{A_2\}\}, \\ &= K^{-1}\{v^2 K\{3xt^2\}\}, \\ &= K^{-1}\{6xv^7\}, \\ &= xt^3, \end{aligned} \tag{4.66}$$

and so on. Thus, on summing the above iterations we get

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = x\left(1 + t + t^2 + t^3 + ...\right), \tag{4.67}$$

which is leading to the exact solution

$$u(x,t) = \frac{x}{1-t}. \tag{4.68}$$

4.7 Example Seven

Consider the one dimensional nonlinear wave-like equation [Ghoreish $at\ al$ [44]]

$$u_{tt} = x^2 \frac{\partial}{\partial x}(u_x u_{xx}) - x^2(u_{xx})^2 - u, \tag{4.69}$$

with the initial conditions

$$u(x,0) = 0,\ and\ u_t(x,0) = x^2. \tag{4.70}$$

Applying the **ZZ transform coupled with ADM** to Eq.(4.69) as described, the above system has the following solution given recursively as

$$\begin{cases} u_0(x,t) = u(x,0) + tu_t(x,0), \quad n = 0 \\ u_{n+1}(x,t) = H^{-1}\{\frac{v^2}{s^2}H\{x^2\frac{\partial}{\partial x}(A_n)\}\} - \\ H^{-1}\{\frac{v^2}{s^2}H\{x^2 B_n\}\} - H^{-1}\{\frac{v^2}{s^2}H\{u_n\}\}, \quad n\ \geq 0. \end{cases} \tag{4.71}$$

where A_n's and B_n's are the Adomian polynomials with few terms expressed from Eq.(1.7) as follows

$$A = u_x u_{xx}, \tag{4.72}$$

$$\begin{aligned} A_0 &= u_{0_x} u_{o_{xx}}, \\ A_1 &= u_{0_x} u_{1_{xx}} + u_{0_{xx}} u_{1_x}, \\ A_2 &= u_{0_x} u_{2_{xx}} + u_{1_x} u_{1_{xx}} + u_{2_x} u_{0_{xx}}, \end{aligned} \tag{4.73}$$

and so on.

$$B = u_{xx}^2, \tag{4.74}$$

and

$$\begin{aligned} B_0 &= u_{0_{xx}}^2, \\ B_1 &= 2u_{0xx} u_{1_{xx}}, \\ B_2 &= 2u_{0_{xx}} u_{2_{xx}} + u_{1_{xx}}^2, \end{aligned} \tag{4.75}$$

and so on. So we get few iterations as follows

$$u_0(x,t) = xt, \tag{4.76}$$

$$\begin{aligned} u_1(x,t) =& H^{-1}\{\frac{v^2}{s^2}H\{x^2\frac{\partial}{\partial x}(A_0)\}\} - H^{-1}\{\frac{v^2}{s^2}H\{x^2 B_0\}\} - H^{-1}\{\frac{v^2}{s^2}H\{u_0\}\}, \\ &= -H^{-1}\{\frac{v^2}{s^2}H\{xt\}\}, \\ &= -H^{-1}\{x\frac{v^3}{s^3}\}, \\ &= -\frac{xt^3}{3!}, \end{aligned} \tag{4.77}$$

$$\begin{aligned} u_2(x,t) =& H^{-1}\{\frac{v^2}{s^2}H\{x^2\frac{\partial}{\partial x}(A_1)\}\} - H^{-1}\{\frac{v^2}{s^2}H\{x^2 B_1\}\} - H^{-1}\{\frac{v^2}{s^2}H\{u_1\}\}, \\ &= -H^{-1}\{\frac{v^2}{s^2}H\{-\frac{xt^3}{3!}\}\}, \\ &= H^{-1}\{x\frac{v^5}{s^5}\}, \\ &= \frac{xt^5}{5!}, \end{aligned} \tag{4.78}$$

$$\begin{aligned}u_3(x,t) =&H^{-1}\{\frac{v^2}{s^2}H\{x^2\frac{\partial}{\partial x}(A_2)\}\} - H^{-1}\{\frac{v^2}{s^2}H\{x^2B_2\}\} - H^{-1}\{\frac{v^2}{s^2}H\{u_2\}\},\\ &= -H^{-1}\{\frac{v^2}{s^2}H\{-\frac{xt^5}{5!}\}\},\\ &= H^{-1}\{-x\frac{v^7}{s^7}\},\\ &= -\frac{-xt^7}{7!},\end{aligned} \tag{4.79}$$

and so on. Thus, on summing the above iterations we get

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = x\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + ...\right), \tag{4.80}$$

which is leading to the exact solution

$$u(x,t) = xsin(t). \tag{4.81}$$

5 Conclusion

In conclusion, the general review of the integral transforms based decomposition methods has been attempted and further applied to solve some nonlinear partial differential equations. The method comprises of coupling a given particular integral transform defined in a time domain with the decomposition method by G. Adomian. Started with the Laplace decomposition method, followed by the recent initiated integral transforms mixed with Adomian decomposition method, such as the Sumudu decomposition method, the Natural decomposition method, the Elzaki decomposition method, the Aboodh decomposition method, the mixture of the new integral transform by Kashuri *et al* with the Adomian decomposition method, and finally the ZZ decomposition method. Finally, the respective methods are highly recommended for solving nonlinear ordinary and partial differential equations as they require no linearization, no discretization and no perturbation among others in comparison with other methods used in solving nonlinear differential equations.

Appendix

Table 1. Properties of Laplace & Sumudu Transform

$u(t)$	Laplace transform	Sumudu Transform
1	$\frac{1}{s}$	1
$t^n, \quad n \geq 0$	$\frac{n!}{S^{n+1}}$	$n!\ v^n$
e^{-at}	$\frac{1}{s+a}$	$\frac{1}{1+av}$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$\frac{av}{1+a^2v^2}$
$cos(at)$	$\frac{s}{s^2+a^2}$	$\frac{1}{1+a^2v^2}$

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Table 2. Properties of Natural & Elzaki Transform

$u(t)$	Natural transform	Elzaki Transform
1	$\frac{1}{s}$	v^2
$t^n, \quad n \geq 0$	$\frac{n! v^n}{s^{n+1}}$	$n!\, v^{n+2}$
e^{-at}	$\frac{1}{s+av}$	$\frac{v^2}{1+av}$
$\sin(at)$	$\frac{av}{s^2+a^2v^2}$	$\frac{av^3}{1+a^2v^2}$
$cos(at)$	$\frac{1}{s^2+a^2v^2}$	$\frac{v^2}{1+a^2v^2}$

Table 3. Properties of Aboodh & New Integral Transform by Kashuri & Fundo

$u(t)$	Aboodh transform	New Integral Transform by Kashuri & Fundo
1	$\frac{1}{v^2}$	v
$t^n, \quad n \geq 0$	$\frac{n!}{v^{n+2}}$	$n! v^{2n+1}$
e^{-at}	$\frac{1}{v^2+av}$	$\frac{v}{1+av^2}$
$\sin(at)$	$\frac{a}{v(v^2+a^2)}$	$\frac{av^2}{1+a^2v^4}$
$cos(at)$	$\frac{1}{(v^2+a^2)}$	$\frac{v^2}{1+a^2v^4}$

Table 4. Properties of ZZ & RG Transform

$u(t)$	ZZ Transform	RG Transform
1	$\frac{v}{s}$	$\frac{v}{s^2}$
$t^n, \quad n \geq 0$	$n! \frac{v^n}{s^n}$	$n! \frac{v^n}{s^{n+1}}$
e^{-at}	$\frac{s}{s+av}$	$\frac{1}{s+av}$
$\sin(at)$	$\frac{avs}{s^2+a^2v^2}$	$\frac{av}{s^2+a^2v^2}$
$cos(at)$	$\frac{s^2}{s^2+a^2v^2}$	$\frac{s}{s^2+a^2v^2}$

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