Palestine Journal of Mathematics

Vol. 7(1)(2018) , 53–59

Coefficient estimates for a subclass of analytic bi-univalent functions by means of Faber polynomial expansions

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Communicated by S.P. Goyal

MSC 2010 Classifications: Primary 30C45, 30C50.

Keywords and phrases: Analytic functions, Univalent functions, Bi-univalent functions, Taylor-Maclaurin series expansion, Coefficient bounds and coefficient estimates, Taylor-Maclaurin coefficients, Faber polynomials.

Abstract In this work, considering a subclass of analytic bi-univalent functions, we determine estimates for the general Taylor-Maclaurin coefficients of the functions in this class. For this purpose, we use the Faber polynomial expansions. In certain cases, our estimates improve some of those existing coeffcient bounds.

1 Introduction

Let $\mathcal{A}$ denote the class of all functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$. We also denote by $\mathcal{S}$ the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.

It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f\left(f^{-1}(w)\right) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function $g = f^{-1}$ is given by

$$\begin{aligned} g(w) &= f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots \\ &= w + \sum_{n=2}^{\infty} A_n w^n. \end{aligned} \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). The class of analytic bi-univalent functions was first introduced and studied by Lewin [25], where it was proved that $|a_2| < 1.51$. Brannan and Clunie [5] improved Lewin's result to $|a_2| \leq \sqrt{2}$ and later Netanyahu [27] proved that $|a_2| \leq 4/3$. Brannan and Taha [6] and Taha [33] also investigated certain subclasses of bi-univalent functions and found non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. For a brief history and interesting examples of functions in the class Σ, see [31] (see also [6]). In fact, the aforecited work of Srivastava *et al.* [31] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years; it was followed by such works as those by Frasin and Aouf [17], Xu *et al.* [35, 36], Hayami and Owa [22], and others (see, for example, [3, 7, 8, 9, 10, 14, 18, 26, 28, 29, 30]).

Not much is known about the bounds on the general coefficient $|a_n|$ for $n > 3$. This is because the bi-univalency requirement makes the behavior of the coefficients of the function f and f^{-1} unpredictable. Here, in this paper, we use the Faber polynomial expansions for a general subclass of analytic bi-univalent functions to determine estimates for the general coefficient bounds $|a_n|$.

The Faber polynomials introduced by Faber [16] play an important role in various areas of mathematical sciences, especially in geometric function theory. The recent publications [19] and [21] applying the Faber polynomial expansions to meromorphic bi-univalent functions motivated us to apply this technique to classes of analytic bi-univalent functions.

In the literature, there is only a few works determined the general coefficient bounds $|a_n|$ for the analytic bi-univalent functions given by (1.1) using Faber polynomial expansions, [2, 11, 12, 13, 20, 23, 24, 32].

Now, we consider a subclass of analytic bi-univalent functions defined by Murugusundaramoorthy *et al.* [26].

Definition 1.1. (See [26]) A function $f \in \Sigma$ given by (1.1) is said to be in the class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$ if the following conditions are satisfied:

$$\Re\left(\frac{zf'(z)}{(1-\lambda) f(z) + \lambda z f'(z)}\right) > \alpha \tag{1.3}$$

and

$$\Re\left(\frac{wg'(w)}{(1-\lambda) g(w) + \lambda w g'(w)}\right) > \alpha \tag{1.4}$$

where $0 \leq \alpha < 1$; $0 \leq \lambda < 1$; $z, w \in \mathbb{U}$ and $g = f^{-1}$ is defined by (1.2).

Note that, for $\lambda = 0$, the class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$ reduces to $\mathcal{S}_{\Sigma}^{*}(\alpha)$ bi-starlike functions of order α $(0 \leq \alpha < 1)$.

Murugusundaramoorthy *et al.* [26] obtained the following coefficient estimates for the functions belonging the class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$.

Theorem 1.2. *[26] Let $f(z)$ given by (1.1) be in the class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$, $0 \leq \alpha < 1$ and $0 \leq \lambda < 1$. Then*

$$|a_2| \leq \frac{\sqrt{2(1-\alpha)}}{1-\lambda}, \tag{1.5}$$

$$|a_3| \leq \frac{4(1-\alpha)^2}{(1-\lambda)^2} + \frac{1-\alpha}{1-\lambda}. \tag{1.6}$$

Later Bulut [9] improved these results as follows:

Theorem 1.3. *[9] Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$. Then*

$$|a_2| \leq \begin{cases} \frac{\sqrt{2(1-\alpha)}}{1-\lambda} & , \quad 0 \leq \alpha \leq \frac{1}{2} \\ \frac{2(1-\alpha)}{1-\lambda} & , \quad \frac{1}{2} \leq \alpha < 1 \end{cases}$$

and

$$|a_3| \leq \begin{cases} \frac{2(1-\alpha)}{(1-\lambda)^2} & , \quad 0 \leq \alpha \leq \frac{3-\lambda}{4} \\ \frac{4(1-\alpha)^2}{(1-\lambda)^2} + \frac{1-\alpha}{1-\lambda} & , \quad \frac{3-\lambda}{4} \leq \alpha < 1 \end{cases}.$$

The object of the present paper is to give an upper bound for the coefficients $|a_n|$ of analytic bi-univalent functions in the class $\mathcal{M}_{\Sigma}(\alpha, \lambda)$ by using Faber polynomials.

2 Coefficient estimates

Using the Faber polynomial expansion of functions $f \in \mathcal{A}$ of the form (1.1), the coefficients of its inverse map $g = f^{-1}$ may be expressed as, [1]:

$$g(w) = f^{-1}(w) = w + \sum_{n=2}^{\infty} \frac{1}{n} K_{n-1}^{-n}(a_2, a_3, \ldots, a_n) w^n, \tag{2.1}$$

where

$$\begin{aligned} K_{n-1}^{-n} &= \frac{(-n)!}{(-2n+1)!\,(n-1)!}a_2^{n-1} + \frac{(-n)!}{(2(-n+1))!\,(n-3)!}a_2^{n-3}a_3 \\ &+\frac{(-n)!}{(-2n+3)!\,(n-4)!}a_2^{n-4}a_4 \\ &+\frac{(-n)!}{(2(-n+2))!\,(n-5)!}a_2^{n-5}\left[a_5+(-n+2)\,a_3^2\right] \\ &+\frac{(-n)!}{(-2n+5)!\,(n-6)!}a_2^{n-6}\left[a_6+(-2n+5)\,a_3a_4\right] \\ &+\sum_{j\geq 7} a_2^{n-j}V_j, \end{aligned} \tag{2.2}$$

such that V_j $(7 \leq j \leq n)$ is a homogeneous polynomial in the variables $a_2, a_3, \ldots, a_n$, [4]. In particular, the first three terms of K_{n-1}^{-n} are

$$K_1^{-2} = -2a_2,$$

$$K_2^{-3} = 3\left(2a_2^2 - a_3\right),$$

$$K_3^{-4} = -4\left(5a_2^3 - 5a_2a_3 + a_4\right).$$

In general, for any $p \in \mathbb{Z} := \{0, \pm 1, \pm 2, \ldots\}$, an expansion of K_n^p is as, [1],

$$K_{n-1}^p = pa_n + \frac{p(p-1)}{2}D_{n-1}^2 + \frac{p!}{(p-3)!\,3!}D_{n-1}^3 + \cdots + \frac{p!}{(p-n+1)!\,(n-1)!}D_{n-1}^{n-1}, \tag{2.3}$$

where $D_{n-1}^p = D_{n-1}^p(a_2, a_3, \ldots, a_n)$, and by [34],

$$D_{n-1}^m(a_2, \ldots, a_n) = \sum \frac{m!}{i_1! \ldots i_{n-1}!} a_2^{i_1} \ldots a_n^{i_{n-1}}$$

and the sum is taken over all non-negative integers $i_1, \ldots, i_{n-1}$ satisfying

$$i_1 + i_2 + \cdots + i_{n-1} = m$$

$$i_1 + 2i_2 + \cdots + (n-1)\,i_{n-1} = n-1.$$

It is clear that $D_{n-1}^{n-1}(a_2, \ldots, a_n) = a_2^{n-1}$.

Consequently, for functions $f \in \mathcal{M}_\Sigma(\alpha, \lambda)$ of the form (1.1), we can write:

$$\frac{zf'(z)}{\mathcal{F}_\lambda(z)} = 1 + \sum_{n=2}^{\infty} F_{n-1}(b_2, b_3, \ldots, b_n)\, z^{n-1}, \tag{2.4}$$

where

$$\mathcal{F}_\lambda(z) = (1-\lambda) f(z) + \lambda z f'(z) = z + \sum_{n=2}^{\infty} b_n z^n$$

with

$$b_n = [1 + (n-1)\lambda]\, a_n.$$

So we get

$$F_{n-1}(b_2, b_3, \ldots, b_n) = (na_n - b_n) + \sum_{j=1}^{n-2} K_j^{-1}(b_2, b_3, \ldots, b_{j+1})\left[(n-j)\,a_{n-j} - b_{n-j}\right].$$

Our first theorem introduces an upper bound for the coefficients $|a_n|$ of analytic bi-univalent functions in the class $\mathcal{M}_\Sigma(\alpha, \lambda)$.

Theorem 2.1. *For* $0 \leq \lambda < 1$ *and* $0 \leq \alpha < 1$, *let the function* $f \in \mathcal{M}_{\Sigma}(\alpha, \lambda)$ *be given by* (1.1). *If* $a_k = 0 \ (2 \leq k \leq n-1)$, *then*

$$|a_n| \leq \frac{2(1-\alpha)}{(n-1)(1-\lambda)} \qquad (n \geq 4).$$

Proof. For the function $f \in \mathcal{M}_{\Sigma}(\alpha, \lambda)$ of the form (1.1), we have the expansion (2.4) and for the inverse map $g = f^{-1}$, considering (1.2), we obtain

$$\frac{wg'(w)}{\mathcal{G}_{\lambda}(w)} = 1 + \sum_{n=2}^{\infty} F_{n-1}(B_2, B_3, \ldots, B_n) w^{n-1} \tag{2.5}$$

where

$$\mathcal{G}_{\lambda}(w) = (1-\lambda) g(w) + \lambda w g'(w) = z + \sum_{n=2}^{\infty} B_n w^n$$

with

$$B_n = [1 + (n-1)\lambda] A_n.$$

On the other hand, since $f \in \mathcal{M}_{\Sigma}(\alpha, \lambda)$ and $g = f^{-1} \in \mathcal{M}_{\Sigma}(\alpha, \lambda)$, by definition, there exist two positive real part functions

$$p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \in \mathcal{A}$$

and

$$q(w) = 1 + \sum_{n=1}^{\infty} d_n w^n \in \mathcal{A},$$

where

$$\Re(p(z)) > 0 \quad \text{and} \quad \Re(q(w)) > 0$$

in $\mathbb{U}$ so that

$$\frac{zf'(z)}{\mathcal{F}_{\lambda}(z)} = \alpha + (1-\alpha) p(z) \tag{2.6}$$

and

$$\frac{wg'(w)}{\mathcal{G}_{\lambda}(w)} = \alpha + (1-\alpha) q(w). \tag{2.7}$$

Note that, by the Caratheodory lemma (e.g., [15]), $|c_n| \leq 2$ and $|d_n| \leq 2$ $(n \in \mathbb{N} := \{1, 2, \ldots\})$. Comparing the corresponding coefficients of (2.4) and (2.6), for any $n \geq 2$, yields

$$(na_n - b_n) + \sum_{j=1}^{n-2} K_j^{-1}(b_2, b_3, \ldots, b_{j+1}) [(n-j) a_{n-j} - b_{n-j}] = (1-\alpha) c_{n-1} \tag{2.8}$$

and similarly, from (2.5) and (2.7) we find

$$(nA_n - B_n) + \sum_{j=1}^{n-2} K_j^{-1}(B_2, B_3, \ldots, B_{j+1}) [(n-j) A_{n-j} - B_{n-j}] = (1-\alpha) d_{n-1}. \tag{2.9}$$

Note that for $a_k = 0 \ (2 \leq k \leq n-1)$, we have $A_n = -a_n$ and so

$$na_n - b_n = (n-1)(1-\lambda) a_n = (1-\alpha) c_{n-1},$$

$$nA_n - B_n = -(n-1)(1-\lambda) a_n = (1-\alpha) d_{n-1}.$$

Taking the absolute values of the above equalities, we obtain

$$|a_n| = \frac{(1-\alpha)|c_{n-1}|}{(n-1)(1-\lambda)} = \frac{(1-\alpha)|d_{n-1}|}{(n-1)(1-\lambda)} \leq \frac{2(1-\alpha)}{(n-1)(1-\lambda)},$$

which completes the proof of the Theorem 2.1. □

The following corollary is immediate consequence of the above theorem.

Corollary 2.2. *Let the function* $f \in \mathcal{S}_{\Sigma}^{*}(\alpha)$ $(0 \le \alpha < 1)$ *be given by* (1.1). *If* $a_k = 0$ $(2 \le k \le n-1)$, *then*

$$|a_n| \le \frac{2(1-\alpha)}{n-1} \qquad (n \ge 4).$$

Theorem 2.3. *For* $0 \le \lambda < 1$ *and* $0 \le \alpha < 1$, *let the function* $f \in \mathcal{M}_{\Sigma}(\alpha, \lambda)$ *be given by* (1.1). *Then one has the following*

$$|a_2| \le \begin{cases} \frac{\sqrt{2(1-\alpha)}}{1-\lambda} & , \quad 0 \le \alpha < \frac{1}{2} \\ \frac{2(1-\alpha)}{1-\lambda} & , \quad \frac{1}{2} \le \alpha < 1 \end{cases}, \tag{2.10}$$

$$|a_3| \le \begin{cases} \frac{2(1-\alpha)}{(1-\lambda)^2} & , \quad 0 \le \alpha \le \frac{3-\lambda}{4} \\ \frac{4(1-\alpha)^2}{(1-\lambda)^2} + \frac{1-\alpha}{1-\lambda} & , \quad \frac{3-\lambda}{4} \le \alpha < 1 \end{cases}. \tag{2.11}$$

Proof. If we set $n = 2$ and $n = 3$ in (2.8) and (2.9), respectively, we get

$$(1-\lambda)\, a_2 = (1-\alpha)\, c_1, \tag{2.12}$$

$$(1-\lambda)\left[2a_3 - (1+\lambda)\, a_2^2\right] = (1-\alpha)\, c_2, \tag{2.13}$$

$$-(1-\lambda)\, a_2 = (1-\alpha)\, d_1, \tag{2.14}$$

$$(1-\lambda)\left[(3-\lambda)\, a_2^2 - 2a_3\right] = (1-\alpha)\, d_2. \tag{2.15}$$

From (2.12) and (2.14), we find (by the Caratheodory lemma)

$$|a_2| = \frac{(1-\alpha)\,|c_1|}{1-\lambda} = \frac{(1-\alpha)\,|d_1|}{1-\lambda} \le \frac{2(1-\alpha)}{1-\lambda}. \tag{2.16}$$

Also from (2.13) and (2.15), we obtain

$$2(1-\lambda)^2 a_2^2 = (1-\alpha)(c_2 + d_2). \tag{2.17}$$

Using the Caratheodory lemma, we get

$$|a_2| \le \frac{\sqrt{2(1-\alpha)}}{1-\lambda},$$

and combining this with the inequality (2.16), we obtain the desired estimate on the coefficient $|a_2|$ as asserted in (2.10).

Next, in order to find the bound on the coefficient $|a_3|$, we subtract (2.15) from (2.13). We thus get

$$a_3 = a_2^2 + \frac{(1-\alpha)(c_2 - d_2)}{4(1-\lambda)}. \tag{2.18}$$

Upon substituting the value of a_2^2 from (2.12) into (2.18), it follows that

$$a_3 = \frac{(1-\alpha)^2 c_1^2}{(1-\lambda)^2} + \frac{(1-\alpha)(c_2 - d_2)}{4(1-\lambda)}.$$

We thus find (by the Caratheodory lemma) that

$$|a_3| \le \frac{4(1-\alpha)^2}{(1-\lambda)^2} + \frac{1-\alpha}{1-\lambda}. \tag{2.19}$$

On the other hand, upon substituting the value of a_2^2 from (2.17) into (2.18), it follows that

$$a_3 = \frac{1-\alpha}{4(1-\lambda)^2}\left[(3-\lambda)c_2 + (1+\lambda)d_2\right].$$

Consequently, (by the Caratheodory lemma) we have

$$|a_3| \leq \frac{2(1-\alpha)}{(1-\lambda)^2}. \tag{2.20}$$

Combining (2.19) and (2.20), we get the desired estimate on the coefficient $|a_3|$ as asserted in (2.11). This evidently completes the proof of Theorem 2.3. □

Note that, Theorem 2.3 gives another proof of Theorem 1.3.

By setting $\lambda = 0$ in Theorem 2.3, we obtain the following consequence.

Corollary 2.4. *Let the function $f \in \mathcal{S}_\Sigma^*(\alpha)$ $(0 \leq \alpha < 1)$ be given by (1.1). Then one has the following*

$$|a_2| \leq \begin{cases} \sqrt{2(1-\alpha)} & , \quad 0 \leq \alpha < \frac{1}{2} \\ 2(1-\alpha) & , \quad \frac{1}{2} \leq \alpha < 1 \end{cases},$$

$$|a_3| \leq \begin{cases} 2(1-\alpha) & , \quad 0 \leq \alpha \leq \frac{3}{4} \\ 4(1-\alpha)^2 + (1-\alpha) & , \quad \frac{3}{4} \leq \alpha < 1 \end{cases}.$$

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Received: October 24, 2016.

Accepted: February 19, 2017.