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NUMERICAL IDENTIFICATION OF ROBIN COEFFICIENT BY ITERATIVE METHOD

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Abstract In this paper, we consider an inverse problem associated with the Laplace's equation of determining the robin coefficient of some specimen material by performing measurements on some part of the boundary. This problem leads to the resolution of a data completion problem to determine the unknown Dirichlet and Neumann conditions on the inaccessible part of the boundary. We propose a variant of an iterative algorithm that perform the standard one to identify the parameter. The algorithm are implemented using the finite element method. The numerical results is performed in the case of a tube 2D represented by an annular domain and irregular square domain showing the effectiveness and the robustness of the proposed algorithm.

1 Introduction

We consider an inverse problem for the Laplace's equation called problem of identifying the robin coefficient. This problem arises and can be encountered as challenge in several areas of engineering. In particular; pipes for transport of water, gas and oil under the sea or even on the surface of the earth are subject to be attacked by an internal and external corrosion phenomenon or cracking, which is the main cause of leaks and ruptures of pipes, sometimes resulting in catastrophic damage either the human level or pollution of the environment.

Such problem arises for instance, in corrosion detection problem. The corrosion may occur in many different forms and several models of this problem are encountered in the literature [1], [2]. Identifying the robin coefficient from the over-determined measures of the boundary turns out to be a way to locate the corroded part in a structure, and possibly evaluate the damage level by an electrical impedance tomography process (EIT) [3].

In this work, we are interested in determining the robin coefficient also called electrical impedance [4], [5], [6] for identifying corrosion occurred in the inaccessible part of boundary, which is the quotient of the extended data, from the observed data on the accessible part. This leads us to calculate the unknown Dirichlet and Neumann conditions on the inaccessible part of the boundary. This problem is called data completion problem.

This parameter has been the subject of several studies. In particular, the stability, i.e. the continuous dependence of the unknown parameter on the measured data which is a crucial issue for numerical applications. This has been the concern of many authors [7], [8], [9], [10]. Some results of identifiability are also shown by [4], [11]. An identification algorithm proposed by [12] based on the least squares minimization, the algorithm consists of comparing solutions corresponding to Robin-Dirichlet and Robin-Neumann boundary conditions, which coincide at the actual solution. This method smooths out possible oscillations in the impedance which may gives information on the regions of corrosion.

Since the robin coefficient may be recovered from the completed cauchy data, this problem reduces to solving a Cauchy problem for Laplace operator. The data completion problem, which aims at recovering missing conditions on some inaccessible part of the boundary from the over specified boundary data on the remaining part, is ill-posed in the sense of Hadamard [13], since existence, uniqueness and stability of the solution are not always assured [14], [15].

Solving this problem by direct method is very difficult and leads to unstable solutions. Then, Many performing numerical methods have been developed to overcome the ill-posed nature of this kind of problem. Among recent approaches to the cauchy problem, we mention the method of quasi-reversibility introduced by Lattés since 1960, and recently by [16], [17], [18], thikhonov method [19] and iterative method [20], [21], [22], [23] .

In this work, we consider an algorithm based on the iterative algorithm addressed by Kozlov, Mazya and Fomin since 1991 [24], also called alternating method to complete Dirichlet and Neumann conditions on the boundary; then, we calculate the desired coefficient. To deal with the large number of iterations required to achieve the convergence with the iterative algorithm, we propose a new variant to overcome this problem with more precision.

The next section is devoted to the mathematical formulation of the problem. In the third section, we present the algorithm for solving the robin coefficient problem based on the altenating method to determine the missing conditions. In addition, we consider a New algorithm and exhibit the relationship it has with the classical one. Finally, section 4 presents a numerical examples showing the feasibility of the alternating formulation and its ability to find an approximate solution more accurately in less iterations.

2 Setting of the problem

Let Ω be a bounded connex domain of $\mathbb{R}^2$ representing the specimen to control and we assume that the boundary $\partial\Omega$ is at least piecewise.
Γ_0 and Γ_1 are two disjoint closed sub-parts of $\partial\Omega$, where Γ_0 is the accessible part; however, Γ_1 is the inaccessible part where the corrosion has occurred.
The electric potential u solves the Laplace equation in Ω:

$$-\Delta u(x) = 0 \quad \text{in} \quad x \in \Omega \tag{2.1}$$

On the accessible part Γ_0, the Dirichlet and Neumann data of the electric potential u are given by:

$$u(x) = f(x) \quad , \quad \partial_n u(x) = g(x) \quad \text{on} \quad x \in \Gamma_0 \tag{2.2}$$

where $\partial_n u$ is the normal derivative of u.
We assume that the corrosion is only happend in the interior boundary of the domain Ω and the corrosion can be described by a non-negative function γ in the boundary condition on the inaccessible part. That is:

$$\partial_n u(x) + \gamma(x)u(x) = 0 \quad \text{on} \quad x \in \Gamma_1 \tag{2.3}$$

We consider that the entire exterior part is accessible and measurements can be carried. However, the inside part is inaccessible.

The inverse problem is to find $\gamma(x)$ from the knowledge of the Cauchy data f and g in the accessible part of the boundary considering two steps:
The first step is to complete the data on the inaccessible part Γ_1, which amounts to solving data completion problem given by:

$$\begin{cases} -\Delta u(x) = 0 & in \quad \Omega \\ u(x) = f(x) & on \quad \Gamma_0 \\ \partial_n u(x) = g(x) & on \quad \Gamma_0 \end{cases} \tag{2.4}$$

where the goal is to get the Cauchy data $u(x)$ and $\partial_n u(x)$ on Γ_1.
The second step consists to get the impedance γ that can be obtained by:

$$\gamma(x) = -\frac{\partial_n u(x)}{u(x)}_{/\Gamma_1} \quad \text{if} \quad u(x) \neq 0 \tag{2.5}$$

where $\gamma(x)$ represents the dammage of corrosion.

3 Resolution of the problem

Taking into account the multiple advantages of the group of iterative methods which allows to any physical constraints to be easily taken into account directly in the scheme of the iterative algorithm and the simplicity of their implementation, we are interested in this paper to investigate the iterative method KMF developed by [24] for solving the data completion problem (4) that is an ill-posed problem, and then calculate the robin coefficient.

3.1 Description of the standard algorithm

The iterative algorithm investigated is based on reducing this ill-posed problem to a sequence of mixed well-posed boundary value problems and consists of the following steps:
Step 1. Specify an initial data u_0 on Γ_1
Step 2. Solve the well posed problem:

$$\begin{cases} \Delta u^{(0)} = 0 & in \quad \Omega \\ u^{(0)} = u_0 & on \quad \Gamma_1 \\ \partial_n u^{(0)} = g & on \quad \Gamma_0 \end{cases} \tag{3.1}$$

to obtain $v_0 = \partial_n u^{(0)}_{/\Gamma_1}$

then $\gamma_0 = -\frac{v_0}{u_0}{}_{/\Gamma_1}$

Step 3. for $n \geq 1$, solve alternatively the following two well-posed problems:

$$\begin{cases} \Delta u^{(2n-1)} = 0 & in \quad \Omega \\ \partial_n u^{(2n-1)} = v_{n-1} & on \quad \Gamma_1 \\ u^{(2n-1)} = f & on \quad \Gamma_0 \end{cases} \quad \text{and} \quad \begin{cases} \Delta u^{(2n)} = 0 & in \quad \Omega \\ u^{(2n)} = u_n & on \quad \Gamma_1 \\ \partial_n u^{(2n)} = g & on \quad \Gamma_0 \end{cases} \tag{3.2}$$

to obtain $\quad u_n = u^{(2n-1)}_{/\Gamma_1}$ to obtain $\quad v_n = \partial_n u^{(2n)}_{/\Gamma_1}$

then $\gamma_n = -\frac{v_n}{u_n}{}_{/\Gamma_1}$
Step 4. Repeat step 3 until the stop criterion E is satisfied.

We have chosen the following stop criterion:

$$E = \|\gamma_n - \gamma_{n+1}\|_{0,\Gamma_1} \leq \epsilon \tag{3.3}$$

where ϵ is a positive real number small enough.

3.2 Convergence of the iterative algorithm

In this section, we recall the convergence results proposed in the work of Kozlov et al. [24] to complete the missing data in the case of a connected open domain.

Theorem 3.1. *For a compatible data, the sequence $(u^k)_k$ converge in $H^1(\Omega)$ to the solution of the Cauchy problem (2.4) for any initial choice $u_0 \in H^{(\frac{1}{2})}(\Gamma_1)$.*

Let us also recall the convergence result given by Baumeister et al. A demonstration can be found in [25].

Theorem 3.2. *(a) If the Cauchy problem has a unique solution $u \in H^1(\Omega)$ then the sequence $(u_n)_{n\geq 0}$ defined in the algorithm converges to $u_{/\Gamma_1}$ for the norm of $H^{(\frac{1}{2})}(\Gamma_1)$.*

(b) If the sequence $(u_n)_{n\geq 0}$ defined in the algorithm converges in $H^{(\frac{1}{2})}(\Gamma_1)$ then it converges to $u_{/\Gamma_1}$ where $u \in H^1(\Omega)$ is the unique solution of the Cauchy problem.

Remark 3.3. (a) The same conclusion can be obtained if at the step 1, one considers a given initial guess the form $v_0 \in H^{-\frac{1}{2}}(\Gamma_0)$ instead of the initial guess $u_0 \in H^{\frac{1}{2}}(\Gamma_0)$, and modifies accordingly problems in the step (3) of the algorithm.

(b) The algorithm would not converge if in the step 3 the mixed problems were replaced by Dirichlet or Neumann problems.

3.3 Description of the proposed algorithm

The standard algorithm was found to produce an accurate and stable numerical solutions for the cauchy problem. However, one possible disadvantage of the method is the large number of iterations necessary to achieve convergence if the initial guess is far from the exact solution.

Following numerical simulations carried out to solve a data completion problem, we observed that the measurement of the inaccessible part influences the results. Thus, the smaller the measurement, the better the results [26]. Hence, the idea of subdividing the inaccessible part into two parts and approaching the unknown data on the two sub-parts in alternative way in the context of identification problem of robin coefficient.

For this, we consider $\Gamma_1 = \Gamma_{1,1} \cup \Gamma_{1,2}$ such that $\Gamma_{1,1} \cap \Gamma_{1,2} = \emptyset$ and $mes(\Gamma_{1,1}) = mes(\Gamma_{1,2})$.

The algorithm consists of the following steps:
Step 1. Specify an initial guess u_0 on Γ_1 and solve:

$$\begin{cases} -\Delta u^{(0)} = 0 & in \quad \Omega \\ u^{(0)} = u_0 & on \quad \Gamma_{1,1} \cup \Gamma_{1,2} \\ \partial_n u^{(0)} = g & on \quad \Gamma_0 \end{cases} \tag{3.4}$$

to obtain $v_{1,0} = \partial_n u^{(0)}_{/\Gamma_{1,1}}$ and $v_{2,0} = \partial_n u^{(0)}_{/\Gamma_{1,2}}$

then $\gamma_0 = -\frac{v_0}{u_0}{}_{/\Gamma_1}$

Step 2. For $n \geq 0$, solve the two well-posed problems:

$$\begin{cases} -\Delta u^{(2n-1)} = 0 & in \quad \Omega \\ \partial_n u^{(2n-1)} = v_{1,n-1} & on \quad \Gamma_{1,1} \\ \partial_n u^{(2n-1)} = v_{2,n-1} & on \quad \Gamma_{1,2} \\ u^{(2n-1)} = f & on \quad \Gamma_0 \end{cases} \quad \text{and} \quad \begin{cases} -\Delta u^{(2n-1)} = 0 & in \quad \Omega \\ u^{(2n-1)} = u_{1,n} & on \quad \Gamma_{1,1} \\ \partial_n u^{(2n-1)} = v_{2,n-1} & on \quad \Gamma_{1,2} \\ u^{(2n-1)} = f & on \quad \Gamma_0 \end{cases} \tag{3.5}$$

to obtain $u_{1,n} = u^{(2n-1)}_{/\Gamma_{1,1}}$ to obtain $u_{2,n} = u^{(2n-1)}_{/\Gamma_{1,2}}$

Step 3. For $n \geq 0$, solve the two well-posed problems:

$$\begin{cases} -\Delta u^{(2n)} = 0 & in \quad \Omega \\ u^{(2n)} = u_{1,n} & on \quad \Gamma_{1,1} \\ u^{(2n)} = u_{2,n} & on \quad \Gamma_{1,2} \\ \partial_n u^{(2n)} = f & on \quad \Gamma_0 \end{cases} \quad \text{and} \quad \begin{cases} -\Delta u^{(2n)} = 0 & in \quad \Omega \\ \partial_n u^{(2n)} = v_{1,n} & on \quad \Gamma_{1,1} \\ u^{(2n)} = u_{2,n} & on \quad \Gamma_{1,2} \\ \partial_n u^{(2n)} = g & on \quad \Gamma_0 \end{cases} \tag{3.6}$$

to obtain $v_{1,n} = \partial_n u^{(2n)}_{/\Gamma_{1,1}}$ to obtain $v_{2,n} = \partial_n u^{(2n)}_{/\Gamma_{1,2}}$

then $\gamma_n = -\frac{v_n}{u_n}{}_{/\Gamma_1} = -\frac{\partial_n u^{(2n)}}{u^{(2n-1)}}{}_{/\Gamma_1}$

Step 4. Repeat the step 3 - 4 until a prescribed stopping criterion is satisfed.

Remark 3.4. It should be noted that:

(a) The developed algorithm used in the identification of the robin coefficient can be seen as two parallel problems of the standard algorithm. These two problems are initialized with the same initial data. Each problem allows obtaining an approximation in each subpart $\Gamma_{1,i}$ where $i = 1, 2$ (for the approximation in $\Gamma_{1,1}$ the two first well-posed problems in (3-5) and (3-6), for the approximation in $\Gamma_{1,2}$ the two second well-posed problems in (3-5) and (3-6).

(b) Each solved problem allows an approximation in one of the inaccessible sub-parts that can be introduced in the other well-posed problems.

(c) The robin coefficient is calculated after each iteration, i.e. after solving the well-posed problems in step 3 and 4.

4 Numerical results and discussion

For the implementation of the proposed algorithm to identify the robin coefficient, we use the finite element method with continuous piecewise linear polynomials to resolve the well-posed problem described in the algorithm which provide simultaneously the unspecified boundary, namely, the Dirichlet and Neumann conditions.
As programming software, we used the FreeFem++ software dedicated to the finite element method.
The convergence of the algorithm may be investigated by evaluating at every iteration the error:

$$e_\gamma = \|\gamma_n - \gamma_{ex}\|_{0,\Gamma_1} \tag{4.1}$$

where, γ_n is the obtained approximation for the function γ after n iterations and γ_{ex} is the exact solution of the problem.
We can also evaluate at every iteration the errors:

$$e_u = \|u_n - u_{ex}\|_{0,\Gamma_1} \quad \text{and} \quad e_v = \|\partial_n u_n - \partial_n u_{ex}\|_{0,\Gamma_1} \tag{4.2}$$

where, u_n is the approximation obtained of the function on the boundary Γ_0 after n iterations and u_{ex} is the exact solution of the Cauchy problem.

4.1 Example 1:

In this example, we consider an annular domain defined by:

$$\Omega = \{(x, y) \in \mathbb{R}^2 / R_1^2 \leq x^2 + y^2 \leq R_0^2\} \tag{4.3}$$

with $R_1 = 0.5$ and $R_0 = 1$.

$$\Gamma_0 = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 = R_0^2\} \tag{4.4}$$

$$\Gamma_1 = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 = R_1^2\} \tag{4.5}$$

The analytic function solution of data completion problem is given by:

$$u_{ex} = exp(y)cos(x) \tag{4.6}$$

The obtained solution with the software FreeFem++ for u is given in Figure 1. showing that the proposed algorithm allows to achieve a good approximation of the exact solution u.

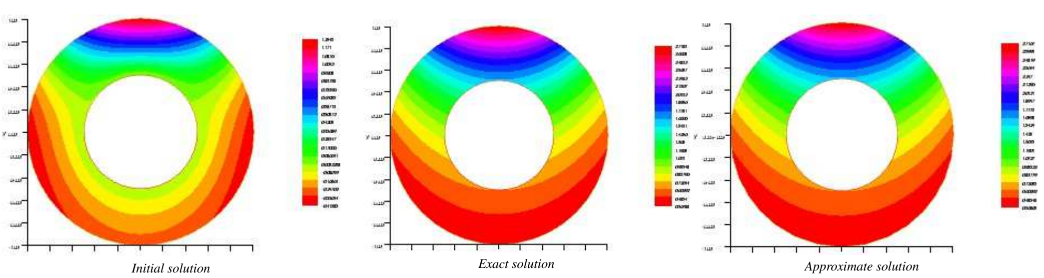
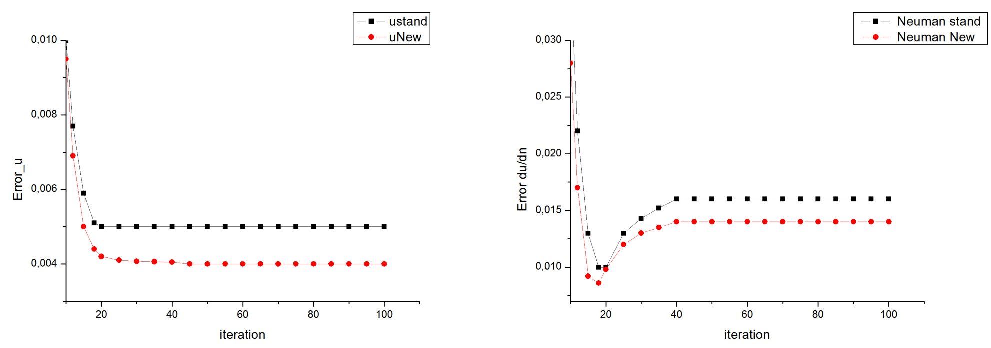


Figure 1. Numerical solution u with the proposed algorithm in comparison with initial solution and exact solution.

The Figure 2. and Figure 3. Present a comparison between the numerical results e_u, e_v and e_γ obtained with the standard algorithm and the developed algorithm.
It can be seen that the algorithm proposed decreases considerably the number of iterations necessary to achieve the convergence that can be reduced, and present a more accurate approximations for both Dirichlet and Neumann missing data.



Error e_u *Error* e_u

Figure 2. The errors e_u and e_v with the standard algorithm and the developed algorithm according to number of iterations.

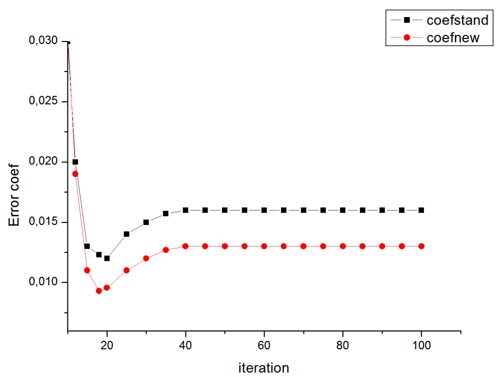


Figure 3. The error e_γ with stand algorithm and the proposed algorithm according to number of iterations.

4.2 Example 2:

In this second example, we consider the example with non-smooth boundary, such as a square $\Omega = (0,1) \times (0,1)$.

Namely, the analytical harmonic potential to be retrieved is given by:

$$u(x,y) = cos(x)cosh(y) + sin(x)sinh(y) \tag{4.7}$$

We consider that the corrosion is produced in the under-specified boundary which is taken to be $\Gamma_0 = \{0\} \times (0,1)$, while the over-specified boundary is $\Gamma_2 = \{1\} \times (0,1)$.
The known boundaries are given by $\Gamma_1 = (0,1) \times \{0\}$ and $\Gamma_3 = (0,1) \times \{1\}$.

It is easy to verify that the known conditions are:
$u_{/\Gamma_1} = cos(x)$
$u_{/\Gamma_2} = cos(1) \times cosh(y) + sin(1) \times sinh(y)$
$\partial_n u_{/\Gamma_3} = cos(x)sinh(1) + sin(x)cosh(1)$

and the robin coefficient is given by:

$$\gamma(y) = \frac{sinh(y)}{cosh(y)} \tag{4.8}$$

For the step 1 of the algorithm, as an initial guess $u_0 \in H^{1/2}(\Gamma_0)$, we have chosen $u_0(y) = 1 + y(-L + sinh(L)) + y^2/2, y \in [0,1]$, which also ensures the continuity of $\partial u/\partial y$ at the corner $\overline{\Gamma_0} \cap \overline{\Gamma_3}$ and provides that the initial guess is not too close to the exact value of u_{ex}.

In this example, we use a finite element method with continuous piecewise linear polynomials to provide simultaneously the unspecified conditions of Dirichlet and Neumann. Then, we calculate the robin coefficient.

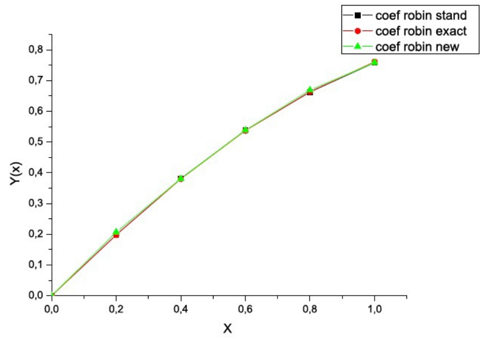


Figure 4. Numerical solution of the robin coefficient with standard algorithm and proposed algorithm in comparison with the exact solution.

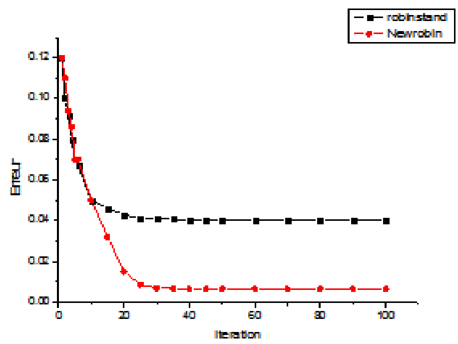


Figure 5. The error e_γ with standard algorithm and the proposed algorithm according to number of iteratins.

Figure 4. shows the numerical results obtained for the coefficient robin algorithm with the standard and the proposed algorithm showing the efficacy and robustness of the two algorithms.

Concerning Figure 5., a comparative study of the error e_γ is made during the iterative process showing that the proposed algorithm can lead to more accurate results in a reduced number of iterations.

Figure 6. shows the evolution of the function γ during iterations, showing that after a reduced number of iterations, with is not very closed initialization, we have already come to a very good approximation of the desired coefficient.

5 Conclusion

In this work, a problem of identification of the robin coefficient is presented. The calculation of this parameter requires the construction of missing data (Dirichlet and Neumann). A developed iterative algorithm is proposed to calculate the parameter in a few number of iterations and more accurately. The numerical results presented in the case of an annular domain and square domain show the effectiveness and the robustness of the proposed algorithm.

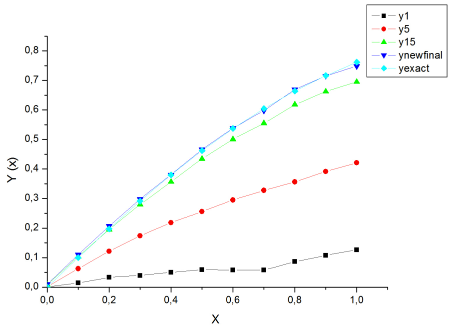


Figure 6. The function $\gamma(x)$ during the iteration process.

References

[1] P. Kaup, F. Santosa and M. vogelius, A method for imaging corrosion damage in thin plates from electrostatic data, *Inverse Problems*, **12**, 279–293 (1996).

[2] P. Kaup and F. Santosa, Nondestructive evaluation of corrosion damage using electrostatic measurements, *J. Nondestruct. Eval.* **14**, 127–136 (1995).

[3] F. Santosa, M. Vogelius and J. M. Xu, An effective nonlinear boundary condition for a corroding surface. Identification of the damage based on steady state electric data, *Z. Angew. Math. Phys.* **49**, 656–679 (1998).

[4] G. Inglese, An inverse problem in corrosion detection, *Inverse Problems* **13**, 977–994 (1997).

[5] S. Chaabane and M. Jaoua, Identification of Robin coefficients by the means of boundary measurements, *Inverse Problems* **25**, 1425–1438 (1999).

[6] S. Chaabane, M. Jaoua and J. Leblond, Parametrer identification for laplace equation and approximation in Hardy classes, *J. Inv. P. Prob.* **66**, 367–383 (2004).

[7] M. Choulli, An inverse problem in corrosion detection stability estimates, *J.Inv. p. Prob.* **12**, 349–367 (2004).

[8] S. Chaabane, C. Elhechmi and M. Jaoua, A stable recovery algorithm for the Robin inverse problem, *Math. and Comput. in Simul.* **11**, 33–57 (2003).

[9] S. Chaabane, I. Fellah, M. Jaoua and J. Leblond, Logarithmic stability estimates for a Robin coefficient in 2D Laplace inverse problems, *Inverse Problems* **20**, 47–59 (2004).

[10] M. Choulli and A. Jbalia, The problem of detecting corrosion by an electric measurement revisited, *Disc. and Cont. Dyn. Sys. - Series S* **9**, 643–650 (2016).

[11] M. Choulli, On the determination of an unknown boundary function in a parabolic equation, *Inverse Problems* **15**, 659–667 (1999).

[12] S. Chaabane, C. Elhechmi and M. Jaoua, A stable recovery algorithm for the Robin inverse problem, *Math. Comput. Simul.* **11**, 33–57 (2003).

[13] R. Gilmer, *Lectures on the Cauchy Problem in Linear Partial Differential Equations*, Yale University Press, New Haven (1923).

[14] V. Isakov, *Inverse Problems for Partial Differential Equations*, Applied Mathematical Sciences, Springer, NewYork (1998).

[15] L. E. Payne, *Improperly Posed Problems in Partial Differential Equations*, SIAM, Philadelphia (1975).

[16] R. Lattes and J. L. Lions, *Méthode de Quasi-reversibilité et Applications*, Dunod, Paris (1967).

[17] M. V. Klibanov and F. Santosa, A computational quasi-reversibility method for Cauchy problems for Laplace's equation, *SIAM J. Appl. Math.* **51**, 1653–1675 (1991).

[18] L. Bourgeois, A mixed formulation of quasi-reversibility to solve the Cauchy problem for Laplaces equation, *Inverse Problems* **21**, 1087–1104 (2005).

[19] A. Cimetière, F. Delvare, M. Jaoua and F. Pons, Solution of the Cauchy problem using iterated Tikhonov regularization, *Inverse Problems* **17**, 553–570 (2001).

[20] D. Lesnic, L. Elliott and D. B. Ingham, An iterative boundary element method for solving numerically the Cauchy problem for the Laplace equation, *Eng. Anal. with Bound. Elem.* **20**, 123–133 (1997).

[21] M. Jourhmane and A. Nachaoui, Convergence of an alternating method to solve the Cauchy problem for Poissons equation, *Appl. Anal.* **81**, 1065–1083 (2002).

[22] M. Jourhmane, D. Lesnic and N. S. Mera, Relaxation procedures for an iterative algorithm for solving the Cauchy problem for the Laplace equation, *Eng. Anal. Bound. Elem.* **28**, 655–665 (2004).

[23] D. Lesnic, L. Elliott and D. B. Ingham, An iterative boundary element method for solving numerically the Cauchy problem for the Laplace equation, *Eng. Anal. Bound. Elem.* **20**, 123–133 (1997).

[24] V. A. Kozlov, V. G. Mazya and D. V. Fomin, An iterative method for solving the Cauchy problem for elliptic equation, *Comput. Math. Phys.* **31**, 45–52 (1991).

[25] J. Baumeister and A. Leitao, Iterative methods for illposed problems modeled by partial differential equations, *J. of Inv. and ill-posed prob.* **9**, 1–17 (2001).

[26] C. Tajani, J. Abouchabaka and O. Abdoun, Numerical simulation of an inverse problem: Testing the influence of data, *J. Math. Comput. Sci.* **9**, 352–364 (2013).

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