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On three dimensional locally ϕ-semisymmetric trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura Connection

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Abstract. The object of the present paper is to characterize the locally ϕ- semisymmetry in three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection. Locally Ricci semisymmetric, Locally projectively ϕ- semisymmetric and Locally conformally ϕ- semisymmetric trans-Sasakian manifolds of dimension three have also been studied with respect to generalized Tanaka Webster Okumura connection.

1 Introduction

Let M be an n-dimensional, $n \geq 3$, connected smooth Riemannian manifold endowed with the Riemannian metric g. Let ∇, R, S and r be the Levi-Civita connection, curvature tensor, Ricci tensor and the scalar curvature of M respectively. The manifold M is called locally symmetric due to cartan ([6], [7]) if the global geodesic symmetry at $p \in M$ is an isometry, which is equivalent to the fact that $\nabla R = 0$. Generalizing the concept of local symmetry , the notion of semisymmetry was introduced by Cartan [8] and fully classified by Szabo ([27], [28], [29]). The manifold M is said to be semisymmetric if

$$(R(U,V).R)(X,Y)Z = 0, \tag{1.1}$$

for all vector fields X, Y, Z, U, V on M, where $R(U,V)$ is considered as the derivation of the tensor algebra at each point of M.

In 1977 Takahashi [30] introduced the notion of local ϕ- symmetry on Sasakian manifolds. A Sasakian manifold is said to be locally ϕ-symmetric if

$$\phi^2((\nabla_W R)(X,Y)Z) = 0, \tag{1.2}$$

for any vector fields X, Y, Z, W on M, where ϕ is the structure tensor of the manifold M. The concept of locally ϕ- symmetry on various structures and their generalizations or extension are studied in ([10], [22], [23], [24], [25]). By extending the notion of semisymmetry and generalizing the concept of local ϕ- symmetry of Takahashi [30] the authors in the paper [26] introduced the notion of locally ϕ-semisymmetric Sasakian manifolds. A Sasakian manifold M, $n \geq 3$, is said to be locally ϕ-semisymmetric if,

$$\phi^2\{(R(U,V).R)(X,Y)Z\} = 0, \tag{1.3}$$

for all horizontal vector fields X, Y, Z, U, V on M.

In 1985 J. A. Oubina [18] introduced a new class of almost contact metric manifolds, called trans-Sasakian manifold, which includes Sasakian, Kenmotso and Cosymplectic structures. The authors in the papers [1], [2], [5] and [9] have studied such manifolds and obtained some interesting results . It is known that [14] trans-Sasakian structure of type (0, 0), (0, β) and (α, 0) are Cosymplectic, β-Kenmotsu and α-Sasakian respectively. The local classification of trans-Sasakian manifold is given by J. C. Marrero [17] and it is proved that a trans-Sasakian manifold of dimension $n \geq 5$ is either Cosymplectic or α-Sasakian or β-Kenmotsu manifold. In the present paper we have studied three dimensional trans-Sasakian manifolds with respect to

generalized Tanaka Webster Okumura connection. The notion of generalized Tanaka Webster Okumura connection was introduced and studied by the authors in the paper [13]. In the present paper we have studied three dimensional locally ϕ-semisymmetric trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection. The present paper is organized as follows:
After introduction in Section 1 we give some preliminaries in Section 2. Section 3 is devoted to the study of locally Ricci semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection. In Sections 4 and 5 we have studied respectively locally ϕ- semisymmetric and locally projectively ϕ- semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection. Section 6 is completed with the study of locally conformally ϕ- semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

2 Preliminaries

Let M be a $(2n+1)$-dimensional connected differentiable manifold endowed with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a tensor field of type $(1,1)$, ξ is a vector field, η is an 1-form and g is a Riemannian metric on M such that [4]

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1. \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \quad X, Y \in T(M) \tag{2.2}$$

Then also

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(X) = g(X, \xi). \tag{2.3}$$

$$g(\phi X, X) = 0. \tag{2.4}$$

$$g(\phi X, Y) = -g(X, \phi Y) \tag{2.5}$$

An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be a trans-Sasakian manifold [18] if $(M^{2n+1} \times R, J, G)$ belongs to the class W_4 [12] of the Hermitian manifolds, where J is the almost complex structure on $M^{2n+1} \times R$ defined by [11]

$$J(Z, f\frac{d}{dt}) = (\phi Z - f\xi, \eta(Z)\frac{d}{dt}), \tag{2.6}$$

for any vector field Z on M^{2n+1} and smooth function f on $M^{2n+1} \times R$ and G is the Hermitian metric on the product $M^{2n+1} \times R$. This may be expressed by the condition [18]

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \tag{2.7}$$

for some smooth functions α and β on M^{2n+1}, and we say that the trans-Sasakian structure is of type (α, β). From equation (2.7), it follows that

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(Y)\xi), \tag{2.8}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y)\xi + \beta g(\phi X, \phi Y). \tag{2.9}$$

The Ricci operator, Ricci tensor and curvature tensor for three dimensional trans-Sasakian manifolds have been studied in the paper [9]. From [9], we have the following for three dimensional trans-Sasakian manifolds:

$$2\alpha\beta + \xi\alpha = 0, \tag{2.10}$$

$$S(X, \xi) = \{2(\alpha^2 - \beta^2)\}\eta(X) - \xi - (\phi X)\alpha\beta, \tag{2.11}$$

$$\begin{aligned} S(X,Y) &= \{\tfrac{r}{2}+\xi\beta-(\alpha^2-\beta^2)\}g(X,Y) \\ &- \{\tfrac{r}{2}+\xi\beta-3(\alpha^2-\beta^2)\}\eta(X)\eta(Y) \\ &- \{Y\beta+(\phi Y)\alpha\}\eta(X)-\{X\beta+(\phi X)\alpha\}\eta(Y), \end{aligned} \tag{2.12}$$

and

$$\begin{aligned} R(X,Y)Z &= \{\tfrac{r}{2}+2\xi\beta-2(\alpha^2-\beta^2)\}\{g(Y,Z)X-g(X,Z)Y\} \\ &- g(Y,Z)[\{\tfrac{r}{2}+\xi\beta-3(\alpha^2-\beta^2)\}\eta(X)\xi \\ &- \eta(X)(\phi grad\alpha-grad\beta)+\{X\beta+(\phi X)\alpha\}\xi] \\ &+ g(X,Z)[\{\tfrac{r}{2}+\xi\beta-3(\alpha^2-\beta^2)\}\eta(Y)\xi \\ &- \eta(Y)(\phi grad\alpha-grad\beta)+\{Y\beta+(\phi Y)\alpha\}\xi] \\ &- [\{Z\beta+(\phi Z)\alpha\}\eta(Y)+\{Y\beta+(\phi Y)\alpha\}\eta(Z) \\ &+ \{\tfrac{r}{2}+\xi\beta-3(\alpha^2-\beta^2)\}\eta(Y)\eta(Z)]X \\ &+ [\{Z\beta+(\phi Z)\alpha\}\eta(X)+\{X\beta+(\phi X)\alpha\}\eta(Z) \\ &+ \{\tfrac{r}{2}+\xi\beta-3(\alpha^2-\beta^2)\}\eta(X)\eta(Z)]Y, \end{aligned} \tag{2.13}$$

where S is the Ricci tensor of type $(0,2)$, R is the curvature tensor of type $(1,3)$ and r is the scalar curvature of the manifold M.
Now if X, Y and Z are orthogonal to ξ then relations (2.12) and (2.13) are changes to

$$S(X,Y) = \{\tfrac{r}{2}+\xi\beta-(\alpha^2-\beta^2)\}g(X,Y), \tag{2.14}$$

and

$$\begin{aligned} R(X,Y)Z &= \{\tfrac{r}{2}+2\xi\beta-2(\alpha^2-\beta^2)\}\{g(Y,Z)X-g(X,Z)Y\} \\ &- g(Y,Z)\{X\beta+(\phi X)\alpha\}\xi+g(X,Z)\{Y\beta+(\phi Y)\alpha\}\xi. \end{aligned} \tag{2.15}$$

Again from (2.15) we have

$$R(\xi,Y)Z = \{\tfrac{r}{2}+2\xi\beta-2(\alpha^2-\beta^2)\}g(Y,Z)\xi-g(Y,Z)(X\beta)\xi. \tag{2.16}$$

and

$$R(X,Y)\xi = 0, \tag{2.17}$$

where X, Y and Z are orthogonal to ξ
The generalized Tanaka Webster Okumura connection [13] $\tilde{\nabla}$ and the Levi-Civita connection ∇ are related by

$$\tilde{\nabla}_X Y = \nabla_X Y + A(X,Y), \tag{2.18}$$

for all vectors fields X, Y on M. Here

$$A(X,Y) = \alpha\{g(X,\phi Y)\xi+\eta(Y)\phi X\}+\beta\{g(X,Y)\xi-\eta(Y)X\}-l\eta(X)\phi Y, \tag{2.19}$$

where l is a real constant.
The Torsion $\tilde{T}$ of the gTWO-connection $\tilde{\nabla}$ is given by

$$\tilde{T}(X,Y) = \alpha\{2g(X.\phi Y)\xi-\eta(X)\phi Y+\eta(Y)\phi X\}+\eta(X)(\beta Y-l\phi Y)-\eta(Y)(\beta X-l\phi X). \tag{2.20}$$

Again relation between the curvature tensors $\tilde{R}$ and R with respect to the generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ and the Levi-Civita connection ∇ respectively is given by [21]

$$\begin{aligned}
&\tilde{R}(X,Y)Z = R(X,Y)Z + \alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z)\xi \\
+&\ \eta(\nabla_Y Z)\phi X - g(Y,\phi\nabla_X Z)\xi - \eta(\nabla_X Z)\phi Y - \eta(Z)\phi[X,Y]\} \\
+&\ \beta\{\eta(\nabla_X Z)Y - \eta(\nabla_Y Z)X + \eta(Z)[X,Y]\} \\
-&\ l\{\eta(X)\phi\nabla_Y Z + \eta(Y)\phi\nabla_X Z + \eta([X,Y])\phi Z\} \\
+&\ \{\alpha g(Y,\phi Z) + \beta g(Y,Z)\}\{\nabla_X\xi + \alpha\phi X + \beta(\eta(X)\xi - X)\} \\
-&\ \{\alpha g(X,\phi Z) + \beta g(X,Z)\}\{\nabla_Y\xi + \alpha\phi Y + \beta(\eta(Y)\xi - Y)\} \\
+&\ (\alpha\phi Y - \beta Y)[\nabla_X\eta(Z) + \alpha\{g(X,\phi\eta(Z))\xi + \eta(\eta(Z))\phi X\} \\
+&\ \beta\{g(X,\eta(Z))\xi - \eta(\eta(Z))X\} - l\eta(X)\phi\eta(Z)] \\
-&\ (\alpha\phi X - \beta X)[\nabla_Y\eta(Z) + \alpha\{g(Y,\phi\eta(Z))\xi + \eta(\eta(Z))\phi Y\} \\
+&\ \beta\{g(Y,\eta(Z))\xi - \eta(\eta(Z))Y\} - l\eta(Y)\phi\eta(Z)] \\
+&\ \alpha\eta(Z)[\nabla_X\phi Y - \nabla_Y\phi X + \beta\{g(X,\phi Y) - g(Y,\phi X)\}\xi + l\{\eta(X)Y - \eta(Y)X\}] \\
-&\ \beta\eta(Z)[\nabla_X Y - \nabla_Y X + \alpha\{g(X,\phi Y)\xi + \eta(Y)\phi X - g(Y,\phi X)\xi \\
+&\ \eta(X)\phi Y\} - \beta\{\eta(Y)X - \eta(X)Y\} + l\{\eta(Y)\phi X - \eta(X)\phi Y\}] \\
-&\ l\eta(Y)[\nabla_X\phi Z + \alpha\{\eta(X)\eta(Z) - g(X,Z)\}\xi + \beta g(X,\phi Z)\xi \\
+&\ l\{\eta(X)Z - \eta(X)\eta(Z)\xi\}] + l\eta(X)[\nabla_Y\phi Z + \alpha\{\eta(Y)\eta(Z) - g(Y,Z)\}\xi \\
+&\ \beta g(Y,\phi Z)\xi + l\{\eta(Y)Z - \eta(Y)\eta(Z)\xi\}] \\
-&\ l\phi Z[\nabla_X\eta(Y) - \nabla_Y\eta(X) + \alpha\{g(X,\phi\eta(Y))\xi + \eta(\eta(Y))\phi X) - g(Y,\phi\eta(X))\xi \\
-&\ \eta(\eta(X))\phi Y\} + \beta\{g(X,\eta(Y))\xi - \eta(\eta(Y))X - g(Y,\eta(X))\xi + \eta(\eta(X))Y\} \\
+&\ l\{\eta(Y)\phi\eta(X) - \eta(X)\phi\eta(Y)].
\end{aligned} \tag{2.21}$$

We suppose that X, Y, Z, $\nabla_X Z$ and $\nabla_Y Z$ are orthogonal to ξ. Then (2.21) becomes

$$\begin{aligned}
\tilde{R}(X,Y)Z &= R(X,Y)Z + \alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z) \\
&- g(Y,\phi\nabla_X Z)\}\xi + \beta g([X,Y],Z)\xi \\
&+ \{\alpha g(Y,\phi Z) + \beta g(Y,Z)\}\{\nabla_X\xi + \alpha\phi X + \beta - X\} \\
&- \{\alpha g(X,\phi Z) + \beta g(X,Z)\}\{\nabla_Y\xi + \alpha\phi Y + \beta - Y\},
\end{aligned} \tag{2.22}$$

3 Locally Ricci semisymmetric three dimensional trans-Sasakian manifolds with generalized Tanaka Webster Okumura connection

In this section we have studied locally Ricci semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 3.1. A trans-Sasakian manifold of dimension three is said to be locally Ricci-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if

$$(\tilde{R}(U,V).\tilde{S})(X,Y) = 0, \tag{3.1}$$

for all horizontal vector fields X, Y, U, V on M.

Thus for a trans-Sasakian manifold of dimension three we obtain by using (2.8) in (2.22)

$$\begin{aligned}
\tilde{R}(X,Y)Z &= R(X,Y)Z + \alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z) \\
&- g(Y,\phi\nabla_X Z)\}\xi + \beta g([X,Y],Z)\xi,
\end{aligned} \tag{3.2}$$

where X, Y and Z are orthogonal to ξ.
Applying ϕ^2 on both side of (3.2) we get

$$\phi^2\{\tilde{R}(X,Y)Z\} = \phi^2\{R(X,Y)Z\} \tag{3.3}$$

Now taking inner product on both side of (3.2) by W we obtain

$$g(\tilde{R}(X,Y)Z,W) = g(R(X,Y)Z,W). \tag{3.4}$$

From relation (3.4) we obtain
$$\tilde{S}(X,W) = S(X,W), \tag{3.5}$$
where W is orthogonal to ξ.
Again we know that for a trans-Sasakian manifolds the Ricci tensor S with respect to Levi Civita connection is
$$S(X,Y) = (\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2))g(X,Y), \tag{3.6}$$
where X and Y are orthogonal to ξ.
i.e.
$$S(X,Y) = Hg(X,Y), \tag{3.7}$$
where
$$H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2). \tag{3.8}$$
Therefore by (3.5) and (3.6) we get
$$\tilde{S}(X,Y) = (\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2))g(X,Y), \tag{3.9}$$
Again we know that the Ricci operator $\tilde{Q}$ with generalized Tanaka Webster Okumura connection is given by
$$\tilde{S}(X,Y) = g(\tilde{Q}X,Y). \tag{3.10}$$
Thus combining (3.9) and (3.10) we get
$$\tilde{Q}X = (\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2))X. \tag{3.11}$$
Now we know that
$$(\tilde{R}(U,V).\tilde{S})(X,Y) = (\tilde{R}(U,V).S)(X,Y). \tag{3.12}$$
or
$$(\tilde{R}(U,V).\tilde{S})(X,Y) = -S(\tilde{R}(U,V)X,Y) - S(X,\tilde{R}(U,V)Y) \tag{3.13}$$
Using (2.14) and (3.2) in relation (3.13) we get
$$(\tilde{R}(U,V).\tilde{S})(X,Y) = -H\{g(R(U,V)X,Y) + g(X,R(U,V)Y)\} \tag{3.14}$$
Again using (2.15) in relation (3.14) we get
$$\begin{aligned}(\tilde{R}(U,V).\tilde{S})(X,Y) &= -H\{\tfrac{r}{2} + 2\xi\beta - 2(\alpha^2 - \beta^2)\}[g(V,X)g(U,Y) \\ &- g(U,X)g(V,Y) + g(U,X)g(V,Y) - g(V,X)g(U,Y)] \\ &= 0.\end{aligned} \tag{3.15}$$
Thus we are in a position to state the following:

Theorem 3.2. *Every trans-Sasakian manifolds of dimension three is locally Ricci semisymmetric with respect to generalized Tanaka Webster Okumura connection.*

4 Locally ϕ-semisymmetric three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection

In this section we have studied locally ϕ-semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 4.1. A trans-Sasakian manifold of dimension three is said to be locally ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if
$$\phi^2\{(\tilde{R}(U,V).\tilde{R})(X,Y)Z\} = 0, \tag{4.1}$$
for all horizontal vector fields X, Y, Z, U, V on M.

Now we know that

$$[(\tilde{R}(U,V).\tilde{R})(X,Y)Z] = \tilde{R}(U,V)\tilde{R}(X,Y)Z - \tilde{R}(\tilde{R}(U,V)X,Y)Z - \tilde{R}(X,\tilde{R}(U,V)Y)Z - \tilde{R}(X,Y)\tilde{R}(U,V)Z \tag{4.2}$$

Now using (3.2) in first and last terms of right hand side of (4.2) we get

$$\begin{aligned}\tilde{R}(U,V)\tilde{R}(X,Y)Z &= R(U,V)R(X,Y)Z \\ &+ [\alpha\{g(V,\nabla_U\phi\tilde{R}(X,Y)Z) - g(U,\nabla_V\phi\tilde{R}(X,Y)Z) \\ &+ g(U,\phi\nabla_V\tilde{R}(X,Y)Z) - g(V,\phi\nabla_U\tilde{R}(X,Y)Z)\} \\ &+ \beta g([U,V],\tilde{R}(X,Y)Z)]\xi.\end{aligned} \tag{4.3}$$

and

$$\begin{aligned}\tilde{R}(X,Y)\tilde{R}(U,V)Z &= R(X,Y)R(U,V)Z \\ &+ [\alpha\{g(Y,\nabla_X\phi\tilde{R}(U,V)Z) - g(X,\nabla_Y\phi\tilde{R}(U,V)Z) \\ &+ g(X,\phi\nabla_Y\tilde{R}(U,V)Z) - g(Y,\phi\nabla_X\tilde{R}(U,V)Z)\} \\ &+ \beta g([X,Y],\tilde{R}(U,V)Z)]\xi.\end{aligned} \tag{4.4}$$

Similarly using (3.2) in second and third terms of right hand side of (4.2) we get

$$\begin{aligned}\tilde{R}(\tilde{R}(U,V)X,Y)Z &= R(R(U,V)X,Y)Z \\ &+ [\alpha\{g(V.\nabla_U\phi X) - g(U,\nabla_V\phi X) \\ &+ g(U,\phi\nabla_V X) - g(V,\phi\nabla_U X)\} \\ &+ \beta g([U,V],X)]R(\xi,Y)Z \\ &+ [\alpha\{g(Y,\nabla_{\tilde{R}(U,V)X}\phi Z) - g(\tilde{R}(U,V)X,\nabla_Y\phi Z) \\ &+ g(\tilde{R}(U,V)X,\phi\nabla_Y Z) - g(Y,\phi\nabla_{\tilde{R}(U,V)X}Z\} \\ &+ \beta g([\tilde{R}(U,V)X,Y],Z)]\xi\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}\tilde{R}(X,\tilde{R}(U,V)Y)Z &= -\tilde{R}(\tilde{R}(U,V)Y,X)Z \\ &= -R(R(U,V)Y,X)Z \\ &- [\alpha\{g(V.\nabla_U\phi Y) - g(U,\nabla_V\phi Y) \\ &+ g(U,\phi\nabla_V Y) - g(V,\phi\nabla_U Y)\} \\ &+ \beta g([U,V],Y)]R(\xi,X)Z \\ &- [\alpha\{g(X,\nabla_{\tilde{R}(U,V)Y}\phi Z) - g(\tilde{R}(U,V)Y,\nabla_X\phi Z) \\ &+ g(\tilde{R}(U,V)Y,\phi\nabla_X Z) - g(X,\phi\nabla_{\tilde{R}(U,V)Y}Z\} \\ &+ \beta g([\tilde{R}(U,V)Y,X],Z)]\xi\end{aligned} \tag{4.6}$$

Using (4.3), (4.4), (4.5) and (4.6) in relation (4.2) we get

$$\begin{aligned}
[(\tilde{R}(U,V).\tilde{R})(X,Y)Z] &= [(R(U,V).(X,Y)Z] \\
&+ [\alpha\{g(V,\nabla_U\phi\tilde{R}(X,Y)Z) - g(U,\nabla_V\phi\tilde{R}(X,Y)Z) \\
&+ g(U,\phi\nabla_V\tilde{R}(X,Y)Z) - g(V,\phi\nabla_U\tilde{R}(X,Y)Z)\} \\
&+ \beta g([U,V],\tilde{R}(X,Y)Z)]\xi \\
&+ [\alpha\{g(V.\nabla_U\phi X) - g(U,\nabla_V\phi X) \\
&+ g(U,\phi\nabla_V X) - g(V,\phi\nabla_U X)\} \\
&+ \beta g([U,V],X)]R(\xi,Y)Z \\
&+ [\alpha\{g(Y,\nabla_{\tilde{R}(U,V)X}\phi Z) - g(\tilde{R}(U,V)X,\nabla_Y\phi Z) \\
&+ g(\tilde{R}(U,V)X,\phi\nabla_Y Z) - g(Y,\phi\nabla_{\tilde{R}(U,V)X}Z\} \\
&+ \beta g([\tilde{R}(U,V)X,Y],Z)]\xi \qquad (4.7)\\
&- [\alpha\{g(V.\nabla_U\phi Y) - g(U,\nabla_V\phi Y) \\
&+ g(U,\phi\nabla_V Y) - g(V,\phi\nabla_U Y)\} \\
&+ \beta g([U,V],Y)]R(\xi,X)Z \\
&- [\alpha\{g(X,\nabla_{\tilde{R}(U,V)Y}\phi Z) - g(\tilde{R}(U,V)Y,\nabla_X\phi Z) \\
&+ g(\tilde{R}(U,V)Y,\phi\nabla_X Z) - g(X,\phi\nabla_{\tilde{R}(U,V)Y}Z\} \\
&+ \beta g([\tilde{R}(U,V)Y,X],Z)]\xi \\
&+ [\alpha\{g(Y,\nabla_X\phi\tilde{R}(U,V)Z) - g(X,\nabla_Y\phi\tilde{R}(U,V)Z) \\
&+ g(X,\phi\nabla_Y\tilde{R}(U,V)Z) - g(Y,\phi\nabla_X\tilde{R}(U,V)Z)\} \\
&+ \beta g([X,Y],\tilde{R}(U,V)Z)]\xi.
\end{aligned}$$

Now applying ϕ^2 on both side of (4.7) we get

$$\begin{aligned}
\phi^2[(\tilde{R}(U,V).\tilde{R})(X,Y)Z] &= \phi^2[(R(U,V).(X,Y)Z] \\
&+ [\alpha\{g(V.\nabla_U\phi X) - g(U,\nabla_V\phi X) \\
&+ g(U,\phi\nabla_V X) - g(V,\phi\nabla_U X)\} \\
&+ \beta g([U,V],X)]\{\phi^2 R(\xi,Y)Z\} \qquad (4.8)\\
&- [\alpha\{g(V.\nabla_U\phi Y) - g(U,\nabla_V\phi Y) \\
&+ g(U,\phi\nabla_V Y) - g(V,\phi\nabla_U Y)\} \\
&+ \beta g([U,V],Y)]\{\phi^2 R(\xi,X)Z\}
\end{aligned}$$

Using (2.16) in (4.8) we get

$$\phi^2[(\tilde{R}(U,V).\tilde{R})(X,Y)Z] = \phi^2[(R(U,V).(X,Y)Z] \qquad (4.9)$$

Thus we are in a position to state the following:

Theorem 4.2. *A trans-Sasakian manifold of dimension three is locally ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is so with respect to Levi-civita connection ∇.*

5 Locally projectively ϕ-semisymmetric three dimensional trans-Sasakian manifolds with generalized Tanaka Webster Okumura connection

In this section we have studied locally projectively ϕ- semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 5.1. For a (2n+1)- dimensional (n>1) manifold the Weyl projective curvature tensor $\tilde{P}$ with respect to generalized Tanaka Webster Okumura connection will be given by

$$\tilde{P}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{2n}[\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y]. \qquad (5.1)$$

In view of (3.2) and (3.9) we get from (5.1)

$$\begin{aligned} \tilde{P}(X,Y)Z &= R(X,Y)Z + \alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) \\ &+ g(X,\phi\nabla_Y Z) - g(Y,\phi\nabla_X Z)\}\xi + \beta g([X,Y],Z)\xi \\ &- \tfrac{H}{2n}\{g(Y,Z)X - g(X,Z)Y\}, \end{aligned} \tag{5.2}$$

where

$$H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2). \tag{5.3}$$

Definition 5.2. A (2n+1)-dimensional (n>1) trans-Sasakian manifold is said to be a Locally projectively ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection if the relation

$$\phi^2[(\tilde{R}(X,Y).\tilde{P})(Z,U)V] = 0, \tag{5.4}$$

holds for all horizontal vector fields X, Y, Z, U and V on M.

Let M be a 3-dimensional connected trans-Sasakian manifold. Now we know that

$$\begin{aligned} [(\tilde{R}(X,Y).\tilde{P})(Z,U)V] &= \tilde{R}(X,Y)\tilde{P}(Z,U)V - \tilde{P}(\tilde{R}(X,Y)Z,U)V \\ &- \tilde{P}(Z,\tilde{R}(X,Y)U)V - \tilde{P}(Z,U)\tilde{R}(X,Y)V \end{aligned} \tag{5.5}$$

Now by using (3.2) and (5.2) on the first term of right hand side of (5.5) we have

$$\begin{aligned} \tilde{R}(X,Y)\tilde{P}(Z,U)V &= R(X,Y)R(Z,U)V + \alpha\{g(Y,\nabla_X\phi R(Z,U)V) \\ &- g(X,\nabla_Y\phi R(Z,U)V) + g(X,\phi\nabla_Y R(Z,U)V) \\ &- g(Y,\phi\nabla_X R(Z,U)V)\}\xi + \beta g([X,Y],R(Z,U)V)\xi \\ &+ \alpha\{g(U,\nabla_Z\phi V) - g(Z,\nabla_U\phi V) \\ &+ g(Z,\phi\nabla_U V) - g(U,\phi\nabla_Z V)\}\xi + \beta g([Z,U],V)\xi \\ &- \tfrac{H}{2n}[g(U,V)\tilde{R}(X,Y)Z - g(Z,V)\tilde{R}(X,Y)U] \end{aligned} \tag{5.6}$$

Using (3.2), (5.1) and (5.2) in the second term of right hand side of (5.5) we get

$$\begin{aligned} \tilde{P}(\tilde{R}(X,Y)Z,U)V &= R(R(X,Y)Z,U)V \\ &+ [\alpha\{g(U,\nabla_{R(X,Y)Z}\phi V) - g(R(X,Y)Z,\nabla_U\phi V) \\ &+ g(R(X,Y)Z,\phi\nabla_U V) - g(U,\phi\nabla_{R(X,Y)Z})V\} \\ &+ \beta g([R(X,Y)Z,U],V)]\xi \\ &+ [\alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z) \\ &- g(Y,\phi\nabla_X Z)\} + \beta g([X,Y],Z)]\tilde{R}(\xi,U)V \\ &- \tfrac{H}{2n}[g(U,V)\tilde{R}(X,Y)Z - g(R(X,Y)Z,V)U]. \end{aligned} \tag{5.7}$$

Similarly for third and fourth terms of right hand side of (5.5), using (3.2), (5.1) and (5.2) we have the following:

$$\begin{aligned} \tilde{P}(Z,\tilde{R}(X,Y)U)V &= R(Z,R(X,Y)U)V \\ &+ [\alpha\{g(R(X,Y)Z,\nabla_Z\phi V) - g(Z,\nabla_{R(X,Y)Z}\phi V \\ &- g(R(X,Y)Z,\phi\nabla_Z V) + g(Z,\phi\nabla_{R(X,Y)Z}V\} \\ &+ \beta g([R(X,Y)Z,U],V)]\xi \\ &+ [\alpha\{g(Y,\nabla_X\phi U) - g(X,\nabla_Y\phi U) + g(X,\phi\nabla_Y U) \\ &- g(Y,\phi\nabla_X U)\} + \beta g([X,Y],U)]\tilde{R}(Z,\xi)V \\ &- \tfrac{H}{2n}[g(R(X,Y)U,V)Z - g(Z,V)\tilde{R}(X,Y)U] \end{aligned} \tag{5.8}$$

and

$$\begin{aligned} \tilde{P}(Z,U)\tilde{R}(X,Y)V &= R(Z,U)R(X,Y)V \\ &+ \alpha\{g(U,\nabla_Z\phi R(X,Y)V) - g(Z,\nabla_U\phi R(X,Y)V) \\ &+ g(Z,\phi\nabla_U R(X,Y)V) - g(U,\phi\nabla_Z R(X,Y)V)\}\xi \\ &+ \beta g([Z,U],R(X,Y)V)\xi \\ &- \tfrac{H}{2n}\{g(U,R(X,Y)V)Z - g(Z,R(X,Y)V)U\}. \end{aligned} \tag{5.9}$$

Using (5.6), (5.7), (5.8) and (5.9) in (5.5) we get,

$$\begin{aligned}
[(\tilde{R}(X,Y).\tilde{P})(Z,U)V] &= [R(X,Y).R)(Z,U)V] \\
&+ \alpha\{g(Y,\nabla_X\phi R(Z,U)V) \\
&- g(X,\nabla_Y\phi R(Z,U)V) + g(X,\phi\nabla_Y R(Z,U)V) \\
&- g(Y,\phi\nabla_X R(Z,U)V)\}\xi + \beta g([X,Y],R(Z,U)V)\xi \\
&+ \alpha\{g(U,\nabla_Z\phi V) - g(Z,\nabla_U\phi V) \\
&+ g(Z,\phi\nabla_U V) - g(U,\phi\nabla_Z V)\}\xi + \beta g([Z,U],V)\xi \\
&+ \alpha\{g(U,\nabla_{R(X,Y)Z}\phi V) - g(R(X,Y)Z,\nabla_U\phi V) \\
&+ g(R(X,Y)Z,\phi\nabla_U V) - g(U,\phi\nabla_{R(X,Y)Z})V\} \\
&+ \beta g([R(X,Y)Z,U],V)]\xi \\
&+ [\alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z) \\
&- g(Y,\phi\nabla_X Z)\} + \beta g([X,Y],Z)]\tilde{R}(\xi,U)V \\
&+ [\alpha\{g(R(X,Y)Z,\nabla_Z\phi V) - g(Z,\nabla_{R(X,Y)Z}\phi V \\
&- g(R(X,Y)Z,\phi\nabla_Z V) + g(Z,\phi\nabla_{R(X,Y)Z}V\} \\
&+ \beta g([R(X,Y)Z,U],V)]\xi \\
&+ [\alpha\{g(Y,\nabla_X\phi U) - g(X,\nabla_Y\phi U) + g(X,\phi\nabla_Y U) \\
&- g(Y,\phi\nabla_X U)\} + \beta g([X,Y],U)]\tilde{R}(Z,\xi)V \\
&+ \alpha\{g(U,\nabla_Z\phi R(X,Y)V) - g(Z,\nabla_U\phi R(X,Y)V) \\
&+ g(Z,\phi\nabla_U R(X,Y)V) - g(U,\phi\nabla_Z R(X,Y)V)\}\xi \\
&+ \beta g([Z,U],R(X,Y)V)\xi
\end{aligned} \tag{5.10}$$

Now applying ϕ^2 on both side of (5.10) we get

$$\begin{aligned}
\phi^2[(\tilde{R}(X,Y).\tilde{P})(Z,U)V] &= \phi^2[R(X,Y).R)(Z,U)V] \\
&+ [\alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z) \\
&- g(Y,\phi\nabla_X Z)\} + \beta g([X,Y],Z)]\{\phi^2\tilde{R}(\xi,U)V\} \\
&+ [\alpha\{g(Y,\nabla_X\phi U) - g(X,\nabla_Y\phi U) + g(X,\phi\nabla_Y U) \\
&- g(Y,\phi\nabla_X U)\} + \beta g([X,Y],U)]\{\phi^2\tilde{R}(Z,\xi)V\}
\end{aligned} \tag{5.11}$$

In view of (3.2) and (2.16) we get from (5.11)

$$\phi^2[(\tilde{R}(X,Y).\tilde{P})(Z,U)V] = \phi^2[R(X,Y).R)(Z,U)V] \tag{5.12}$$

Thus we are in a position to state the following:

Theorem 5.3. *A trans-Sasakian manifold of dimension three is locally projectively ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is locally ϕ-semisymmetric with respect to Levi-civita connection ∇.*

6 Locally conformally ϕ-semisymmetric three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection

In this section we have studied locally conformally ϕ- semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 6.1. For a $(2n+1)$ dimensional Riemannian manifold the Weyl conformal curvature tensor with respect to generalized Tanaka Webster Okumura connection is defined by

$$\begin{aligned}
\tilde{C}(X,Y)Z &= \tilde{R}(X,Y)Z - \tfrac{1}{2n-1}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y \\
&+ g(Y,Z)\tilde{Q}X - g(X,Z)\tilde{Q}Y\} \\
&+ \tfrac{\tilde{r}}{2n(2n-1)}\{g(Y,Z)X - g(X,Z)Y\}.
\end{aligned} \tag{6.1}$$

Using (3.9) and (3.11) in (6.1) we get

$$\tilde{C}(X,Y)Z = \tilde{R}(X,Y)Z + \{\tfrac{r}{2n(2n-1)} - \tfrac{2H}{2n-1}\}\{g(Y,Z)X - g(X,Z)Y\}, \tag{6.2}$$

where $H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)$.
i.e.

$$\tilde{C}(X,Y)Z = \tilde{R}(X,Y)Z + N\{g(Y,Z)X - g(X,Z)Y\}, \tag{6.3}$$

where $N = \{\frac{r}{2n(2n-1)} - \frac{2H}{2n-1}\}$.
Using (3.2) in (6.3) we obtain,

$$\begin{aligned}\tilde{C}(X,Y)Z &= R(X,Y)Z + [\alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z)\\ &+ g(X,\phi\nabla_Y Z) - g(Y,\phi\nabla_X Z)\} + \beta g([X,Y],Z)]\xi\\ &+ N\{g(Y,Z)X - g(X,Z)Y\},\end{aligned} \tag{6.4}$$

Definition 6.2. A trans-Sasakian manifold of dimension three is said to be locally conformally ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if

$$\phi^2\{(\tilde{R}(U,V).\tilde{C})(X,Y)Z\} = 0, \tag{6.5}$$

for all horizontal vector fields X, Y, Z, U, V on M.

Now we know that

$$\begin{aligned}[(\tilde{R}(U,V).\tilde{C})(X,Y)Z] &= \tilde{R}(U,V)\tilde{C}(X,Y)Z - \tilde{C}(\tilde{R}(U,V)X,Y)Z\\ &- \tilde{C}(X,\tilde{R}(U,V)Y)Z - \tilde{C}(X,Y)\tilde{R}(U,V)Z\end{aligned} \tag{6.6}$$

Using (3.2), (2.17) and (6.3) in each terms of right hand side of (6.6) and by straightforward calculation we obtain the followings:

$$\begin{aligned}\tilde{R}(U,V)\tilde{C}(X,Y)Z &= R(U,V)C(X,Y)Z\\ &+ [\alpha\{g(Y,\nabla_X\phi Z) - g(X,\nabla_Y\phi Z) + g(X,\phi\nabla_Y Z)\\ &- g(Y,\phi\nabla_X Z)\} + \beta g([X,Y],Z)]\xi\\ &+ [\alpha\{g(V,\nabla_U\phi\tilde{C}(X,Y)Z) - g(U,\nabla_V\phi\tilde{C}(X,Y)Z)\\ &+ g(U,\phi\nabla_V\tilde{C}(X,Y)Z) - g(V,\phi\nabla_U\tilde{C}(X,Y)Z)\}\\ &+ \beta g([U,V],\tilde{C}(X,Y)Z)]\xi\\ &+ N\{g(Y,Z)R(U,V)X - g(X,Z)R(U,V)Y\},\end{aligned} \tag{6.7}$$

$$\begin{aligned}\tilde{C}(\tilde{R}(U,V)X,Y)Z &= C(R(U,V)X,Y)Z\\ &+ [\alpha\{g(V,\nabla_U\phi X) - g(U,\nabla_V\phi X) + g(U,\phi\nabla_V X)\\ &- g(V,\phi\nabla_U X)\} + \beta g([U,V],X)]R(\xi,Y)Z\\ &+ [\alpha\{g(Y,\nabla_{\tilde{R}(U,V)X}\phi Z) - g(\tilde{R}(U,V)X,\nabla_Y\phi Z)\\ &+ g(\tilde{R}(U,V)X,\phi\nabla_Y Z) - g(Y,\phi\nabla_{\tilde{R}(U,V)X}Z)\\ &+ \beta g([\tilde{R}(U,V)X,Y],Z)]\xi\\ &+ N\{g(Y,Z)\tilde{R}(U,V)X - g(R(U,V)X,Z)Y\},\end{aligned} \tag{6.8}$$

$$\begin{aligned}\tilde{C}(X,\tilde{R}(U,V)Y)Z &= C(X,R(U,V)Y\\ &+ [\alpha\{g(V,\nabla_U\phi Y) - g(U,\nabla_V\phi Y) + g(U,\phi\nabla_V Y)\\ &- g(V,\phi\nabla_U Y)\} + \beta g([U,V],Y)]R(X,\xi)Z\\ &+ [\alpha\{g(\tilde{R}(U,V)Y,\nabla_X\phi Z) - g(X,\nabla_{\tilde{R}(U,V)Y}\phi Z)\\ &+ g(X,\phi\nabla_{\tilde{R}(U,V)Y}Z) - g(\tilde{R}(U,V)Y,\phi\nabla_X Z)\}\\ &+ \beta g([X,\tilde{R}(U,V)Y],Z)]\xi\\ &+ N\{g(R(U,V)Y,Z)X - g(X,Z)\tilde{R}(U,V)Y\},\end{aligned} \tag{6.9}$$

and

$$\begin{aligned}\tilde{C}(X,Y)\tilde{R}(U,V)Z &= R(X,Y)R(U,V)Z \\ &+ [\alpha\{g(Y,\nabla_X\phi\tilde{R}(U,V)Z)-g(X,\nabla_Y\phi\tilde{R}(U,V)Z) \\ &+ g(X,\phi\nabla_Y\tilde{R}(U,V)Z)-g(Y,\phi\nabla_X\tilde{R}(U,V)Z)\} \\ &+ \beta g([X,Y],\tilde{R}(U,V)Z)]\xi \\ &+ N\{g(R(U,V)Z,Y)X-g(R(U,V)Z,X)Y\}.\end{aligned} \tag{6.10}$$

Now using (6.7), (6.8), (6.9) and (6.10) in (6.6) we get

$$\begin{aligned}[(\tilde{R}(U,V).\tilde{C})(X,Y)Z] &= [(R(U,V).C)(X,Y)Z] \\ &+ [\alpha\{g(Y,\nabla_X\phi Z)-g(X,\nabla_Y\phi Z)+g(X,\phi\nabla_Y Z) \\ &- g(Y,\phi\nabla_X Z)\}+\beta g([X,Y],Z)]\xi \\ &+ [\alpha\{g(V,\nabla_U\phi\tilde{C}(X,Y)Z)-g(U,\nabla_V\phi\tilde{C}(X,Y)Z) \\ &+ g(U,\phi\nabla_V\tilde{C}(X,Y)Z)-g(V,\phi\nabla_U\tilde{C}(X,Y)Z)\} \\ &+ \beta g([U,V],\tilde{C}(X,Y)Z)]\xi \\ &+ N\{g(Y,Z)R(U,V)X-g(X,Z)R(U,V)Y\} \\ &+ [\alpha\{g(V,\nabla_U\phi X)-g(U,\nabla_V\phi X)+g(U,\phi\nabla_V X) \\ &- g(V,\phi\nabla_U X)\}+\beta g([U,V],X)]R(\xi,Y)Z \\ &+ [\alpha\{g(Y,\nabla_{\tilde{R}(U,V)X}\phi Z)-g(\tilde{R}(U,V)X,\nabla_Y\phi Z) \\ &+ g(\tilde{R}(U,V)X,\phi\nabla_Y Z)-g(Y,\phi\nabla_{\tilde{R}(U,V)X}Z) \\ &+ \beta g([\tilde{R}(U,V)X,Y],Z)]\xi+Ng(Y,Z)\tilde{R}(U,V)X \\ &+ [\alpha\{g(V,\nabla_U\phi Y)-g(U,\nabla_V\phi Y)+g(U,\phi\nabla_V Y) \\ &- g(V,\phi\nabla_U Y)\}+\beta g([U,V],Y)]R(X,\xi)Z \\ &+ [\alpha\{g(\tilde{R}(U,V)Y,\nabla_X\phi Z)-g(X,\nabla_{\tilde{R}(U,V)Y}\phi Z) \\ &+ g(X,\phi\nabla_{\tilde{R}(U,V)Y}Z)-g(\tilde{R}(U,V)Y,\phi\nabla_X Z)\} \\ &+ \beta g([X,\tilde{R}(U,V)Y],Z)]\xi-Ng(X,Z)\tilde{R}(U,V)Y\} \\ &+ [\alpha\{g(Y,\nabla_X\phi\tilde{R}(U,V)Z)-g(X,\nabla_Y\phi\tilde{R}(U,V)Z) \\ &+ g(X,\phi\nabla_Y\tilde{R}(U,V)Z)-g(Y,\phi\nabla_X\tilde{R}(U,V)Z)\} \\ &+ \beta g([X,Y],\tilde{R}(U,V)Z)]\xi.\end{aligned} \tag{6.11}$$

Applying ϕ^2 on both side of (6.11) and using (2.3), (3.3) and (2.16) we obtain

$$\phi^2[(\tilde{R}(U,V).\tilde{C})(X,Y)Z] = \phi^2[(R(U,V).C)(X,Y)Z]. \tag{6.12}$$

Thus we are in a position to state the following:

Theorem 6.3. *A trans-Sasakian manifold of dimension three is locally conformally ϕ-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is locally ϕ-semisymmetric with respect to Levi-civita connection ∇.*

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