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NUMERICAL STUDY OF UNSTEADY MHD POISEUILLE FLOW WITH TEMPERATURE-DEPENDENT VISCOSITY THROUGH A POROUS CHANNEL UNDER AN OSCILLATING PRESSURE GRADIENT

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Abstract In the present paper, we examine the unsteady heat transfer and incompressible magnetohydrodynamic Poiseuille flow of a viscous liquid through a porous channel. The fluid, which has temperature-dependent viscosity, is bounded by two horizontal plates and influenced by an oscillating pressure gradient. The governing Brinkman equation and energy equation are solved using Galerkin's Finite Element Method. The influence of Hartmann number, amplitude of the pressure gradient, oscillation frequency, viscosity parameter, suction/injection parameter, Darcy number and time are discussed. The results revealed that the aforementioned parameters have a marked influence on flow velocity and fluid temperature. An increase in the volumetric flow rate can be achieved by reducing the suction/injection parameter and Hartmann number, and by increasing the Darcy number, viscosity parameter and Prandtl number. It was also determined that enhancements of the Nusselt number ocurrs with decreasing suction/injection parameter and Prandtl number. The obtained results are consistent with the existing literature.

Nomenclature

A	dimensionless amplitude
B_0	magnetic field
c	specific heat capacity, $Jkg^{-1}K^{-1}$
Da	Darcy number
h	half-width of channel, m
Ha	Hartmann number
K	permeability, m^2
Nu	Nusselt number
P	pressure, Pa
Pr	Prandtl number
Q	volumetric flow rate
t	time, s
S	suction/injection parameter
T	temperature, K
T_1	lower plate temperature, K
T_2	upper plate temperature, K
u	velocity, m/s
V_0	suction/injection velocity, m/s
x,y	space coordinates, m
α	viscosity variation parameter
ε	porosity
γ	empirical viscosity constant, s^{-1}
κ	thermal conductivity, $Wm^{-1}K^{-1}$
μ	viscosity, $kgm^{-1}s^{-1}$
μ_0	reference viscosity, $kgm^{-1}s^{-1}$
ρ	density, kgm^{-3}
σ	electrical conductivity $\Omega^{-1}m^{-1}$
ω	dimensionless frequency

1 Introduction

The study of magnetohydrodynamics (MHD) with heat transfer has received considerable focus by researchers over the past decade due to its applications in industry, such as geothermal energy technology and the design of MHD power generators [1, 2, 3, 4] . In particular, fluid flows between two parallel plates are applicable to aerodynamic heating technology, and the polymer and petroleum industries [5, 6]. Attia and Abdeen [7] studied the unsteady heat transfer and incompressible magnetohydrodynamic flow of a viscous electrically-conducting liquid through parallel porous plates. The flow was driven by a uniform magnetic field, an exponentially decreasing pressure gradient, and constant suction/injection. The authors came to the conclusion that increasing the magnetic field strength reduces flow velocity. One finding reached by Falade et al. [8] in their investigation of oscillatory MHD flow through a porous channel was that increasing the suction/injection parameter improved the wall slip flow velocity and the fluid temperature distribution. Ahmed et. al [9] examined the oscillatory MHD flow through a channel with heat transfer and suction effects. They discovered that increased magnetic field strength increases the velocity, temperature, rate of heat transfer and skin friction. Also, an increase in suction reduces fluid flow, and causes a reduction in skin friction on the bottom wall and an increase on the top wall. It was also noted that the fluid suction and the magnetic field can control the fluid velocity and temperature; this is particularly important in the flow of liquid metals through rectangular channels in fusion power reactors and flow of coolant fluids through channels in micro fluid devices. In a mixed convection study within a vertical microchannel, Jha and Aina [10] discovered that fluid suction enhances heat transfer and skin-friction coefficient, whereas injection decreases both. The rate of heat transfer reduces as the suction/injection rate increases, while the volumetric flow rate increases. Mondal et al. [11] looked at the effects of entropy generation and radiative heat transfer on MHD generalized Couette fluid flow in a microchannel. Suction/injection, fluid slip, free convection, wall temperature ratio, and viscous dissipation all played a role in their study.

The finite element technique is a powerful numerical method for solving real-world problems that are modelled using partial differential equations. This numerical method has been utilized in the areas of heat transfer, fluid flow, mass transport and many other fields of physics and geometry. Reddy et al. [12] investigated the unsteady MHD natural convection rotating Couette flow and associated heat transfer of a viscous incompressible fluid using the finite element technique. The authors concluded that increasing the Prandtl number and magnetic parameter reduces the velocity of the fluid. Moreover, the fluid temperature is reduced when the Prandtl number is increased. Job and Gunakala [13] used the finite element method to examine unsteady Couette flow through two permeable parallel plates with the influence of free convection, exponentially decreasing pressure gradient, thermal radiation, Lorentz force and viscous and Joule dissipations. The influence of Grashof number, Eckert number, magnetic parameter, suction/injection parameter, Prandtl number and radiation parameter on the temperature and velocity distributions was examined for the cases of uniformly-accelerated motion and impulsive movement of the upper plate. Job and Gunakala [14] also investigated the mixed convective MHD flow of magnetite ferrofluid within a corrugated channel with internal wall-mounted porous blocks. The finite element technique was used to solve the constructed mathematical model, and the influence of several parameters on fluid flow and heat transfer were investigated. Rasheed et al. [15] considered the Generalized Couette, Couette and Poiseuille flows of a third-grade liquid through two parallel infinite plates with heat transfer. The finite element method was applied to solve the equations that governed their mathematical model. It was found that the flow velocity increases as the values of the pressure gradient and non-Newtonian parameter are increased. Furthermore, the temperature of the fluid is increased when the non-Newtonian parameter, pressure gradient and fluid thermal conductivity increase.

The Brinkman equation is a combination of the Navier-Stokes equations and the Darcy Equation, and it is one of the most widely-used models of porous media flows [16, 17]. Almalki and Hamden [18] examined Brinkman's effective fluid viscosity in a porous medium so that the impact of a change in fluid viscosity under Poiseuille flow may be determined. A threshold value corresponding to a value of porosity that is higher than 98% was determined for the conversion from Darcy to Brinkman flow regime. This confirmed the effectiveness of Brinkman's equation for high values of porosity. The time-dependent MHD fluid flow through a porous channel with Hall effect was studied by Sa'adAldin and Qatanani [19] using the finite element method. The porous media flow was modeled by Brinkman's equation. The authors performed a comparison between the finite difference and finite element methods, and found that the finite element method is more accurate. Verma and Gupta [20] used the Brinkman equation to model steady MHD Poiseuille and Couette-Poiseuille flows in a horizontal porous channel, where the wall suction/injection is uniform. A uniform transverse magnetic field was applied and the induced magnetic field was ignored. The authors observed that the Hartmann number and the permeability and suction/injection parameter both have a strong influence on the characteristics of flow. Furthermore, since injection decreases skin friction it increases flow, whereas suction decreases fluid flow.

All the aforementioned studies have made the assumption that the fluid thermophysical properties are constant. However, in many engineering applications, these properties can be strongly dependent on the fluid temperature [21]. Consequently, the temperature dependence of viscosity, electrical conductivity and thermal conductivity has been investigated by several researchers. The steady heat transfer and MHD Couette-Poiseuille flow with temperature-dependent electrical conductivity and viscosity was examined by the authors Makinde and Onyejekwe [22]. The results of this study revealed that increases in the Hartmann number and viscosity exponent reduce the flow velocity. Increasing the viscosity exponent causes an increase in heat transfer rate at the non-stationary wall and a decrease at the stationary wall. Moreover, increasing the electrical conductivity exponent increases the flow rate, heat transfer rate and skin friction. Kumar et al. [23] investigated the influence of the temperature-dependent thermal conductivity and viscosity on the MHD flow and convection heat transfer of two immiscible fluids through a vertical channel. It was determined that the dependence of temperature on the thermal conductivity and the viscosity significantly influences fluid flow, but does not significantly affect the fluid temperature. Also, increases in the viscosity and thermal conductivity of the fluid result in enhanced heat transfer.

In our present study, we investigate the MHD Poiseuille flow of fluid and transfer of heat through a horizontal porous channel with an oscillating pressure gradient and temperature-dependent viscosity. A transverse magnetic field and suction/injection are also applied in this work. The effects of Hartmann number, amplitude of the pressure gradient, oscillation frequency, viscosity parameter, Darcy number, suction/injection parameter and time on the fluid temperature and velocity are examined and discussed with the aid of graphs.

2 Description of the Problem

We consider the unsteady laminar, incompressible and hydromagnetic flow of an electrically-conducting and viscous liquid in a porous medium which has constant permeability K and width $2h$ ($h > 0$) between two horizontal and infinitely-long parallel plates. The Cartesian coordinate system chosen is in terms of x and y, where the x-axis is along the centre of the medium. The upper and lower plates are stationary, with the upper plate positioned at $y = h$ and the lower plate at $y = -h$, as shown in Fig. 1. The fluid flow is influenced by uniform injection at the bottom plate and uniform suction at the top plate, with suction/injection velocity V_0. The lower plate is isothermally heated with temperature T_1, whereas the upper plate is isothermally cooled with temperature T_2 ($T_1 > T_2$). An oscillating horizontal pressure gradient is applied and a constant vertical magnetic field B_0 is imposed. We assume that the fluid operates like a homogeneous Newtonian fluid with constant electrical conductivity σ and density ρ. The fluid viscosity is related to the fluid temperature by the equation [22]

$$\mu = \frac{\mu_0}{1 + \gamma(T - T_2)} \tag{2.1}$$

where μ_0 and γ are empirical constants.

According to the schematic diagram and the physical description of the problem, the following assumptions are considered:

(i) The fluid has magnetic Reynolds number much smaller than 1.

(ii) The porous medium is described using Brinkman's model.

(iii) The effective viscosity

$$\mu_{eff} = \frac{\mu}{\varepsilon}$$

of the porous medium (ε is the porosity) is assumed to be equal to the dynamic viscosity of the fluid; i.e., $\mu_{eff} = \mu$.

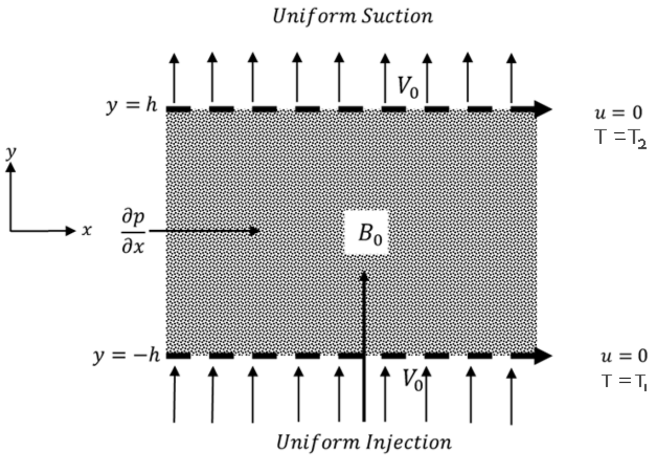

Figure 1. Schematic Diagram of the Problem

From the above assumptions, the governing equations are as follows [23, 24]:
Brinkman Equation:

$$\rho\left(\frac{\partial u}{\partial t} + V_0\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \sigma B_0^2 u - \frac{\mu}{K}u \tag{2.2}$$

Energy Equation:

$$\rho c\left(\frac{\partial T}{\partial t} + V_0\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

where ρ, σ, c and κ are the fluid density, electrical conductivity, specific heat capacity and thermal conductivity respectively.
The initial condition and boundary conditions are

$$u = 0,\ T = T_2 \text{ at } t = 0 \tag{2.4}$$

$$u = 0,\ T = T_1 \text{ at } y = -h \text{ for } t > 0 \tag{2.5}$$

$$u = 0,\ T = T_2 \text{ at } y = h \text{ for } t > 0 \tag{2.6}$$

The problem described by equations (2.1) to (2.6) is simplified by using the following non-dimensional variables:

$$x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ u^* = \frac{\rho h u}{\mu_0}, P^* = \frac{P\rho h^2}{\mu_0^2},\ t^* = \frac{t\mu_0}{\rho h^2},\ T^* = \frac{T - T_2}{T_1 - T_2} \tag{2.7}$$

On dropping the * for convenience and considering an oscillating pressure gradient of the form

$$\frac{\partial P}{\partial x} = -A[1 - \text{sin}XXXXXX(2\pi\omega t)] \tag{2.8}$$

the governing equations (2.2) and (2.3) with initial and boundary conditions (2.4) to (2.6) are:

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{1+\alpha T}\frac{\partial u}{\partial y}\right) - \left(Ha^2 + \frac{1}{Da}\right)u + A[1 - \text{sin}XXXXXX(\pi\omega t)] \tag{2.9}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} \tag{2.10}$$

where $Da = \frac{K}{h^2}$ is the Darcy number, $S = \frac{\rho h V_0}{\mu_0}$ is the suction/injection parameter, $\alpha = \gamma(T_1 - T_2)$ is the (dimensionless) viscosity parameter, A is the (non-dimensional) pressure gradient

amplitude and ω is the (dimensionless) pressure gradient oscillation frequency, $Ha = B_0 h\sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number and $Pr = \frac{\mu_0 c}{\kappa}$ is the Prandtl number.
The non-dimensional initial and boundary conditions become:

$$u = T = 0 \text{ at } t = 0 \tag{2.11}$$

$$u = 0,\ T = 1 \text{ at } y = -1 \text{ for } t > 0 \tag{2.12}$$

$$u = T = 0 \text{ at } y = 1 \text{ for } t > 0 \tag{2.13}$$

The time-dependent volumetric flow rate (dimensionless) is given by

$$Q(t) = \int_{-1}^{1} u(y,t)dy \tag{2.14}$$

whereas the Nusselt number on the heated lower plate is defined as

$$Nu(t) = -\frac{\partial T}{\partial y}(-1,t) \tag{2.15}$$

3 Numerical Solution of the Problem

The non-linear system of equations (2.9) and (2.10) with the initial condition and boundary conditions (2.11) to (2.13) are spatially discretized using the Galerkin finite element technique with three-node quadratic elements, and the Crank-Nicolson technique is utilized for the time discretization. The finite elements are assembled and the computer software MATLAB is used to solve the resulting system of algebraic equations. The numerical procedure was implemented using 100 quadratic elements and 200 time steps.

A grid-independence test is conducted so that a suitable finite element mesh is obtained for the present study. The space-averaged velocity u_{ave} and temperature T_{ave} are computed at time $t = 5$ for 200 time-steps using five different meshes with 25, 50, 100, 200 and 400 quadratic elements (Table 1). It was found that the u_{ave} and T_{ave} relative errors between two successive meshes are less than 0.5%. Based on this grid-independence criterion, a finite element mesh with 100 elements was selected for the present work.

Table 1. Grid-Independence Test Values of u_{ave} and T_{ave} for $Ha = A = \alpha = \omega = S = 1$, $Da = 0.1$, $Pr = 7$ and $t = 5$

No. of Elements	u_{ave}	T_{ave}	Computational Time (s)
25	0.0989	0.9195	6.32
50	0.0999	0.9240	10.48
100	0.1004	0.9262	21.27
200	0.1007	0.9273	40.03
400	0.1008	0.9279	83.65

In order to validate the numerical algorithm used in this study, graphical comparisons (Figs. 2 and 3) of the velocity u and volumetric flow rate Q (when $\alpha = \omega = 0$) are made with the analytical solution obtained by Verma and Gupta [20] for the MHD Poiseuille flow of fluid through a porous channel with constant wall suction/injection. From these graphs, we determine that the results obtained from the present numerical code is in excellent agreement with the work of Verma and Gupta [20].

4 Discussion of Results

The velocity and temperature distributions are represented graphically in Figs. 4 to 11 for varying Ha, A, ω, α, S, Da, t and Pr. Figs. 4 and 5 show profiles of velocity and temperature for varying suction/injection parameter S with $Ha = \omega = \alpha = A = 1$, $Da = 0.1$, $t = 5$ and $Pr = 7$. We observe that the velocity profiles (Fig. 4) are non-symmetrical for $S \neq 0$ and are skewed in the direction of the upper plate at $y = 1$, where there is suction. It can also be seen that the flow velocity u is increased as the suction/injection parameter decreases; this is caused by a reduction in flow resistance within the channel. These results agree with similar work done by Verma and Gupta [20] in their study of MHD porous channel flow with constant injection/suction. From Fig. 5, we see that the temperature of the liquid is decreased with a reduction in the suction/injection parameter due to increased thermal advection associated with the observed increase in flow velocity.

Verma and Gupta [20] Present Algorithm

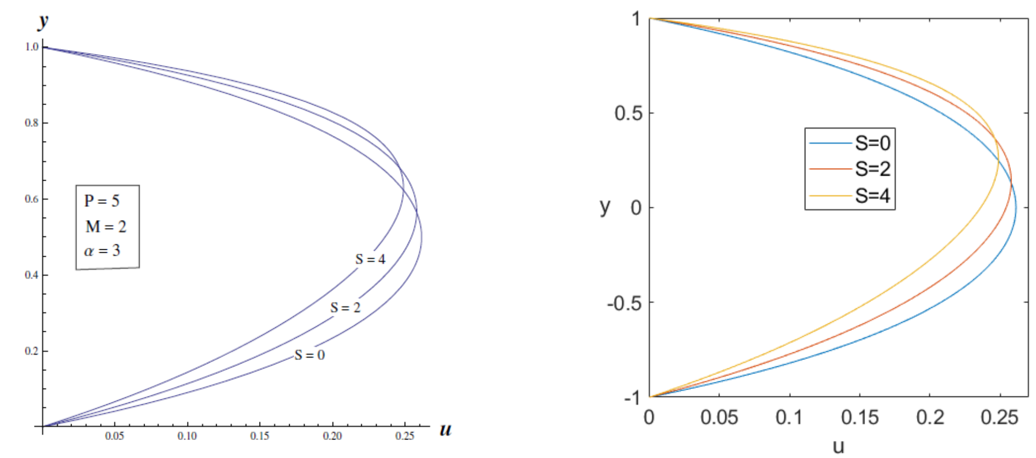

Figure 2. Comparison of Velocity Profiles

Verma and Gupta [20] Present Algorithm

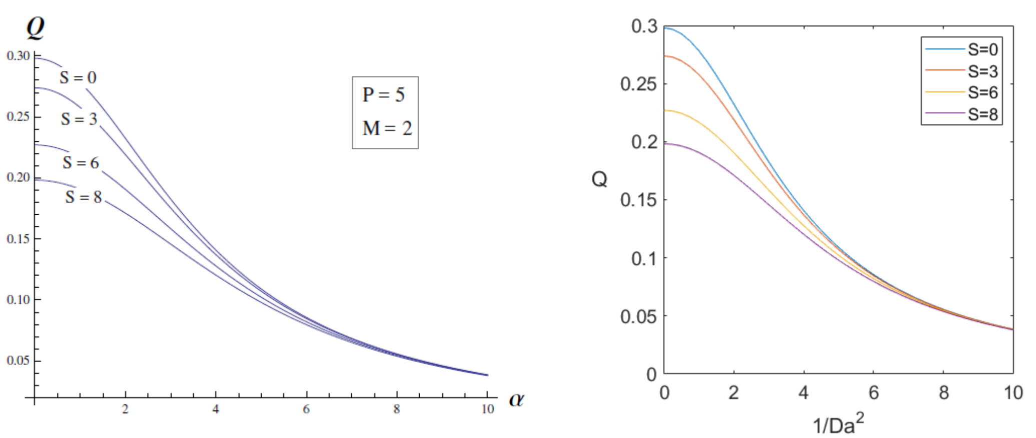

Figure 3. Comparison of Volumetric Flow Rate

Fig. 6 displays the velocity profiles for varying viscosity parameter α with $Ha = \omega = S = A = 1$, $t = 5$, $Da = 0.1$ and $Pr = 7$. We observe that the flow velocity within the channel is enhanced with increased viscosity parameter α. When the viscosity parameter α is increased, the influence of fluid temperature on the viscosity of the fluid is more pronounced. This causes a reduction in the viscosity of the fluid, and leads to the observed increase in flow velocity. We also see that the velocity profile near the channel centre is increasingly flattened as the viscosity parameter α increases.

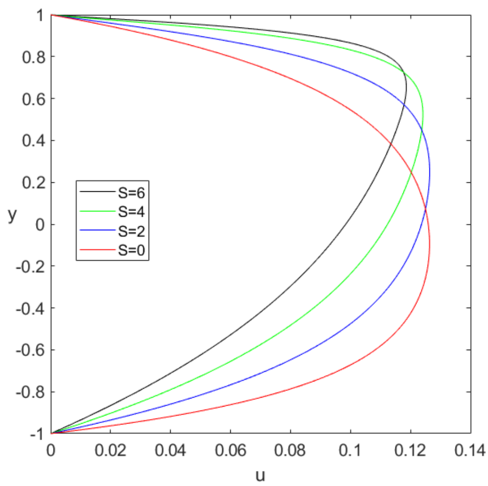

Figure 4. Profiles of velocity for varying S

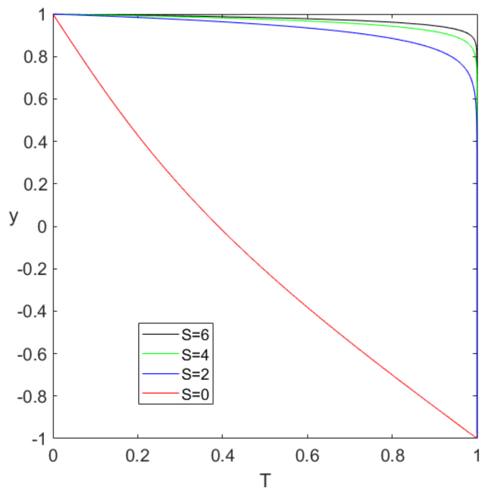

Figure 5. Profiles of temperature for varying S

Fig. 7 shows the profiles of velocity u for varying Darcy number Da with $Ha = \omega = \alpha = S = A = 1$, $t = 5$ and $Pr = 7$. It can be seen in the aforementioned figure that the velocity increases when Da increases. This occurs since an increase in the Darcy number signifies that there is an increase in the porous material's permeability, and therefore velocity increases. This is in good agreement with the study done by Verma and Gupta [20] on magnetohydrodynamic flow in a channel which is porous with constant injection and suction at the boundaries.

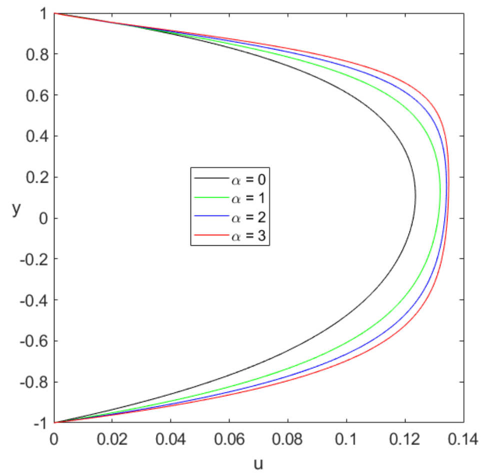

Figure 6. Profiles of velocity for varying α

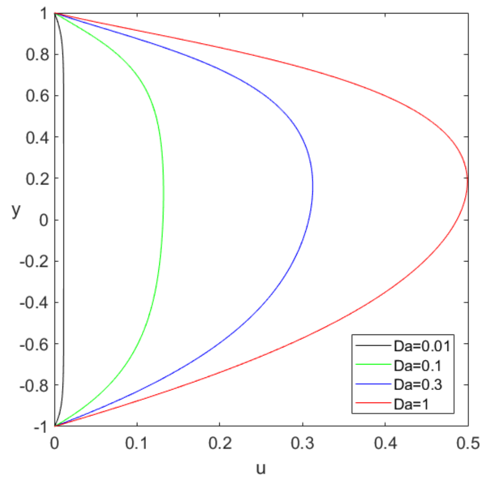

Figure 7. Profiles of velocity for varying Da

Fig. 8 displays the profiles of velocity for Poiseuille flow for varying values of Hartmann number Ha with $\omega = \alpha = S = A = 1$, $t = 5$, $Da = 0.1$ and $Pr = 7$. This figure reveals that as Ha decreases, the velocity u is increased. This occurs because a decrease in the Hartmann number means that the Lorentz force acting in the direction opposite to the flow decreases. As the Hartmann number decreases, the magnetic damping force decreases and therefore the velocity u increases. This result is in good accord with work carried out by Moniem and Hassanin [26], Verma and Gupta [20] and Gupta and Jain [27]. This result is also in agreement with work done by Manyonge, Kiema and Iyaya [28] in their study of steady magnetohydrodynamic Poiseuille flow between two parallel permeable plates of infinite length under the influence of a transverse magnetic field.

Fig. 9 shows the profiles of velocity for varying frequency of oscillation ω with $Ha =$

$\alpha = S = A = 1$, $t = 5$, $Da = 0.1$ and $Pr = 7$. From this graph, we observe that as the frequency of oscillation ω increases, the flow velocity increases. It can be inferred that the fluid pressure decreases as the frequency of oscillations increases, and this causes an increase in the flow velocity [29].

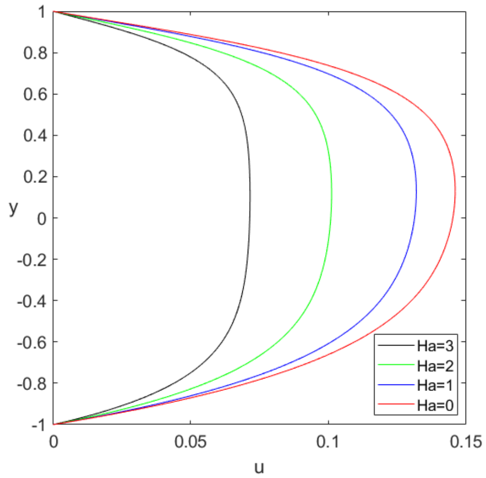

Figure 8. Profiles of velocity for varying Ha

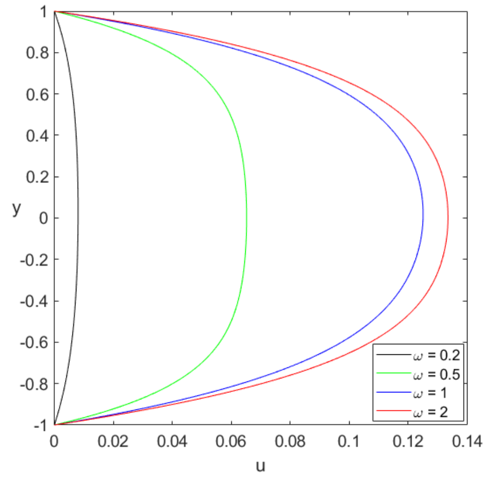

Figure 9. Plot of u vs. y for varying ω

Fig. 10 shows the impact of varying the amplitude A of the pressure gradient on the velocity u of the fluid with $\omega = \alpha = S = A = 1$, $t = 5$, $Da = 0.1$ and $Pr = 7$. The graph shows that as the amplitude A increases, so does the velocity u. This occurs due to a reduction in fluid pressure with increasing amplitude of the pressure gradient [28].

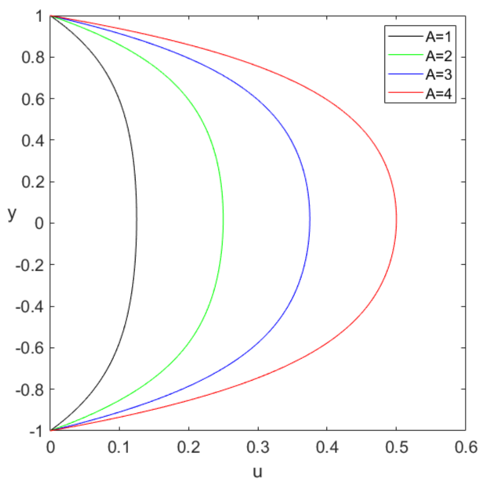

Figure 10. Velocity profiles for varying A

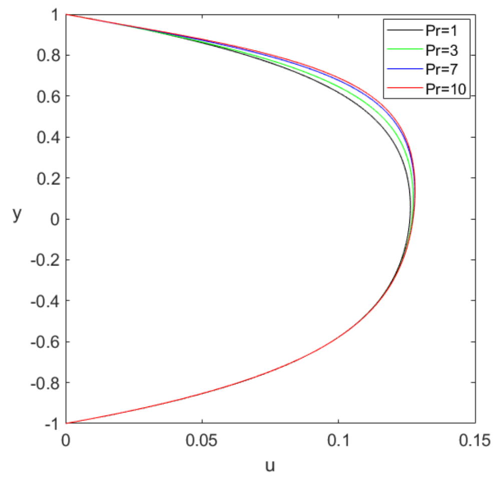

Figure 11. Velocity profile for varying Pr

Figs. 11 and 12 display profiles of flow velocity and fluid temperature for varying Prandtl number Pr with $Ha = \alpha = \omega = S = A = 1$, $t = 5$ and $Da = 0.1$. From these figures, we see that increasing the Prandtl number causes increases in velocity and temperature. This occurs since an increase in the Prandtl number corresponds to a reduction in the thermal diffusivity of the fluid, which causes an increase in fluid temperature. The associated reduction in fluid viscosity produces the observed increase in flow velocity. We also see that the influence of the Prandtl number on fluid flow and heat transfer is more pronounced in the upper section of the channel; this is due to the advective effects of suction/injection through the permeable plates.

The effects of the Darcy number Da, Hartmann number Ha, suction/injection parameter S, Prandtl number Pr and viscosity parameter α on the time-dependent volumetric flow rate Q

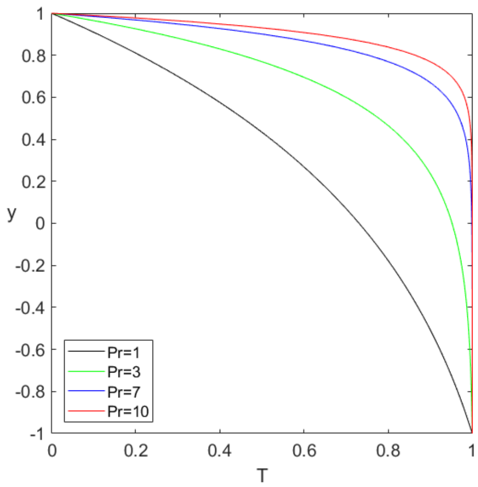

Figure 12. Temperature profile for varying Pr

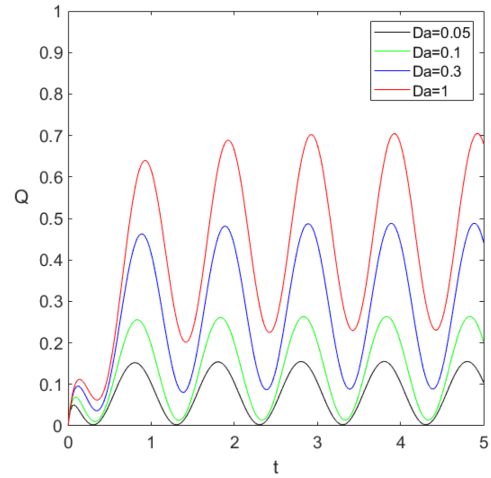

Figure 13. Plot of Q vs. t for varying Da

are shown in Figs. 13 to 17. These figures demonstrate the oscillatory nature of the fluid flow within the channel, which can be attributed to the time-dependent behaviour of the pressure gradient. We found that the volumetric flow rate Q increases when the parameters Da, α and Pr are increased, and when the parameters S and Ha are decreased; these trends occur due to reductions in flow resistance within the channel. The increase in Q with increased Da occurs due to increased permeability in the channel. Increasing the values of α and Pr reduces the viscosity and diffusivity of the fluid, which leads to the observed increase in the volumetric flow rate Q. Decreasing S and Ha increase the volumetric flow rate Q via a reduction in wall suction/injection the Lorentz force acting on the fluid. These results are consistent with the work of Shit and Roy [30], in which it was found that the application of an external magnetic field causes a reduction in the volumetric flow rate.

Figs. 18 and 19 show the time-dependent Nusselt number Nu for varying suction/injection parameter S and Prandtl number Pr. It was determined that for each value of S and for each value of Pr, the Nusselt number decreases over time. This occurs since the transfer of heat from the heated lower plate due to thermal conduction and fluid injection approaches an equilibrium as time progresses. Moreover, due to an increase in fluid thermal diffusivity and a reduction in thermal advection near the lower plate, the heat transfer on the surface of this plate is increased. Therefore, the Nusselt number is increased with reductions in S and Pr.

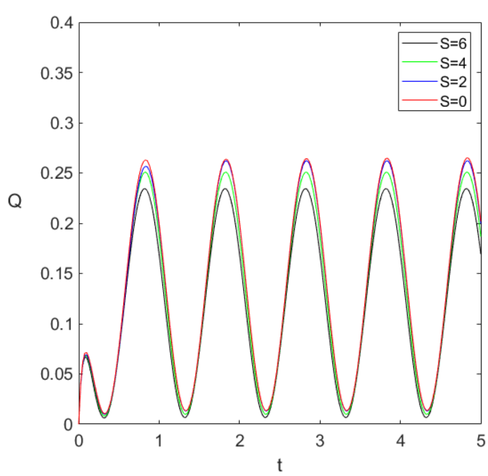

Figure 14. Plot of Q vs. t for varying S

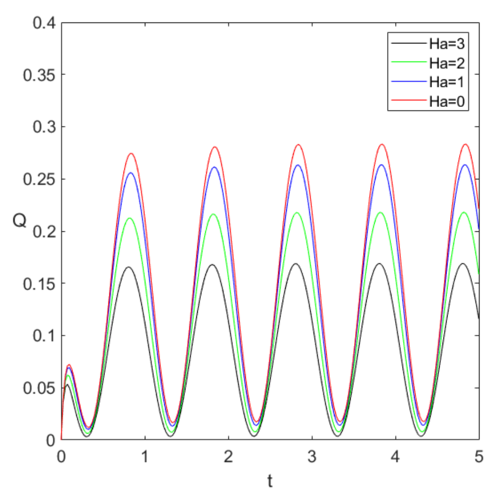

Figure 15. Plot of Q vs. t for varying Ha

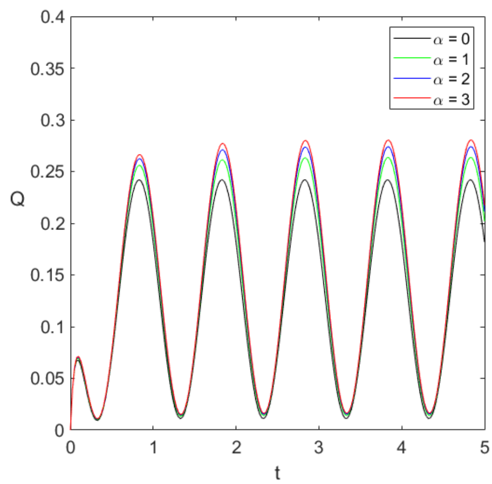

Figure 16. Plot of Q vs. t for varying α

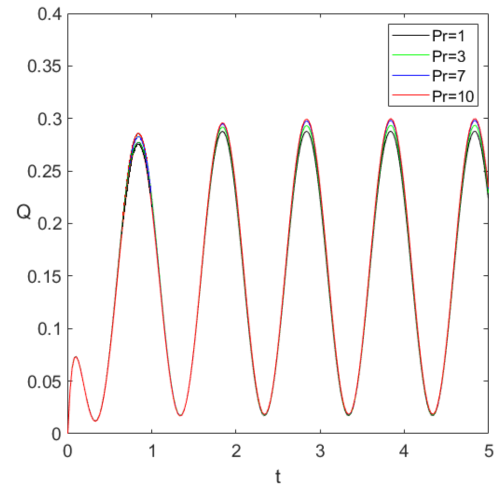

Figure 17. Plot of Q vs. t for varying Pr

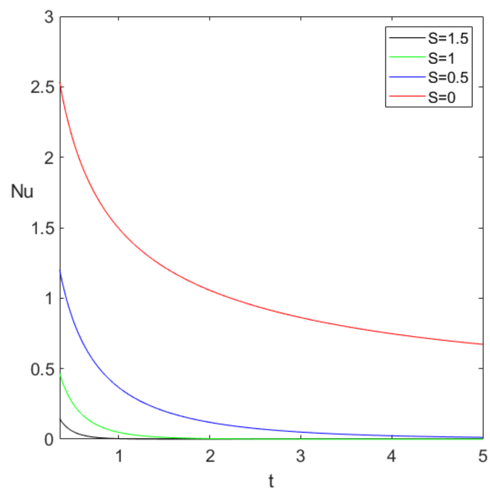

Figure 18. Plot of Nu vs. t for varying S

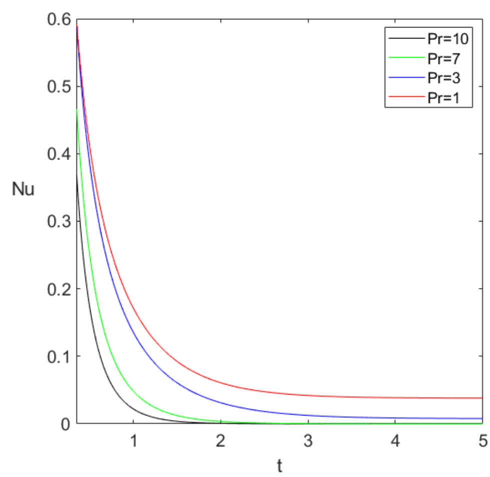

Figure 19. Plot of Nu vs. t for varying Pr

Conclusion

In this study, the unsteady oscillatory MHD Poiseuille convective flow through a horizontal porous channel with uniform suction and injection was investigated. The parameters which were varied to show the behaviour of this type of flow were the suction/injection parameter, Hartmann number, Darcy number, frequency of oscillation, amplitude of the pressure gradient, viscosity parameter and time. The influence of relevant parameters on the time-dependent volumetric flow rate and Nusselt number were also examined. The finite element technique was employed in order to obtain numerical solutions to the problem, and this method was implemented in MATLAB.

The main findings of the present work are as follows:

- The flow velocity is increased when the Darcy number, pressure gradient amplitude, pressure gradient frequency of oscillation, viscosity parameter and Prandtl number are increased, and when the suction/injection parameter and Hartmann number are decreased.
- The temperature of the fluid is increased by enhancing the Prandtl number and decreasing the suction/injection parameter.

- The volumetric flow rate can be enhanced by raising the Darcy number, viscosity parameter and Prandtl number, and decreasing the Hartmann number and suction/injection parameter.
- The Nusselt number can be enhanced by decreasing the suction/injection parameter and Prandtl number.

Conflict of Interest

The authors declare that they have no conflict of interest.

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