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PERISTALTIC FLOW OF COUPLE-STRESS FLUID IN DOUBLY CONNECTED REGION WITH REFERENCE TO ENDOSCOPE

Rashmi K. R., Indira Ramarao and Jagadeesha S.

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Abstract
In the present study, a situation is created by the insertion of endoscope into esophagus is modeled as a concentric annular region. The motion of fluid or semi solid fluid bolus inside the esophagus will be due to peristaltic motion. Since the fluid swallowed will be consisting of solid suspended particles, it is modeled as couple-stress fluid. The peristaltic transport is assumed to be under low Reynolds number and long wavelength approximation. Effect of magnetic field is also considered in the study. The resulting governing equations are solved analytically and velocity profile, rate of flow and pressure gradient are obtained. The velocity and pressure gradient are numerically evaluated and graphically depicted for various flow parameters such as couple-stress parameter, Hartmann number etc. The results coincide with Newtonian case as the couple-stress parameter $\alpha \to \infty$.

1 Introduction

Peristaltic motion occurs in biological system like gastro-intestinal tract, esophagus, intra uterine fluid motion etc. It becomes necessary for diagnostic purpose to insert catheter or endoscope. Catheters are needed for angioplasty, injecting fluid etc. where as endoscope is required to identify problems in stomach, intestine etc. This type of insertion creates annular region which may be eccentric or concentric.

Study of peristalsis in concentric annular region has gained importance in recent years. Nadeem and Akbar [1] have studied effect of heat and mass transfer on flow of Jeffrey six constant fluid under peristalsis. Same authors in another paper [2] have considered Carreau fluid in vertical annulus. Vajravelu et. al. [3] have studied peristaltic motion of Herschel-Buckley fluid in channel. Hayat et. al. [4] have studied flow in tubes with endoscope under peristalsis. Nadeem et. al. [5] have analysed peristaltic flow of a Prandlt fluid in presence of an endoscope. Pandey and Chaube [6] have investigated peristaltic flow of a couple stress fluid to study wall properties. Shit and Roy [7] have considered hydromagnetic effect on flow of Couple-stress fluid in an inclined channel subjected to peristalsis to obtain pressure gradient and stream function. A theoretical study is conducted by Singh et. al. [8] to understand effect of heat transfer on peristaltic transport assuming Robinowitsch fluid model. Mekheimer and Elmaboud[9] have studied peristaltic flow through an annular region assuming porous media with application to endoscope. A mathematical model to study peristaltic transport in an eccentric annular region to obtain perturbation solution is developed by Mekheimer et. al. [10].

In the present study a flow of couple stress fluid in a doubly connected annular region is considered under peristalsis. A couple stress fluid flow is analyzed obtaining series solution for governing equation. Pressure gradient is estimated and graphically depicted.

2 Mathematical Formulation

Concentric cylindrical tubes are considered with inner rigid tube and outer cylinder flexible. A sinusoidal wave is applied on the wall. The fluid considered is couple stress. It is assumed that the annular region is porous medium filled with fluid. Following assumptions are made. A moving frame of reference is considered. Flow is under low Reynolds number and long wavelength approximation is applied.

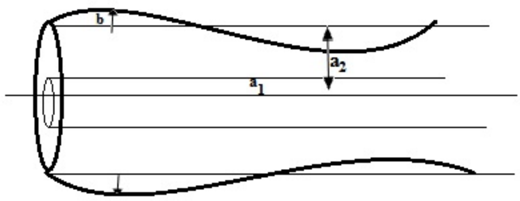

Figure 1. Physical Configuration

After non dimensionalising and applying above assumptions the governing equation takes the form,

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$\frac{\partial p}{\partial z} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]^2 w - \alpha^2\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \frac{\alpha^2}{k}(w+1)\right] \tag{2.2}$$

where k is the non dimensional permeability and α^2 is the couple stress parameter.

The inner cylinder radius is given by $r_1 = \epsilon$ and outer cylinder radius is $r_2 = 1 + \phi cos 2\pi z$, where ϕ is the amplitude ratio. The boundary conditions are assumed in the non dimensional form as $w = 1$ on $r = r_1$ and $r = r_2$.

Solving the resulting fourth order equation using Frobenious method velocity profile is obtained as,

$$w(r,z) = -\left\{1 + \alpha^2\frac{\partial p}{\partial z} - \frac{1}{k}\right\}[b_1Y_1(r) + b_2Y_2(r) + b_3Y_3(r) + b_4Y_4(r) - 1] - 1 \tag{2.3}$$

the constants are listed in the appendix.

The volume flow rate $Q(z)$ is given by,

$$Q(z) = 2\int_{r_1}^{r_2} rw(r,z)dr. \tag{2.4}$$

The average mean volume flow rate is given by,

$$\bar{Q} = Q(z) + \left(1 + \frac{\phi^2}{2}\right) - \epsilon^2 \tag{2.5}$$

see Shapiro et. al. [11].

Substituting the value of $w(r,z)$ in the equation (2.4), we get,

$$Q(z) = \left\{-1 + \alpha^2\frac{\partial p}{\partial z} - \frac{\alpha^2}{k}\right\}\left[b_1\phi_1 + b_2\phi_2 + b_3\phi_3 + b_4\phi_4 - \left(\frac{r_2^2 - r_1^2}{2}\right)\right] - \left(\frac{r_2^2 - r_1^2}{2}\right) \tag{2.6}$$

where $\phi_1 = \int_{r_1}^{r_2} rY_i(r)dr$.

Simplifying the equation (2.6), the pressure gradient is obtained as,

$$\frac{dp}{dz} = \frac{Q(z) + \left(\frac{r_2^2 - r_1^2}{2}\right)}{\left[b_1\phi_1 + b_2\phi_2 + b_3\phi_3 + b_4\phi_4 - \left(\frac{r_2^2 - r_1^2}{2}\right)\right]\alpha^2} + \frac{1}{\alpha^2} + \frac{1}{k}. \tag{2.7}$$

Substituting for $Q(z)$ in the equation (2.5), we get,

$$\frac{dp}{dz} = \frac{\bar{Q} + \epsilon^2 - \left(1 + \frac{\phi^2}{2}\right) + \left(\frac{r_2^2 - r_1^2}{2}\right)}{\left[b_1\phi_1 + b_2\phi_2 + b_3\phi_3 + b_4\phi_4 - \left(\frac{r_2^2 - r_1^2}{2}\right)\right]\alpha^2} + \frac{1}{\alpha^2} + \frac{1}{k}. \tag{2.8}$$

The pressure rise is,

$$\triangle p = \int_0^1 \frac{dp}{dz} dz. \tag{2.9}$$

The frictional force on outer and inner tubes are given by,

$$F_1 = \int_0^1 R_2^2 \left(\frac{dp}{dz}\right) dz \tag{2.10}$$

and

$$F_2 = \int_0^1 R_1^2 \left(\frac{dp}{dz}\right) dz. \tag{2.11}$$

3 Results and Discussions

A flow of couple stress fluid is considered in an annulus filled with porous media. The flow is subjected to long wave length approximation. The solution obtained analytically is computed numerically and graphically depicted. The parameter arising out of the study are assumed to take following values. The value of amplitude ratio of the wave is taken as 0.1, 0.2, 0.3 and r_1 is taken as 0.2 and 0.1. The average flow rate $\bar{Q}$ is assumed as 0.4 and 0.6.The permeability k is assumed as 0.25 and 0.5 and couple stress parameter α is considered as 1.0, 2.0 and 5.0.

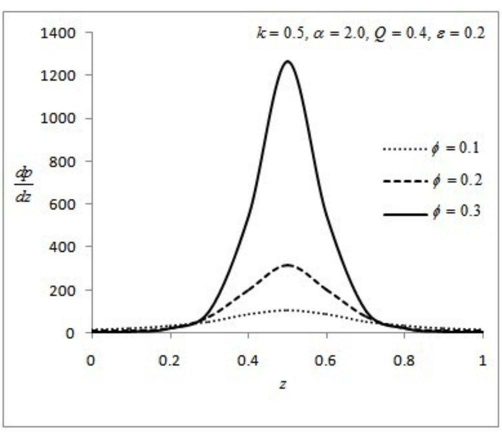

Figure 2. Axial pressure gradient for different values of wave number

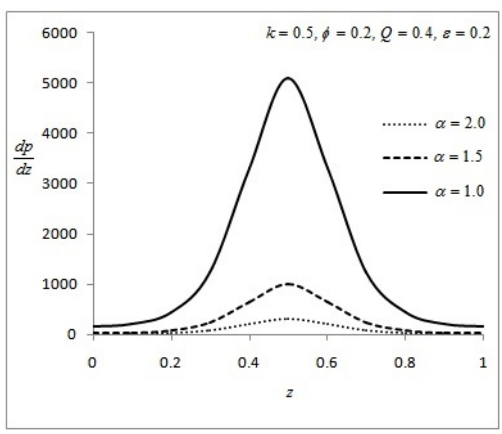

Figure 3. Axial pressure gradient for different values of couple stress parameter

Figure 3 shows effect of couple stress parameter α on pressure gradient. α is inversely proportional to spin of the suspended particle. Hence the fluid nature tends to become Newtonian as α increases. The pressure gradient decreases with increasing in α and this decrease is significant. These results indicate the resistance to flow is higher in case of couple stress fluid.

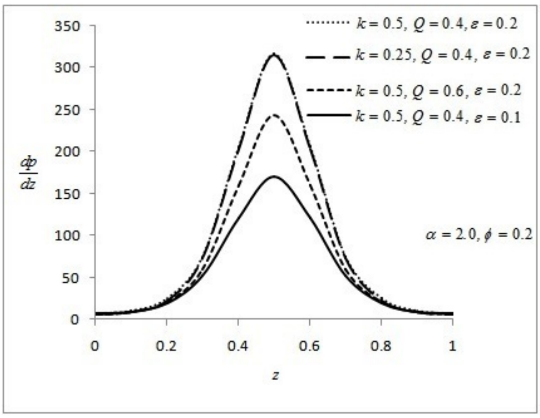

Figure 4. Axial pressure gradient

Figure 4 shows variation of permeability k, average rate of flow $\bar{Q}$ and inner circle radius $\epsilon(r_1)$. The effect of permeability is highly insignificant which can be visualized by top two curves. Increase in rate of flow decreases pressure. Increase in radius of inner tube also results are in coherence with the fact that decrease of ϵ increases area of cross-section increasing flow rate, thereby decreasing $\frac{dp}{dz}$.

Figures 5 and 6 show plot of axial velocity w against radial and axial directions respectively. Velocity shows sinusoidal nature in the figure. As the couple stress parameter α increases velocity increase due to decrease in resistance to flow. The effect of wave amplitude ϕ is significant on velocity. As ϕ increases the wavelength decreases but the peak increases.

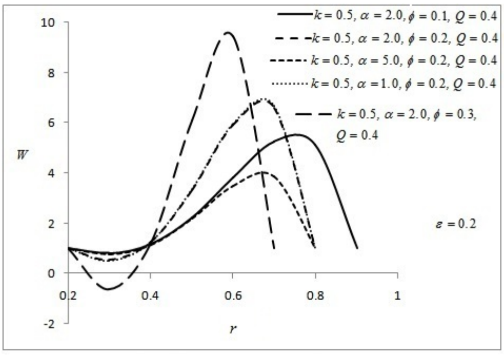

Figure 5. Axial velocity vs. Radial direction

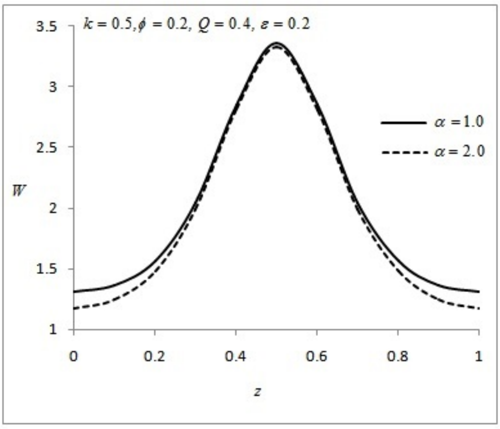

Figure 6. Axial velocity vs. z

Figure 6 shows plot of velocity W against axial direction. The velocity profile is analogous to pressure gradient. The effect of increase in α is to decrease velocity in the axial direction.

4 Conclusions

The present study analyses the effect of couple stresses on the peristaltic flow in a porous annulus under the effect of endoscope analytically. The effect of couple stresses is significant on pressure gradient $\frac{dp}{dz}$, also significant in radial direction but not so significant in axial direction in case of velocity. Permeability is the parameter which influences the flow very significantly. The velocity profile is analogous to pressure profile. The results of Hayat et. al. [4] are obtained in the limit $\alpha \to \infty$.

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Appendix

$$b_1 = \frac{1-b_2Y_2(r_1)-b_3Y_3(r_1)-b_4Y_4(r_1)}{Y_1(r_1)},\ b_2 = \frac{G_1-g_{13}b_3-g_{14}b_4}{g_{12}},\ b_3 = \frac{H_1-h_{14}b_4}{h_{13}},\ b_4 = \frac{L_1}{l_{14}},$$

$$L_1 = h_{23}H_1 - h_{13}H_2,\ l_{14} = h_{23}h_{14} - h_{13}h_{24},$$

$$H_1 = g_{21}G_1 - g_{12}f_{11},\ H_2 = g_{31}G_1 - g_{32}f_{21},\ G_1 = Y_1(r_2) - Y_1(r_1),$$

$$h_{13} = g_{24}g_{13} - g_{12}g_{23},\quad h_{14} = g_{21}g_{14} - g_{12}g_{14},\quad h_{23} = g_{31}g_{13} - g_{12}g_{22},\ h_{24} = g_{31}g_{14} - g_{12}g_{14},$$

$$g_{1i} = Y_1(r_2)Y_i(r_1) - Y_i(r_2)Y_1(r_1),\ g_{2i} = f_{11}Y_i(r_1) - Y_i(r_2)f_{1i},\ g_{3i} = f_{21}Y_i(r_1) - Y_i(r_2)f_{2i},$$

$$f_{1i} = Y_i''(r_1) + \frac{1}{r_1}Y_1'(r_1),\ f_{2i} = Y_i''(r_2) + \frac{1}{r_2}Y_1'(r_2),$$

$$Y_1(r) = 1 + \frac{8}{75}\alpha^2 r^4 + \Sigma\frac{\alpha^{2n-2}2^{4n}\left\{\alpha^2 n! - \frac{75}{k^{n-1}2^{2n-2}n!}\right\}}{(n+1)!\{5.13......(8n-3)\}\{15.23......(8n+7)\}} r^{2n+2},$$

$$Y_2(r) = 1 + \frac{8}{75}\alpha^2 r^4 \left(logr - \frac{49}{60}\right) + \Sigma\frac{\alpha^{2n-2}2^{4n}\left\{\alpha^2 n! - \frac{75}{k^{n-1}2^{2n-2}n!}\right\}}{(n+1)!\{5.13......(8n-3)\}\{15.23......(8n+)\}} \left[logr - \psi(n)\right] r^{2n+2},$$

$$Y_3(r) = 1 + \frac{11}{171}\alpha^2 r^{\frac{27}{4}} + \Sigma\frac{\alpha^{2n-2}\left\{\alpha^2[11.19......(8n+3)]^2 - \frac{19.11.4.13}{k^{n-1}}\right\}}{\{1.9.27.....(8n+11)\}\{11.19......(8n+3)\}n!\{13.17......(4n+5)\}} r^{2n+\frac{11}{4}},$$

$$Y_4(r) = 1 + \frac{1}{9}\alpha^2 r^{\frac{9}{4}} + \Sigma\frac{\alpha^{2n-2}\left\{\alpha^2[1.9......(8n-7)]^2 - \frac{9.1.3.4}{k^{n-1}}\right\}}{\{9.17......(8n+1)\}\{1.9......(8n-7)\}n!\{3.7......(4n-5)\}} r^{2n+\frac{1}{4}}.$$

Author information

Rashmi K. R., Indira Ramarao and Jagadeesha S., Department of Mathematics, Nitte Meenakshi Institute of Technology, Bengaluru, Karnataka - 560064, India.
E-mail: jagadeeshas31@gmail.com

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