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FULLY DEVELOPED MIXED CONVECTION IN A VERTICAL CHANNEL FILLED WITH NANOFLUIDS WITH HEAT SOURCE OR SINK

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Abstract In this paper, we have analysed the fully developed mixed convection in a vertical channel filled with nanofluid with heat source or sink. The effect of Brownian motion and thermophoresis are taken into account as the effective of all slip mechanisms concerned with nanofluids for mathematical modelling. The solutions are then obtained analytically for velocity, temperature and concentration. The effects of Brownian motion coefficient, thermophoresis coefficient and heat source or sink parameters are analysed graphically. Also the profiles of Nusselt number and Sherwood numbers at the left boundary are discussed.

1 Introduction

Convective flows in vertical channels are significant in the improvement of cooling systems in heat exchangers, solar cells, nuclear reactors and many other electrical and industrial appliances. Free convection or combined free and forced convective heat transfer have been the subject of investigations for many years. Tao L. N. [1] have studied the heat transfer problem of combined free and forced convection by a fully developed laminar flow in a vertical channel with constant wall temperature. Aung W. and Worku G. [2] gave insights into the characteristics of fully developed flow and condition for flow reversal with asymmetric wall temperature condition. Barletta A. [3] analysed the laminar mixed convection in a vertical channel in the fully developed region by taking into account the effect of viscous dissipation and analysed the existence of dual solutions of the local balance equations [4]. The effect of viscous dissipation on fully developed mixed convection is analyzed for the laminar flow in a parallel plate vertical channel whose walls exchange heat with an external fluid, with third kind boundary conditions has been studied by E. Zanchini [5].

The presence of heat source or sink in a flowing field affects its velocity and temperature to an appreciable extent. This study is important because of its applications in many industries like heat exchangers in cooling processes etc. A. J. Chamkha [6] examined the influence of the magnetic, heat generation or absorption co-efficient and the viscous parameters on the characteristics of combined forced-free convection flow inside a square cavity. The effects of heat source or sink and other parameters on natural convection flow between periodically heated vertical plates have been investigated by Basant K. Jha [7]. Patil Mallikarjun B et al. [8] studied the effect of magneto convection flow in vertical channel with heat source or sink.

Nanofluids are the fluids in which nanosized (1-100nm) particles such as metallic oxides suspended in base fluids such as water, ethylene glycol etc. The term “Nanofluids” was coined by U. S. Choi [9]. Now a days nanofluids are considered as the better heat transfer fluids in various heat transfer applications such as heat exchangers, cooling of electronic equipment, nuclear reactors etc. This is because the presence of nanoparticles in the fluids considerably enhances the heat transfer characteristics [10], [11] and [12]. J. Buongiorno [13] gave the explanation for the abnormal heat transfer enhancement observed in nanofluids. He identified that Brownian motion and thermophoresis are the important of all the slip mechanisms concerned with nanofluids and gave the two component non-homogeneous equilibrium model for transport phenomenon in nanofluids. Also he noted that the velocity of nanofluids can be viewed as the sum of the base fluid velocity and slip velocity.

Oztop H. F. and Abu-Nada E. [14] have studied natural convection in partially heated rectangular enclosures filled with nanofluis numerically. Kuznetsov et al. [15] examined the influence of nanoparticles on natural convection boundary layer flow over a vertical plate and gave the results of effects on reduced nusselt number. Kang H. U. et al. [16] estimated the thermal conductivities of nanofluid using transient hot-wire method. They used the effective volume of nanoparticles instead of real volume to predict the enhancement of thermal conductivity of nanofluid. T. Grosan and I.Pop [17] have studied the fully developed and laminar mixed convection flow in a vertical channel filled with nanofluid using the model in which Brownian motion and thermophoresis effects are taken into account. J. C. Umavathi et al. [18] analysed the natural convective heat transfer in a vertical rectangular duct filled with a nanofluid. The results are drawn for various nanoparticles and concluded that inclusion of nanoparticles into pure water improves its heat transfer performance and there is an optimum solid volume fraction which maximizes the heat transfer rate. K. Sushma et al. [19] studied the mixed convection flow of a Jeffrey nanofluid in a vertical channel. J. C. Umavathi et al. [20] analysed the mixed convection in a porous open ducts filled with nanofluid for immiscible fluids sandwiched between porous medium filled with conventional fluid. Lavanya and Nagasasikala investigated the radiation and dissipaton on steady convective flow of a nanofluid in a vertical channel.Flow of nanofluids inside a vertical tube subjected by constant heat flux from the wall was numerically analyzed using Ansys Fluent release 17.2. by Mohammed Saad Kamel and Ferenc Lezsovits[22]. Many researchers made a vast contribution in the study of nanofluids in channel because of their tremendous application s in engineering field [23] to [24].

In this paper, the effect of heat source or sink on fully developed mixed convection flow in a parallel plate vertical channel filled with nanofluid is studied.

2 MATHEMATICAL FORMULATION

Consider a parallel plate vertical channel parted by a distance L in which nanofluid is flowing steadily. Let us fix the coordinate system in which x-axis is aligned parallel to the direction of flow and which is opposite to the direction of gravitational field. The y-axis is taken perpendicular to the channel walls as presented in Fig.1. An assumption is made that temperature and nanoparticles concentrations at the wall $y = 0$ are T_1 and C_1 and at $y = L$ are T_2 and C_2 respectively. At the channel entrance, the velocity is assumed to be u_0.

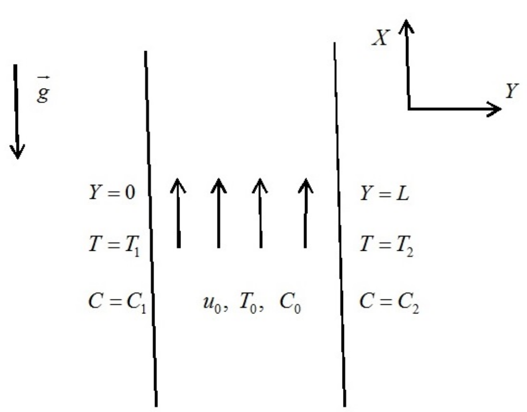

Figure 1. Physical Configuration of the model

The conservation of mass, momentum, energy and nanoparticle concentration equations governing the considered flow along with the Boussinesq approximation in vectorial form [see Ref.[15] and [17]] are given as:

$$\nabla.\vec{q} = 0 \tag{2.1}$$

$$\rho_f(\vec{q}.\nabla\vec{q}) = -\nabla p + \mu\nabla^2\vec{q} + \{C\rho_p + (1-C)[\rho_f(1-\beta(T-T_0))]\}\vec{g} \tag{2.2}$$

$$(\rho c)_f(\vec{q}.\nabla T) = k\nabla^2 T + (\rho c)_p[D_B\nabla T.\nabla C + (D_T/T_0)\nabla T.\nabla T] \pm Q(T-T_0) \tag{2.3}$$

$$\vec{q}.\nabla C = D_B\nabla^2 C + (D_T/T_0)\nabla^2 T \tag{2.4}$$

Where T_0 is the reference temperature assumed to be $T_0 = (T_1 + T_2)/2$, C_0 is the reference nanoparticles concentration which is $C_0 = (C_1 + C_2)/2$, ρ_p is the nanoparticle mass density, ρ_f is the density of the fluid, $(\rho C)_f$ is the heat capacity of the fluid and $(\rho C)_p$ is the heat capacity of the nanoparticle material.

With the assumptions that flow is parallel to x-axis, steady and fully developed, the following relations can be drawn.

$$v = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial C}{\partial x} = 0$$

$$\frac{\partial T}{\partial x} = 0, \frac{\partial p}{\partial x} = 0, \frac{dp}{dx} = 0 \tag{2.5}$$

Therefore, equations (2.2) to (2.4) becomes

$$-\frac{dp}{dx} + \mu\frac{d^2u}{dy^2} + [(1-c_0)\rho_{f_0}\beta(T-T_0) - (\rho_f - \rho_{f_0})(C-C_0)]g = 0 \tag{2.6}$$

$$K\frac{d^2T}{dy^2} + (\rho C)_p\left[D_B\frac{dC}{dy}\frac{dT}{dy} + \left(\frac{D_T}{T_0}\right)\left(\frac{dT}{dy}\right)^2\right] \pm Q(T-T_0) = 0 \tag{2.7}$$

$$D_B\frac{d^2C}{dy^2} + \left(\frac{D_T}{T_0}\right)\frac{d^2T}{dy^2} = 0 \tag{2.8}$$

Subjected to the boundary Conditions

$$u = 0, \qquad T = T_1, \qquad C = C_1 \qquad at \qquad y = 0$$

$$u = 0, \qquad T = T_2, \qquad C = C_2 \qquad at \qquad y = 1 \tag{2.9}$$

Mass flux conservation M is given by

$$\int_0^L u dy = M \tag{2.10}$$

Then we define the following dimensionless parameters

$$Y = \frac{y}{L}, \quad U(Y) = \frac{u(y)}{u_0}, \quad \theta(Y) = \frac{T-T_0}{T_2-T_0}, \quad \phi(Y) = \frac{C-C_0}{C_2-C_0},$$

$$Gr = \frac{(1-C_0)g\beta(T_2-T_0)L^3}{\nu^2}, \quad Re = \frac{u_0 L}{\nu}, \quad Nb = \frac{D_B(C_2-C_0)}{k/(\rho c)_p},$$

$$Nr = \frac{g(\rho_f - \rho_{f_0})(C_2 - C_0)L^2}{\mu u_0}, \qquad Nt = \frac{D_T(T_2 - T_0)}{T_0 k/(\rho c)_p}, \qquad = \frac{QL^2}{K} \tag{2.11}$$

Applying these dimensionless variables in Equations (2.6) to (2.8), we get the non-dimensional governing equations as

$$\frac{d^2U}{dY^2} + \frac{Gr}{Re}\theta - Nr\phi + \alpha = 0 \tag{2.12}$$

$$\frac{d^2\theta}{dY^2} + Nb\frac{d\theta}{dY}\frac{d\phi}{dY} + Nt\left(\frac{d\theta}{dY}\right)^2 \pm \psi\theta = 0 \tag{2.13}$$

$$\frac{d^2\phi}{dY^2} + \frac{Nt}{Nb}\frac{d^2\theta}{dY^2} = 0 \tag{2.14}$$

and the boundary conditions become

$$U(0) = 0, \quad \theta(0) = -1, \quad \phi(0) = -1$$

$$U(1) = 0, \quad \theta(1) = 1, \quad \phi(1) = 1 \tag{2.15}$$

Mass flux conservation equation (2.10) becomes

$$\int_0^1 U dY = 1 \tag{2.16}$$

Where Gr is the Grashof number, Re is the Reynolds number, Nb is the buoyancy ratio parameter, Nt is the thermophoresis parameter, Nr is the buoyancy ratio parameter, $\alpha = dp/dx$ is the pressure gradient parameter and we have taken $Q = U_0 L$.
The physical quantities like Nusselt number (Nu) and Sherwood number (Sh) are defined as

$$Nu = -\frac{L}{T_2 - T_0}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad Sh = -\frac{L}{C_2 - C_0}\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{2.17}$$

Using non-dimensional parameters, we get the dimensionless Nusselt number and Sherwood number as

$$Nu = -\theta'(0) \qquad Sh = -\phi'(0) \tag{2.18}$$

3 Solution

Equations (2.12) to (2.14) with the boundary conditions (2.15) are solved analytically. Equations (2.13) and (2.14) can be decoupled and obtain the expressions for temperature and nanoparticle concentration. Using these in (2.12) and the expression of mass flux conservation, velocity can be evaluated. The solution for velocity, temperature, concentration and pressure gradient are found to be

$$\theta(Y) = A_3 e^{m_1 Y} + A_4 e^{m_2 Y}$$

$$\phi(Y) = -\frac{Nt}{Nb}\theta(Y) + A_1 Y - \frac{A_1}{2}$$

$$\begin{aligned} U(Y) \;=\; & \left(6 + \frac{A_6}{12} + \frac{2A_5A_3}{m_1^2}(2 + e^{m_1}) + \frac{2A_5A_4}{m_2^2}(2 + e^{m_2}) + \frac{6A_5A_3}{m_1^3}(1 - e^{m_1}) + \frac{6A_5A_4}{m_2^3}(1 - e^{m_2})\right) Y + \\ & \left(6 + \frac{A_6}{4} + \frac{3A_5A_3}{m_1^2}(1 + e^{m_1}) + \frac{3A_5A_4}{m_2^2}(1 + e^{m_2}) + \frac{6A_5A_3}{m_1^3}(1 - e^{m_1}) + \frac{6A_5A_4}{m_2^3}(1 - e^{m_2})\right) Y^2 \\ & + \frac{A_6}{6}Y^3 + \frac{A_5A_3}{m_1^2}e^{m_1 Y} + \frac{A_5A_4}{m_2^2}e^{m_2 Y} - \frac{A_5A_3}{m_1^2} - \frac{A_5A_4}{m_2^2} \end{aligned}$$

$$\alpha = 12 + \frac{6A_5A_3}{m_1^2}(1+e^{m_1}) + \frac{6A_5A_4}{m_2^2}(1+e^{m_2}) + \frac{12A_5A_3}{m_1^3}(1-e^{m_1}) + \frac{12A_5A_4}{m_2^3}(1-e^{m_2})$$

where

$$m_1 = \frac{-A_2+\sqrt{A_2^2-4}}{2}, m_2 = \frac{-A_2-\sqrt{A_2^2-4}}{2},$$

$$A_1 = 2\left(1+\frac{Nt}{Nb}\right), A_2 = NbA_1, A_3 = \frac{-e^{m_2}-1}{e^{m_2}-e^{m_1}},$$

$$A_4 = \frac{1+e^{m_1}}{e^{m_2}-e^{m_1}}, A_5 = \left(\frac{Gr}{Re}+\frac{NrNt}{Nb}\right), A_6 = -NrA_1$$

4 Results and Discussion

The problem of fully developed mixed convection in a parallel plate vertical channel filled with nanofluid with heat source or sink is analysed in this paper. The effect of different physical parameters such as Buoyancy ratio parameter Nr, Brownian motion parameter Nb, thermophoresis parameter Nt and heat source or sink parameter on velocity, temperature and concentration are analysed graphically.

For a fixed value of mixed convection parameter Gr/Re=500, in figs. 2-4 we can observe that with increase in Buoyancy ratio parameter Nr, Brownian motion parameter Nb, thermophoresis parameter Nt, velocity of the fluid enhances near cold wall and suppresses near hot wall and flow reversal can also be observed near both the walls. If Nr , Nb and Nt are taken to be zero, the results will be for normal conventional fluids. The same effect can be observed for variation of heat source parameter as shown in fig.7.

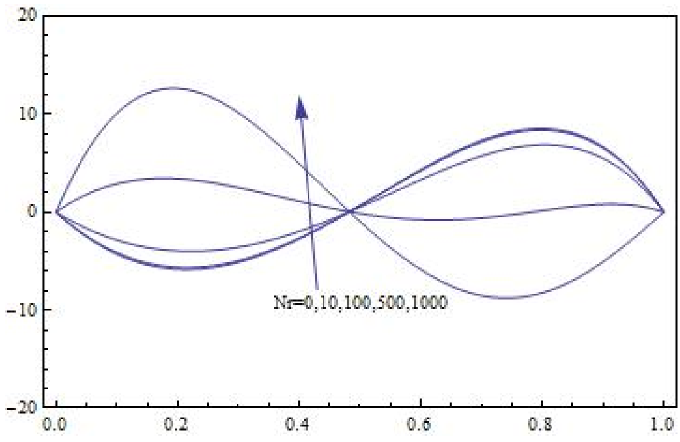

Figure 2. Velocity profiles for $Nr = 0, 10, 100, 500, 1000$ and $Nt = Nb = 0.5$

Figures 5 and 6 represents the temperature and nanoparticle concentration profiles for different values of Nb and Nt. It can be noted that temperature profiles increases with increase in both Brownian motion parameter and thermophoresis parameter because of increase in their collision with the fast moving nanosized particles in the fluid. Nanoparticle concentration increases with Nb and decreases with Nt. From figs. 8 and 9 , we can observe that temperature profiles increases and nanoparticle concentration decreases with increase in heat source parameter.

In figures 10 and 11, Nusselt number and Sherwood numbers are plotted against Brownian motion parameter Nb and thermophoresis parameter Nt. It can noted that Nusselt number decreases with increase in both Nb and Nt, while Sherwood number increases with increase in Nt and decreases with increases in Nt, which is same as observed for temperature and nanoparticle

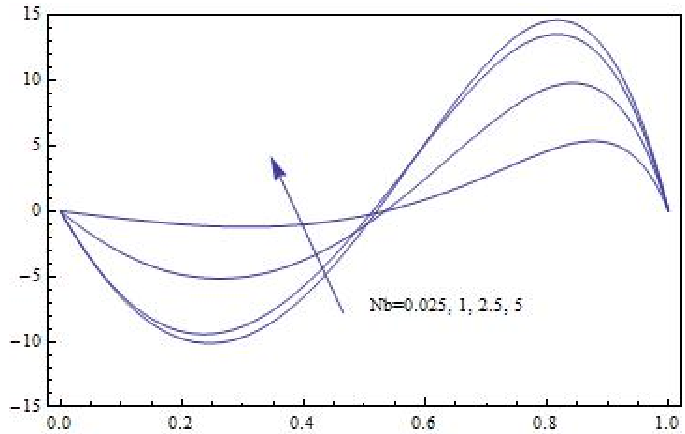

Figure 3. Velocity profiles for $Nb = 0.025, 1, 2.5, 5, Nr = 100$ and $Nt = 0.5$

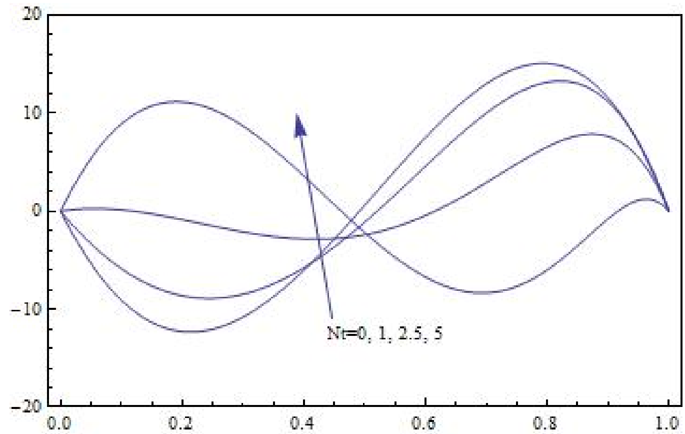

Figure 4. Velocity profiles for $Nt = 0, 1, 2.5, 5, Nr = 100$ and $Nb = 0.5$

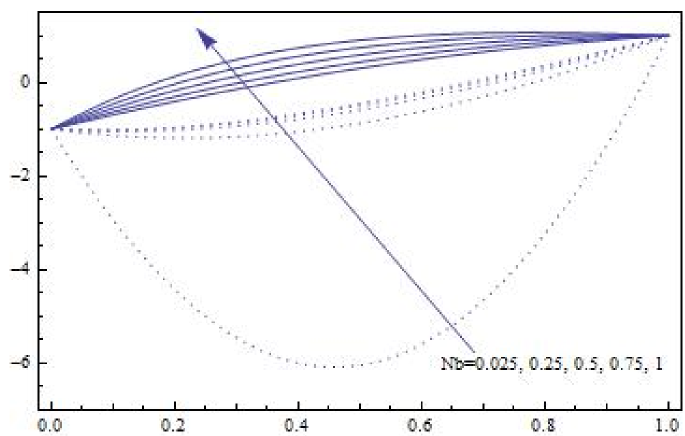

Figure 5. Temperature profiles(Full line) and Concentration profiles (Dotted line) for $Nb = 0.025, 0.25, 0.5, 0.75, 1, Nr = 100$ and $Nt = 0.5$

concentration profiles. Since the momentum equation is uncoupled with the energy and nanoparticle volume fraction equations, Nusselt number and Sherwood numbers are not influenced by the mixed convection parameter $\frac{Gr}{Re}$, buoyancy ratio parameter Nr and pressure gradient parameter α.

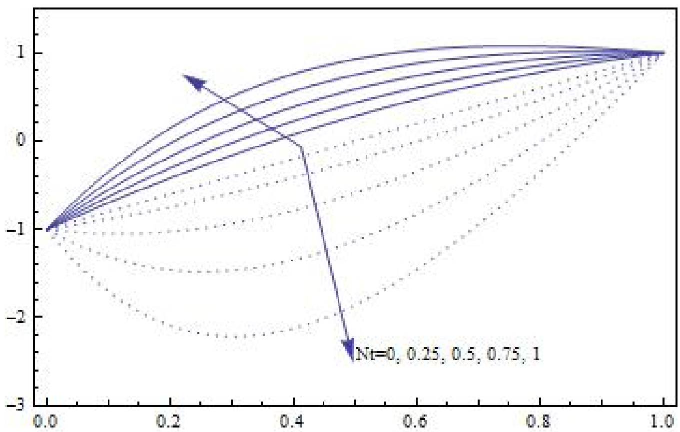

Figure 6. Temperature profiles(Full line) and Concentration profiles (Dotted line) for $Nt = 0, 0.25, 0.5, 0.75, 1, Nr = 100$ and $Nb = 0.5$

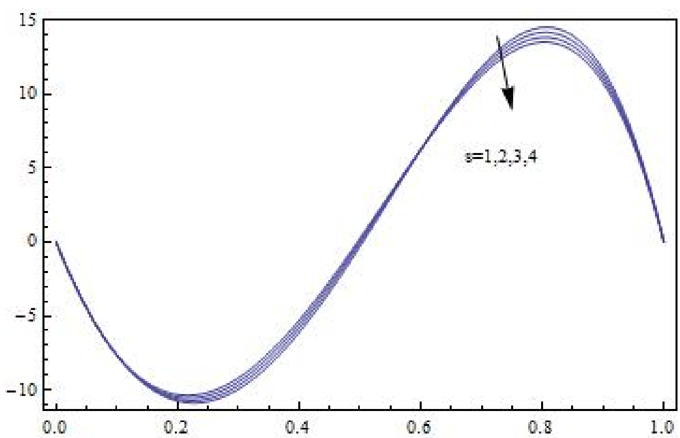

Figure 7. Velocity profiles for $\psi = 1, 2, 3, 4$

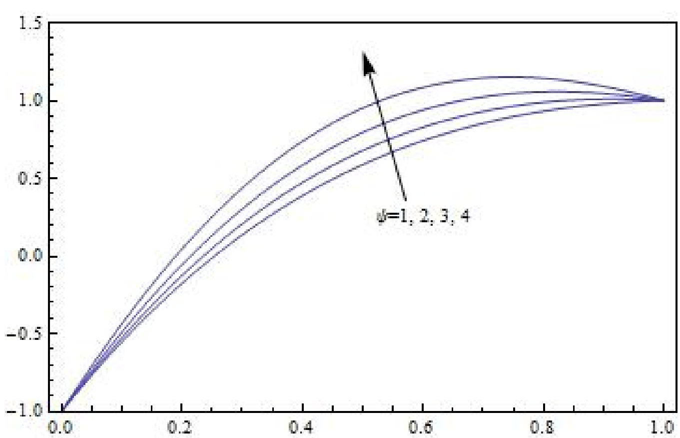

Figure 8. Temperature profiles for $\psi = 1, 2, 3, 4$

5 Conclusion

In this paper, we have analysed the effect of nanofluids on fully developed mixed convection flow in a vertical channel with heat generation or absorption in which Brownian motion and thermophoresis effects are taken into account. The dimensionless governing equations are solved analytically for velocity, temperature and nanoparticles volume fraction. The main results drawn

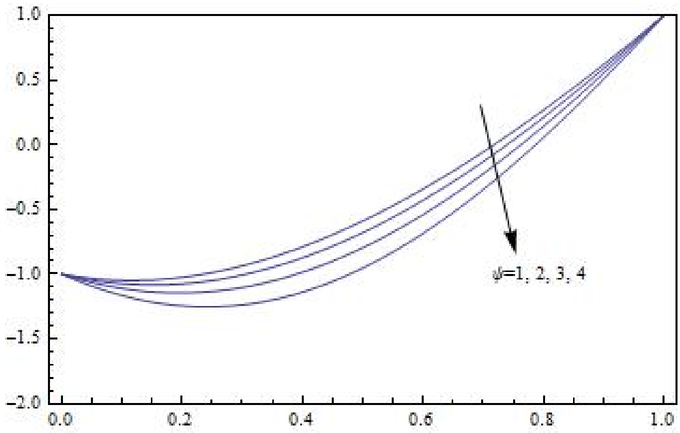

Figure 9. Concentration profiles for $\psi = 1, 2, 3, 4$

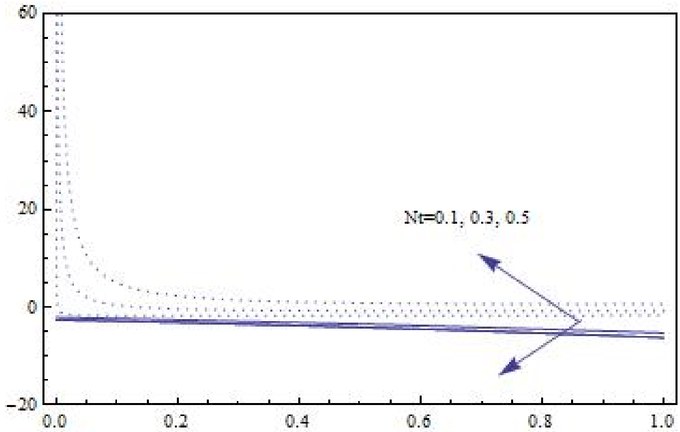

Figure 10. Nusselt number (Full line) and Sherwood number (Dotted line) for $Nt = 0.1, 0.3, 0.5$

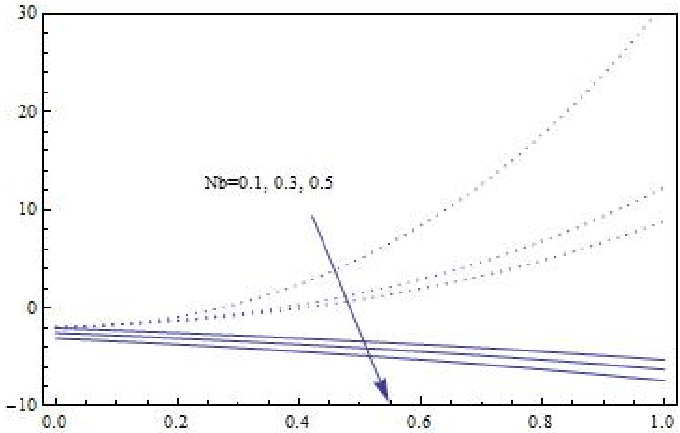

Figure 11. Nusselt number (Full line) and Sherwood number (Dotted line) for $Nb = 0.1, 0.3, 0.5$

are as follows,

- The buoyancy ratio parameter, Brownian motion parameter and thermophoresis parameters enhances the velocity near cold wall and suppresses near hot wall.
- The temperature gradient in the nanofluid creates a thermophoretic force. This pushes the particles from the hot wall towards the cold wall in the direction of heat transfer which leads to a rise in the cold wall's nanoparticle concentration and decrease in the nanoparticles

concentration near the hot wall.

- Nusselt number decreases with increase in Nb and Nt. But Sherwood number decreases with increase in Nb and increases with increase in Nt.

References

[1] Tao L. N., 1960, “On combined free and forced convection in channels”, ASME, Trans. J. Heat Transfer, 82, 233-238.

[2] Aung. W. and Workee G., 1986, “Theory of fully developed combined convection including flow reversal”, ASME Trans. J. Heat Transfer, 108, PP.485-488.

[3] Barletta A., 1998, “Laminar mixed convection with viscous dissipation in a vertical channel”, Int. Journal of Heat and Mass transfer, 41, PP.3501-3513.

[4] Barletta A., Magyari E. and Keller B., 2005, “Dual mixed convection flows in a vertical channel”, Int. Journal of Heat and mass transfer, 48, 4835-4845.

[5] Zanchini E. (1998). Effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of the third kind. International Journal of Heat and Mass Transfer 3949-3959.

[6] A. J. Chamkha, 2002, “Hydrodynamic combined convection flow in a vertical lid-driven cavity enclosure with internal heat generation or absorption”, Numerical Heat Transfer, 529-546.

[7] Basant K. Jha and Abiodun O. Ajibade, “Free convective flow of heat generating or Absorbing fluid between vertical porous plates with periodic heat input”, International Communications in Heat and Mass Transfer, 36, 2009; 624-631.

[8] Patl Mallikarjun B., Umavathi J. C. and Narasimhamurthy S. K., 2013, ”On laminar magnetoconvection flow in a vertical channel in the presence of heat generation or heat absorption”, Int. Journal of Heat Transfer, ASME, 135, 042503-1 to 042503-8.

[9] U. S. Choi, 1995, Enhancing thermal conductivity of fluids with nanoparticles”, ASME, 66, PP.99-105.

[10] J. A. Eastman, U. S. Choi, S. Li, W. Yu and L. J. Thompson, 2001, “Anomalously increased thermal conductivities of ethylene glycol based nanofluids containing Cu nanoparticles”, Appl. Phys. Lett., 78(6), PP.718-720.

[11] S. K. Das, N. Putra, P. Thiesen and W. Roetzel, 2003, “Temperature Dependance of Thermal Conductivity Enhancement for nanofluids”, Journal of Heat Transfer, 125(4),PP.567-574.

[12] Y. Xuan and Q. Li, 2003, “Investigation on convective heat transfer and flow features of nanofluids”, Journal of Heat Transfer, 125, PP.151-155.

[13] J. Buongiorno, 2006, “Convective Transport in nanofluids”, Journal of Heat Transfer, 128, PP.240-250.

[14] Oztop H. F. and Abu-Nada E., 2008, “Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids”, Int. J. Heat Fluid Flow, 29, 1326-1336.

[15] A. V. Kuznetsov and D. A. Nield, 2010, “Natural Convective Boundary layer flow of nanofluid past a vertical plate”, International Journal of Thermal Sciences, 49, PP.243-247.

[16] Kang H. U., Kim S. H. and Oh J. M., 2006, “Estimation of Thermal Conductivity of nanofluid using experimental effective particle volume”, Exp. Heat Transfer, 19, 181-191.

[17] T. Grosan and I. Pop, 2012, “Fully Developed mixed convection in a vertical channel filled by a nanofluid”, ASME Journal of Heat Transfer, 134, 082501-5.

[18] J. C. Umavathi, I-Chung Liu and M. A. Sheremet, 2015, “Convective heat transfer in a vertical rectangular duct filled with a nanofluid”, Heat Transfer Asian Research, Wiley Periodicals, Inc.

[19] K. Sushma, S. Sreenadh and P. Dhanalakshmi, 2017, “Mixed convection flow of a Jeffrey nanofluid in a vertical channel”, Middle-East Journal of Scientific Research, 25(5), 950-959.

[20] J. C. Umavathi, K. Hemavathi and G. Janardhana Reddy, 2017, “Heat Transfer of saturated nanofluid sandwiched between permeable fluids”, Int. J of Engg. Science and Mathematics,6, 28-44.

[21] B. Lavanya, M. Nagasasikala, 2018, "Effects of Dissipation and Radiation on Heat Transfer Flow of a Convective Rotating Cuo-Water Nano-fluid in a Vertical Channel", Journal of advanced Research in Fluid mechanics and Thermal sciences, 50(2), 108-117.

[22] Mohammed Saad Kamel, Ferenc Lezsovits, 2018, "Simulation of nanofluids laminar flow in a vertical channel" Pollack Periodica, AK Journals, 13(2), 147-158.

[23] Amir Mostafazadeh, Davood Toghraie, Ramin Mashayekhi, Omid Ali Akbari, 2019, "Effect of radiation on laminar natural convection of nanofluid in a vertical channel with single- and two-phase approaches", Journal of Thermal Analysis and Calorimetry, 138, 779–794.

[24] J. C. Umavathi, M. A. Sheremet, 2020, "Heat transfer of viscous fuid in a vertical channel sandwiched between nanofuid porous zones", Journal of Thermal Analysis and Calorimetry, Springer.

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