Palestine Journal of Mathematics

Vol. 10(Special Issue I)(2021) , 93–102

AN ANALYSIS ON THE PERFORMANCE OF AN INCLINED PLANE POROUS SLIDER BEARING LUBRICATED WITH COUPLE STRESS FLUID AND HAVING SLIP VELOCITY

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Communicated by Serkan Araci

MSC 2010 Classifications: 74A55,76D08

Keywords and phrases: Couplestress fluid, Inclined slider bearing, Porous, Slip Velocity.

Abstract The article aims to study the performance of a couplestress fluid based porous inclined plane slider bearing with slip velocity. The expression for pressure, centre of pressure, load carrying capacity, Force of friction, coefficient of friction and rise in temperature is derived. The dependence of pressure, load carrying capacity, Force of friction and coefficient of friction on slip parameter, couplestress parameter, permeability parameter have been derived and studied graphically.

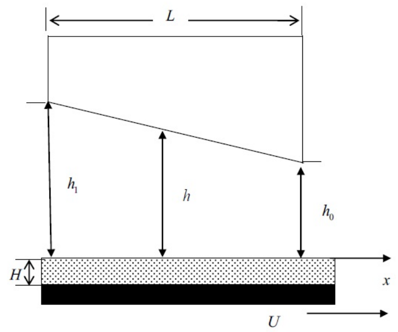
1 Introduction

No slip boundary condition was accepted as the exact boundary condition from nineteenth century. This condition means that the fluid near to the surface has zero velocity when the surface is at rest and when the surface is moving the fluid velocity equals to the surface velocity. Reynolds equation is based on one of this basic principle While deriving the Reynolds equation it was assumed that there is no boundary slip at liquid and solid interface. But it is possible to observe the slip of the fluid over a solid surface on micro scale. Hence the classical Reynolds equation became irrelevant. In reducing fluid friction, proper lubrication is essential and hence the literature of Henck [1], Spikes [2] has achieved great attention. But classical Reynolds equation is a powerful tool in designing and analysing bearing performance. Hence researchers have extended Reynolds equation by considering boundary slip. Many researches related to slip velocity has been carried out viz., the slider bearing by Wu et al.,[3], Salant and Fortier [4], Ahmad and Singh[5]. Patel and Gupta [6] have investigated the problem on an inclined slider bearing with slip condition at porous boundary and found that load capacity can be increased by minimizing slip condition. The effect of velocity slip with porous inclined slider bearing lubricated with a ferrofluid was studied theoretically by Shah and Bhat [7]. Many investigators has used couple stress fluid theory to analyse the performance characteristics of various bearings like., slider bearing by Bujurke et. al.,[8], Naduvinamani et al., [9], long partial journal bearing by Lin[10], Squeeze films by Bujurke and Jayaraman [11]. In all these studies use of couple stress fluid increases load, force of friction, decreases coefficient of friction and delays time of approach. Porous bearings do not need an external supply of lubricant during their operation; therefore, their structures are simple and cost is reduced. Morgan and Cameron [12] was the first to give an analytical survey about the study of porous bearings with the support of hydrodynamic conditions. Since then many investigations has happened to examine the performance of couplestress fluid on porous bearings by Bujurke. et.al.,[13, 14, 15]. Many authors namely Beavers and Joseph [16], Kalavathi. et.al., [16], Patel and Dehri[18] has studied the effect of slip velocity on a porous/fluid interface. Oladeinde and Akpobi [19] has studied the effect of slip velocity on slider bearing lubricated with couple stress fluid.

To the best of authorâĂŹs knowledge so far no research has been made to study effect of slip velocity on porous plane slider bearing lubricated with couple stress fluid. So in the present paper an attempt is made to study the same.

2 Mathematical formulation

The slider bearing represented in figure 1 has film thickness h and inclined part length L. The equations governing the flow with basic assumptions are



The physical configuration of inclined slider bearing.

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{2.1}$$

$$\mu\frac{\partial^2 u}{\partial z^2}-\eta\frac{\partial^4 u}{\partial z^4}=\frac{\partial p}{\partial x} \tag{2.2}$$

$$\frac{\partial p}{\partial z}=0 \tag{2.3}$$

By Darcy's law the flow through porous matrix is

$$q^*=-\frac{k}{\mu(1-\beta)}\nabla p^* \tag{2.4}$$

As $\beta^+\to 0$; equation (2.5) reduces to the usual Darcy law and the pressure, in the porous region, due to continuity, satisfies the Laplace equation:

$$\frac{\partial^2 p^*}{\partial x^2}+\frac{\partial^2 p^*}{\partial z^2}=0 \tag{2.5}$$

At the upper surface$y=h$

$$u=U \tag{2.6a}$$

$$\frac{\partial^2 u}{\partial z^2}=0 \tag{2.6b}$$

$$v=0 \tag{2.6c}$$

At the lower surface $y=0$

$$u=\left(\frac{1}{s}\frac{\partial u}{\partial z}\right)_{z=0} \tag{2.7a}$$

$$\frac{\partial^2 u}{\partial z^2}=0 \tag{2.7b}$$

$$w=w^* \tag{2.7c}$$

Based on the boundary conditions (2.6a,2.6b) and (2.7a,2.7b) we get from equation (2.2) $'u'$ as

$$u=\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[(z-h)\left\{z+\frac{h}{3}\xi_1-2l\xi_0\tanh\frac{h}{2l}+2l^2\left(1-\frac{\cosh(\frac{2z-h}{2l})}{\cosh\frac{h}{2l}}\right)\right\}\right] + \left(\frac{1}{1+sh}+\frac{zs}{1+sh}\right)U \tag{2.8}$$

where

$$\xi_0 = \frac{1}{1+sh}, \xi_1 = \frac{3}{1+sh}, l = (\eta/\mu)^{1/2}$$

Integrating Equation (5) w.r.to z and applying the boundary conditions $\frac{\partial p^*}{\partial z^*} = 0$ at $z = -H$we obtain

$$\left.\frac{\partial p^*}{\partial z}\right|_{z=0} = -\int_{z=0}^{-H} \frac{\partial^2 p^*}{\partial x^2} dz \tag{2.9}$$

The porous layer thickness H is assumed to be very small and applying the pressure$p = p^*$ continuity condition at the interface $y = 0$ of porous matrix and fluid film, equation (2.9) reduces to:

$$\left.\frac{\partial p^*}{\partial z}\right|_{z=0} = -H\frac{\partial^2 p}{\partial x^2} \tag{2.10}$$

Integrating equation (2.1) and applying conditions (2.6c), (2.7c), (2.8), (2.9) we get modified Reynolds equation

$$\frac{\partial}{\partial x}\left[\left\{f_1(h,s,l) + \frac{12kH}{(1-\beta)}\right\}\frac{\partial p}{\partial x}\right] = 6\mu U \frac{d}{dx}\left(\frac{2h+h^2 s}{(1+sh)}\right) \tag{2.11}$$

where

$$f_1(h,s,l) = h^3\left(1+\xi_1\right) - 6lh^2\xi_0 \tanh\frac{h}{2l} - 12l^2\left(h - 2l\tanh\frac{h}{2l}\right)$$

Non-dimensionalizing, we get

$$x^* = \frac{x}{L}, \psi = \frac{kH}{h_0^3}, h^* = \frac{h}{h_0}, s^* = sh_0, P = \frac{{h_0}^2 p}{\mu U L}, l^* = \frac{2l}{h_0}$$

Hence equation (2.11) becomes

$$\frac{\partial}{\partial x^*}\left[\left\{f_1(h^*,s^*,l^*) - \frac{12\psi}{(1-\beta)}\right\}\frac{\partial P}{\partial x^*}\right] = \frac{d}{dx^*}G(h,^*s^*) \tag{2.12}$$

where

$$\xi_0{}^* = \frac{1}{1+s^*h^*}; \xi_1{}^* = \frac{3}{1+s^*h^*}, G(h,^*s^*) = 6\left(\frac{2h^* + h^{*2}s^*}{(1+s^*h^*)}\right)$$

$$f_1(h^*,s^*,l^*) = h^{*3}\left(1+\xi_1{}^*\right) - 3l^*h^{*2}\xi_0^* \tanh\frac{h^*}{l^*} - 3l^{*2}\left(h^* - l^*\tanh\frac{h^*}{l^*}\right)$$

The non-dimensional film thickness is given by
$h^* = \alpha + (1-\alpha)x^*$ where $\alpha = \frac{h_1}{h_0}$is taper ratio. The pressure field boundary conditions are

$$p^* = 0 \quad \text{at} \quad x^* = 0, 1(\text{Ambient Pressure}) \tag{2.13}$$

Solving equation (2.12) subject to conditions of (2.13) pressure becomes

$$p^* = \int_{x^*=0}^{x^*} \frac{G(h^*,s^*) - Q}{f_1(h^*,s^*,l^*) - \frac{12\psi}{(1-\beta)}} dx^* \tag{2.14}$$

where

$$Q = \frac{\int\limits_{x^*=0}^{1} \frac{G(h,^* s^*)}{f_1(h^*,s^*,l^*) - \frac{12\psi}{(1-\beta)}} dx^*}{\int\limits_{x^*=0}^{1} \frac{1}{f_1(h^*,s^*,l^*) - \frac{12\psi}{(1-\beta)}} dx^*}$$

By letting the minimum film height be constant and letting squeezing velocity be zero, the dimensionless load carrying capacity$w^*$is

$$W^* = \int\limits_{x^*=0}^{1} p^* dx^* = \int\limits_{0}^{1} \int\limits_{x^*=0}^{x^*} \frac{G(h^*, s^*) - Q}{f_1(h^*, s^*, l^*) - \frac{12\psi}{(1-\beta)}} dx^* dx^* \tag{2.15}$$

The non dimensional frictional force is

$$F^* = \int\limits_{0}^{1} \left\{ \frac{s^*}{(1 + s^* h^*)} - \left(\frac{\left(h^* (1 + s^* h^*) - l^* \tanh \frac{h^*}{l^*}\right) \{G(h^*, s^*) - Q\}}{2(1 + s^* h^*) \left\{ f_1(h^*, s^*, l^*) + \frac{12\psi}{(1-\beta)} \right\}} \right) \right\} dx^* \tag{2.16}$$

The coefficient of friction is

$$C = \frac{F^*}{W^*} \tag{2.17}$$

The location of the centre of pressure where the resultant force acts is

$$X = \frac{1}{W^*} \int\limits_{0}^{1} p^* x^* dx^* \tag{2.18}$$

The temperature rise in non-dimensional form is

$$\Delta T = \frac{g J \rho c h_0^2}{2\mu U L} \Delta t = \frac{F^*}{C_1} \tag{2.19}$$

3 Results and discussion

The performance of the porous plane slider bearing is analyzed with the aid of various non-dimensional parameters viz; slip parameter s^*, permeability parameter ψ and the couplestress parameter l^*.

Non-dimensional Pressure(p^*)

The graphs illustrated in Fig:2 and Fig :3 represents variation in Non-dimensional pressure p^* against $x^*$and the comparison is shown between slips$s^* = 0.5$ and non slip $s^* = \infty$ cases and is viewed that more pressure generated in non-slip case. In Fig: 2, by fixing $\psi = 0.01$, $\beta = 0.3$, $a^* = 1.2$and by varying l^* the pressure $p^*$is found to be increasing. In Fig:3 by fixing $l^* = 0.3$, $\beta = 0.2$, $a^* = 1.2$and increasing ψ values through $(0, 0.01, 0.1)$,p^* is found to be decreasing.

Non -dimensional load (W^*)

Fig. 4 and Fig. 5 illustrates the variation of W^*against $a^*$by considering slip and non-slip conditions. In Fig :4 it is observed that as l^* increases through the values (0,0.3,0.6) W^*increases and in Fig.5 ,W^*decreases with increasing ψ from (0.0 to 0.1). It is also visible that non-slip condition is having better load carrying capacity.

Non-dimensional friction (F^*)

Fig 6 and Fig:7 illustrates the variation of F^*against a^*. It is seen F^* increases with increasing values of l^* and decreases with increasing values of ψ. It is also clearly visible that F^* is more for non-slip conditions.

Coefficient of Friction(C)

Fig.8 and Fig.9 illustrates the variation of coefficient of friction against a^*.It is seen that C decreases with increasing values of l^* and increases with increasing values of ψ. It is also visible that coefficient of friction is more in non-slip conditions.

4 Conclusion

The capacity for bearing the load (W^*), Force of friction(F^*), coefficient of friction(C), centre of pressure(X)rise in temperature (ΔT), depending on various non-dimensional parameters like permeability parameter(ψ), couplestress parameter l^*, slip parameter(s^*) is analysed . Based on it the following conclusions are made:

- (W^*, F^*, C)decreases with increasing value of (s^*). Therefore the slip should be minimised for the overall augment performance of the bearing.

- W^* andF^* increase with increasing values of l^*, but decreases C.

- W^* and F^*decreases with increasing values of ψ. Therefore the adverse effect of porosity can be mitigated by suitably choosing additives of proper size.

5 Nomenclature

L Length ofthe inclined part of the bearing
H porous layer thickness
k permeability of the porous medium
l^* couplestress parameter
p pressure in the porous surface
$s = \alpha/\sqrt{k}$ slip parameter
W^* Non-dimensional load
F^* Non-friction coefficient of friction
u, v velocity components of the lubricant
α dimensionless slip parameter
$\beta = \eta/\mu$ Ratio of microstructure to porous size
μ_0 viscosity
μ Newtonian viscosity coefficient
ψ Permeability parameter

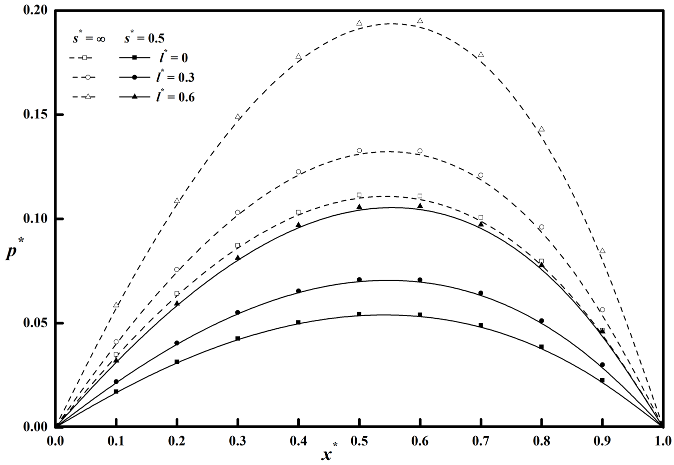


Figure 2. Variation of nondimensional pressure p^* with x^* for different values of l^* and s^* with $\psi = 0.001$, $\beta = 0.3$, $a^* = 1.2$.

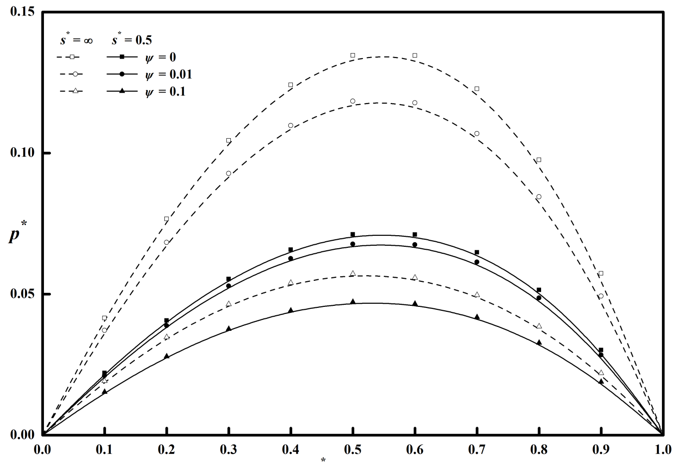


Figure 3. Variation of nondimensional pressure p^* with x^* for different values of ψ and s^* with $l^* = 0.3$, $\beta = 0.2$, $a^* = 1.2$.

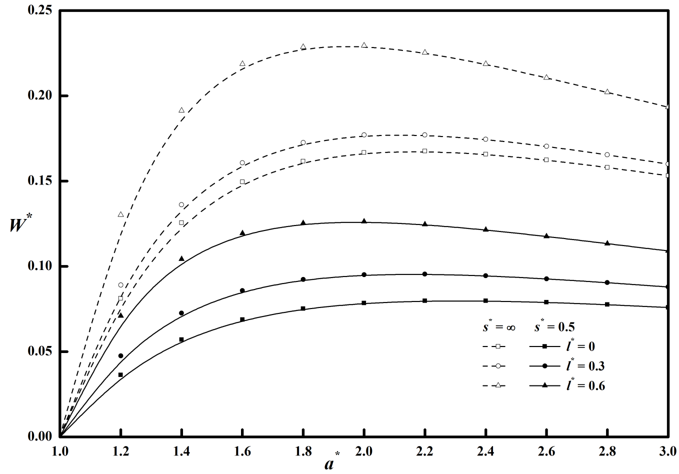


Figure 4. Variation of nondimensional load W^* with a^* for different values of l^* and s^* with ψ = 0.001, β = 0.2.

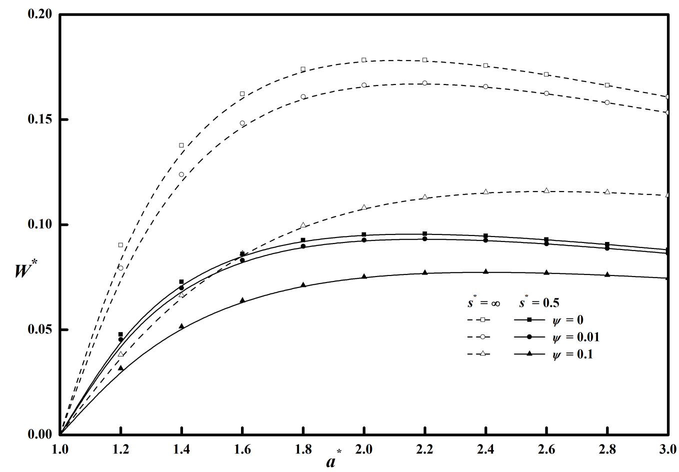


Figure 5. Variation of nondimensional load W^* with a^* for different values of ψ and s^* with l^* = 0.3, β = 0.2.

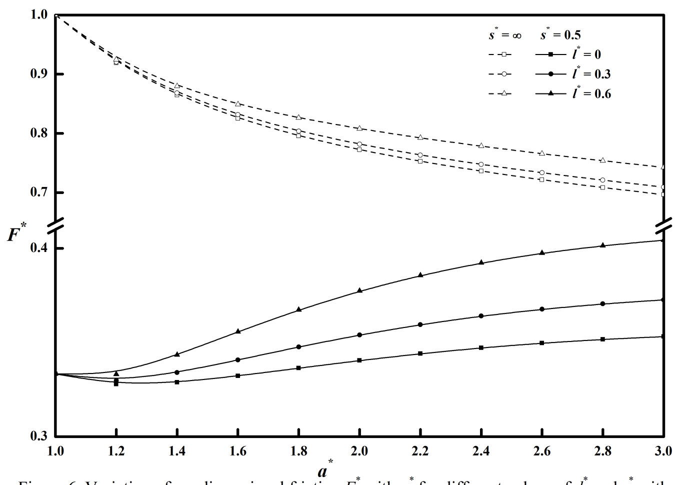


Figure 6. Variation of nondimensional friction F^* with a^* for different values of l^* and s^* with $\psi = 0.001$, $\beta = 0.2$.

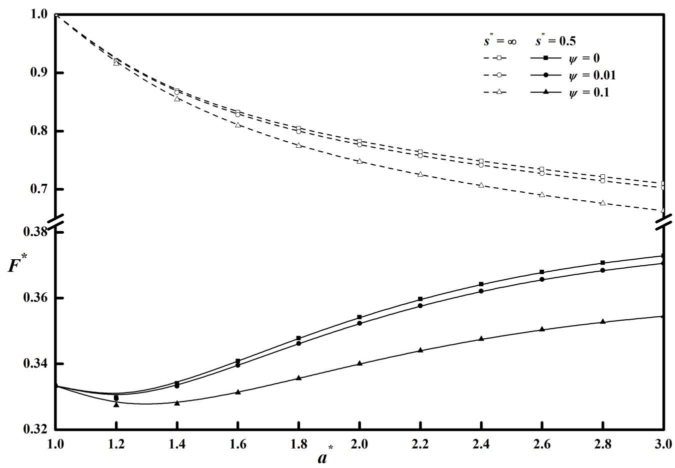


Figure 7. Variation of nondimensional friction F^* with a^* for different values of ψ and s^* with $l^* = 0.3$, $\beta = 0.2$.

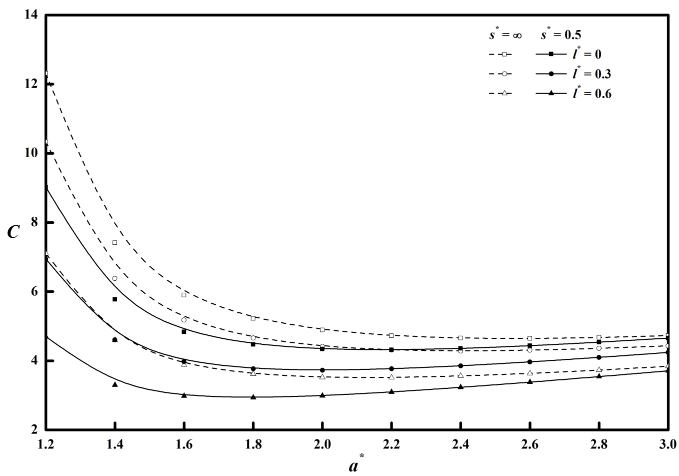


Figure 8. Variation of coefficient of friction C with a^* for different values of l^* and s^* with $\psi = 0.001$, $\beta = 0.2$.

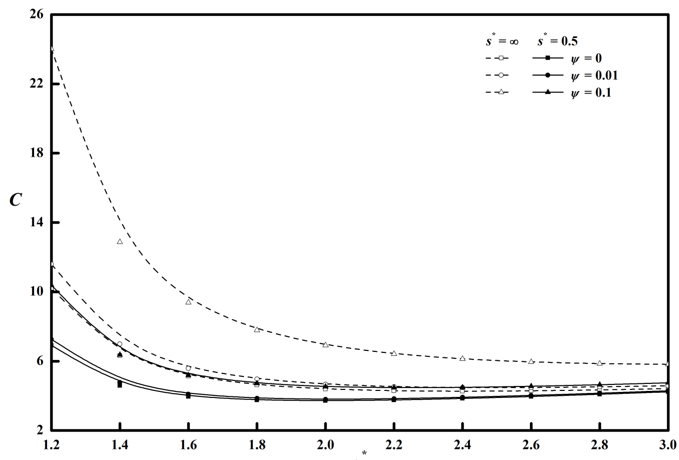


Figure 9. Variation of coefficient of friction C with a^* for different values of ψ and s^* with $l^* = 0.3$, $\beta = 0.2$.

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Received: Dec 23, 2020.
Accepted: Mar 15, 2021.