Palestine Journal of Mathematics

Vol. 12(Special Issue II)(2023) , 20–25

C-AVERAGE ECCENTRIC GRAPHS

T. Sathiyanandham and S. Arockiaraj

Communicated by S. Monikandan

MSC 2010 Classifications: 05C12.

Keywords and phrases: C-average eccentric vertex, C-average eccentric graph.

Abstract The C-average eccentric graph $AE_C(G)$ of a graph G has the vertex set as in G and any two vertices u and v are adjacent in $AE_C(G)$ if either they are at a distance $\left\lceil \frac{e(u)+e(v)}{2} \right\rceil$ while G is connected or they belong to different components while G is disconnected. A graph G is called a C-average eccentric graph if $AE_C(H) \cong G$ for some graph H. The main aim of this paper is to find a necessary and sufficient condition for a graph to be a C-average eccentric graph.

1 Introduction

Throughout this paper, a graph means a non trivial simple graph. For other graph theoretic notation and terminology, we follow [3, 5]. The distance $d(u, v)$ between a pair of vertices u and v in a graph G is the length of a shortest path joining them. The eccentricity $e(u)$ of a vertex u is the distance to a vertex farthest from u. The radius $r(G)$ of G is the minimum eccentricity among the eccentricities of the vertices of G and the diameter $d(G)$ of G is the maximum eccentricity among the eccentricities of the vertices of G. The concept of average eccentricity (also called as eccentric mean) was introduced by Buckley and Harary [3]. A graph G for which $r(G) = d(G)$ is called a self-centered graph of radius $r(G)$. A vertex v is called an eccentric vertex of a vertex u if $d(u, v) = e(u)$. A vertex v of G is called an eccentric vertex of G if it is the eccentric vertex of some vertex of G. The concept of antipodal graph was initially introduced by Singleton [6] and was further expanded by Aravamuthan and Rajendran [1, 2]. The antipodal graph of a graph G, denoted by $A(G)$, is the graph on the same vertices as of G, two vertices being adjacent if the distance between them is equal to the diameter of G. A graph is said to be antipodal if it is the antipodal $A(H)$ of some graph H. The concept of radial graph was introduced by Kathiresan and Marimuthu [4]. The radial graph $R(G)$ based on G has the vertex set as in G and two vertices are adjacent if the distance between them is equal to the radius of G while G is connected. If G is disconnected, then two vertices are adjacent in $R(G)$ if they belong to different components of G. A graph G is called a radial graph if $R(H) = G$ for some graph H. Motivated by these works, we introduce a new graph called C-average eccentric graph. Two vertices u and v of a graph are said to be C-average eccentric to each other if $d(u, v) = \left\lceil \frac{e(u)+e(v)}{2} \right\rceil$. The C-average eccentric graph of a graph G, denoted by $AE_C(G)$, has the vertex set as in G and any two vertices u and v are adjacent in $AE_C(G)$ if either they are at a distance $d(u, v) = \left\lceil \frac{e(u)+e(v)}{2} \right\rceil$ while G is connected or they belong to different components while G is disconnected. A graph G is called a C-average eccentric graph if $AE_C(H) \cong G$ for some graph H. $K_1 \cup (K_4 - e)$ is neither an antipodal graph nor a radial graph but it is a C-average eccentric graph since $AE_C(K_{1,4} \cup \{e\})$ is isomorphic to $K_1 \cup (K_4 - e)$. So the notion of C-average eccentric graph, radial graph and antipodal graph are different.

In this paper, we obtain a necessary and sufficient condition for a graph to be a C-average eccentric graph.

Theorem 1.1. [5] *If G is a simple graph with diameter at least 3, then* $\overline{G}$ *has diameter at most 3.*

Theorem 1.2. [5] *If G is a simple graph with diameter at least 4, then* $\overline{G}$ *has diameter at most 2.*

Theorem 1.3. [5] *If G is a simple graph with radius at least 3, then $\overline{G}$ has radius at most 2.*

Theorem 1.4. [3] *If G is a self centered graph with radius at least 3, then $\overline{G}$ is a self centered graph of radius 2.*

Theorem 1.5. [4] *Let C_n be any cycle on $n \geq 4$ vertices. Then $R(C_n) = \frac{n}{2}K_2$ if n is even and $R(C_n) \cong C_n$ if n is odd.*

Theorem 1.6. [2] *$A(G) = G$ if and only if G is complete.*

Let $F_{11}, F_{12}, F_{22}, F_{23}, F_{24}, F_3$ denote the set of all connected graphs G for which $r(G) = d(G) = 1$, $r(G) = 1$ and $d(G) = 2$, $r(G) = d(G) = 2$, $r(G) = 2$ and $d(G) = 3$, $r(G) = 2$ and $d(G) = 4$, $r(G) \geq 3$ respectively and F_4 denote the set of all disconnected graphs.

2 C-average eccentric graph of some classes of graphs

Remark 2.1. If G is either a self centered graph or a disconnected graph, then $AE_C(G) = R(G) = A(G)$.

Proposition 2.2. *Let P_n be any path on $n \geq 1$ vertices. Then*

$$AE_C(P_n) = \begin{cases} P_n, & \text{if } n = 1, 2 \\ P_2 \cup \overline{K}_{n-2}, & \text{if } n \geq 3. \end{cases}$$

Proof. When $n = 1, 2$, $AE_C(P_n) = P_n$. Let G be a path $v_1v_2v_3...v_n$ with $n \geq 3$ vertices. Then $e(v_i) = n - i$ for $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $e(v_i) = i - 1$ for $\lceil \frac{n}{2} \rceil + 1 \leq i \leq n$ and $d(v_i, v_j) = j - i$ for $1 \leq i, j \leq n$. This implies that the C-average eccentric pairs in G is (v_1, v_n) and the remaining pairs are not C-average eccentric pairs in G. Hence the C-average eccentric pair of vertices v_1 and v_n form the graph $AE_C(G)$. In $AE_C(G)$, v_1v_n is a path on 2 vertices and the remaining vertices form $\overline{K}_{n-2}$. □

Proposition 2.3. *Let C_n be any cycle on $n \geq 3$ vertices. Then*

$$AE_C(C_n) \cong \begin{cases} \frac{n}{2}K_2, & \text{if } n \text{ is even} \\ C_n, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let $v_1, v_2, ..., v_n$ be the n vertices of the cycle C_n. If $n = 3$, then $r(C_3) = 1$ and $e(v_i) = 1$ for $i = 1, 2, 3$. Hence $AE_C(C_3) \cong C_3$. If $n \geq 4$, then the result follows from Remark 2.1 and Theorem 1.5. □

Remark 2.4. Let G_i be a connected graph with r_i vertices for $i = 1, 2, ..., n$. If G is the union of $G_1, G_2, ..., G_n$, then $AE_C(G) = K_{r_1, r_2, ..., r_n}$.

Proposition 2.5. $AE_C(K_{r_1, r_2, ..., r_n}) = K_{r_1} \cup K_{r_2} \cup ... \cup K_{r_n}$ *where* $r_1, r_2, ..., r_n \geq 2$.

Proof. Since radius and diameter are 2, any two vertices are non adjacent in $AE_C(K_{r_1, r_2, ..., r_n})$ whenever the vertices are in different partitions and adjacent whenever they are in same partition. □

Proposition 2.6. $AE_C(K_r + K_{r_1, r_2, ..., r_n}) = K_r \cup K_{r_1} \cup K_{r_2} \cup ... \cup K_{r_n}$ *where* $r_1, r_2, ..., r_n \geq 2$ *and* $r \geq 1$.

Proof. Let $G = K_r + K_{r_1, r_2, ..., r_n}$ and $u_1, u_2, ..., u_r$ be the full degree vertices in G. By definition, u_i and u_j are adjacent in $AE_C(G)$ for $1 \leq i, j \leq r$. For each non full degree vertex v in G, each u_i, $1 \leq i \leq r$ is non adjacent to v in $AE_C(G)$. For any two non full degree vertices v and w in G, $\left\lceil \frac{e(v)+e(w)}{2} \right\rceil = 2 \neq d(v, w)$ while $vw \in E(G)$ and is equal to $d(v, w)$ while $vw \neq E(G)$. Thus $AE_C(G) = K_r \cup K_{r_1} \cup K_{r_2} \cup ... \cup K_{r_n}$ □

Lemma 2.7. *If G is a connected graph having no full degree vertex, then $AE_C(G)$ has no full degree vertex.*

Proof. Since G has no full degree vertex, $e(u) \geq 2$ for all $u \in V(G)$. $uv \notin E(AE_C(G))$ whenever $uv \in E(G)$. Hence $AE_C(G)$ has no full degree vertex. □

Theorem 2.8. *Let G be a graph on n vertices. Then a vertex is a full degree vertex in $AE_CG)$ if and only if either it is an isolated vertex in G or G is complete.*

Proof. If v is an isolated vertex in G, then by definition, v is the full degree vertex in $AE_C(G)$. If G is complete, then by Remark 2.1 and Theorem 1.6, $AE_C(G) = G$.

Suppose v is a full degree vertex in $AE_C(G)$. If G is a disconnected graph having m components say $H_1, H_2, ..., H_m$ with $|H_i| = n_i > 1$ for $i = 1, 2, ..., m$, then by Remark 2.4, $AE_C(G) = K_{n_1,n_2,...,n_m}$, a contracdiction. So G is disconnected with at least one isolated vertex and v is one among them. Suppose G is a connected graph with no full degree vertex. Then by Lemma 2.7, $AE_C(G)$ has no full degree vertex, a contradiction. Hence G should have a full degree vertex. Let w be a full degree vertex in G. If v is not a full degree vertex in G, then $e(w) = 1$ and $e(v) = 2$ in G. Hence $1 = d(v, w) \neq \left\lceil \frac{e(v)+e(w)}{2} \right\rceil$, a contradiction to the fact that $vw \in AE_C(G)$. If v is a full degree vertex in G and w is a non full degree vertex in G, then $e(v) = 1$ and $e(w) = 2$ in G and hence $vw \notin E(AE_C(G))$, a contradiction. Hence all the vertices in G are the full degree vertices in G. □

Theorem 2.9. *Let G be a graph on $n \geq 3$ vertices and m is any positive integer less than n. Then $AE_C(G)$ has exactly m full degree vertices if and only if G has exactly m isolated vertices.*

Proof. Suppose $AE_C(G)$ has exactly m full degree vertices. Let $v_1, v_2, ..., v_m$ be the full degree vertices in $AE_C(G)$. Then by Theorem 2.8, either G is complete or $v_1, v_2, ..., v_m$ are the isolated vertices in G. If G is complete, then by Theorem 1.6, $AE_C(G)$ is complete, a contradiction. If $w \neq v_i$ is an isolated vertex in G for $i = 1, 2, ..., m$, then by Theorem 2.8, w is a full degree vertex in $AE_C(G)$. So $AE_C(G)$ has $m + 1$ full degree vertices, a contradiction. Hence G has exactly m isolated vertices.

Suppose G has exactly m isolated vertices, say $v_1, v_2, ..., v_m$. Then by Theorem 2.8, $v_1, v_2, ...,$ v_m are the full degree vertices in $AE_C(G)$. If $w \neq v_i$ is a full degree vertex in $AE_C(G)$ for $i = 1, 2, ..., m$, then by Theorem 2.8, w is an isolated vertex in G or G is complete. So G has more than m isolated vertices, a contradiction. Hence $AE_C(G)$ has exactly m full degree vertices. □

Theorem 2.10. *Let G be a graph on n vertices. If G has $r(< n)$ number of full degree vertices $v_1, v_2, ..., v_r$, then $AE_C(G) = K_r \cup (\overline{G - \{v_1, v_2, ..., v_r\}})$.*

Proof. Let $v_1, v_2, ..., v_r$ be the full degree vertices and $v_{r+1}, v_{r+2}..., v_n$ be the remaining vertices of G. Let $wv \in E(AE_C(G))$. If $w, v \in \{v_1, v_2, ..., v_r\}$, then $wv = v_iv_j \in E(G)$ for some i and j. If none of w and v is in $\{v_1, v_2, ..., v_r\}$, then $e(w) = e(v) = 2$. This implies that $wv \notin E(G)$. Therefore $wv \notin E(G - \{v_1, v_2, ..., v_r\})$ and hence $wv \in E(\overline{G - \{v_1, v_2, ..., v_r\}})$

Suppose $wv \in E(K_r \cup (\overline{G - \{v_1, v_2, ..., v_r\}}))$=$E(\overline{G - \{v_1, v_2, ..., v_r\}}) \cup E(K_r)$. If $wv \in E(K_r)$, then $wv \in E(AE_C(G))$. If $wv \in E(\overline{G - \{v_1, v_2, ..., v_r\}})$, $wv \notin E(G - \{v_1, v_2, ..., v_r\})$. This implies that $wv \notin E(G)$. Then $d(w, v) = 2 = \left\lceil \frac{e(w)+e(v)}{2} \right\rceil$ and hence $wv \in E(AE_C(G))$. Thus $AE_C(G) = K_r \cup (\overline{G - \{v_1, v_2, ..., v_r\}})$. □

Corollary 2.11. *If $G \in F_{12}$ has at least two full degree vertices, then $\overline{G} \subset AE_C(G)$.*

Theorem 2.12. *Let G be a graph on n vertices. Then for any positive integer $l \leq n$, $AE_C(G) = K_l + \overline{K}_{n-l}$ if and only if any one of the following conditions hold*
(1) G is totally disconnected
(2) G is complete
(3) G is $lK_1 \cup H$ where H is a connected component with $|H| = n - l \geq 2$.

Proof. If (1) holds, then by definition, $AE_C(G) = K_n$. If (2) holds, then by Remark 2.1 and Theorem 1.6, $AE_C(G) = K_n$. If (3) holds, then by Remark 2.4, $AE_C(G) = K_l + \overline{K}_{n-l}$.

Suppose $AE_C(G) = K_l + \overline{K}_{n-l}$. If $l = n$, then by Theorem 2.8, either G or $\overline{G}$ is K_n. Assume that $l < n$. By Theorem 2.9, G has exactly l isolated vertices. Then G is disconnected with $r < n$ components out of which exactly l components are K_1. Let $H_i = \{v_i\}$ for $1 \leq i \leq l$, $H_{l+1}, H_{l+2}, ..., H_r$ be the components of G. If $r > l + 1$, then the vertices of H_{l+1} are adjacent to the vertices of H_{l+2} in $AE_C(G)$, a contradiction. Thus $r = l + 1$. □

3 C-average eccentric graphs

Proposition 3.1. *If $r(G) \geq 2$, then $AE_C(G) \subseteq \overline{G}$.*

Proof. By definition, $V(AE_C(G)) = V(\overline{G}) = V(G)$. If $e = uv \in E(AE_C(G))$ but does not belong to $E(\overline{G})$, then $uv \in E(G)$ and by definition, $e(u) = 1 = e(v)$, a contradiction. So $E(AE_C(G)) \subseteq E(\overline{G})$. Hence $AE_C(G) \subseteq \overline{G}$. □

Theorem 3.2. *Let G be a graph of order n. Then $AE_C(G) = G$ if and only if $G \in F_{11}$.*

Proof. Suppose $AE_C(G) = G$. If G is disconnected with $r \geq 2$ components, then by Remark 2.4, $AE_C(G)$ is a complete r partite graph, a contradiction. So G is connected. If $r(G) \geq 2$, then by Proposition 3.1, $AE_C(G) \subseteq \overline{G}$, a contradiction. If $G \in F_{12}$, then by Theorem 2.10, $AE_C(G) \neq G$, a contradiction.

If $G \in F_{11}$, then by Remark 2.1 and Theorem 1.6, $AE_C(G) = G$. □

Proposition 3.3. *If either G has only one full degree vertex or $G \in F_{22}$, then $AE_C(G) = \overline{G}$.*

Proof. If G has only one full degree vertex, then by Theorem 2.10, $AE_C(G) = \overline{G}$. If $G \in F_{22}$, by Proposition 3.1, $AE_C(G) \subseteq \overline{G}$. If $uv \in E(\overline{G})$, then u and v are at a distance 2 in G so that $d(u, v) = \left\lceil \frac{e(u)+e(v)}{2} \right\rceil$. Hence $uv \in AE_C(G)$. □

Proposition 3.4. *For any positive integer $n \neq 4$, path P_n is a C-average eccentric graph.*

Proof. When $n = 1, 2, P_n$ is a C-average eccentric graph of itself. When $n = 3$, $AE_C(K_1 \cup K_2) = P_3$. It has been found P_4 is not a C-average eccentric graph of any graph on four vertices. If $n \geq 5$, $d(P_n) = n - 1$ and by Theorem 1.2, $d(\overline{P}_n) \leq 2$. Also $\overline{P}_n$ has no full degree vertex. So $\overline{P}_n \in F_{22}$ and by Proposition 3.3, $AE_C(\overline{P}_n) = P_n$. □

Proposition 3.5. *For any positive integer $n \geq 3$, cycle C_n is a C-average eccentric graph.*

Proof. If $n = 3$ and $H = C_3$ or $\overline{K}_3$, then by Proposition 2.3 and Remark 2.4, $AE_C(H) = C_3$. If $n \geq 4$, then for a cycle C_n, $e(u) = 2$ for all $u \in V(\overline{C}_n)$. By Proposition 3.3, $AE_C(\overline{C}_n) = C_n$. □

4 C-average eccentric graphs

Theorem 4.1. *Let G be a graph on n vertices. Then $AE_C(G) = \overline{G}$ if and only if either of the following conditions holds.*
(1) G has only one full degree vertex;
(2) $G \in F_{22}$; and
(3) G is disconnected with each component complete.

Proof. If either (1) or (2) hold, then by Proposition 3.3, $AE_C(G) = \overline{G}$. Suppose (3) holds. If G is totally disconnected, then by definition, $AE_C(G) = K_n = \overline{G}$. Suppose G has at least one component H with $|H| \geq 2$. Then $uv \in E(AE_C(G))$ if and only if u and v belong to different components of G if and only if $uv \in E(\overline{G})$. Hence $AE_C(G) = \overline{G}$.

Suppose $AE_C(G) = \overline{G}$. If $G \in F_{11}$, then by Theorem 3.2, $AE_C(G) \neq \overline{G}$. If G has at least two full degree vertices, then by Theorem 2.10, $AE_C(G) \neq \overline{G}$. If $G \in F_{23}$ (or F_{24} or F_3, respectively), then there exists two non adjacent vertices u and v in G such that $e(u) = 2$, $e(v) = 3$ and $d(u, v) = 2$. So $uv \notin E(AE_C(G))$ but $uv \in E(\overline{G})$, a contradiction. If G is disconnected with at least one non complete component H, then every pair of non adjacent vertices u and v in H are adjacent in $\overline{G}$. But by definition, $uv \notin E(AE_C(G))$, a contradiction. □

Corollary 4.2. *If $G \in F_{24} \cup F_3$, then G is a C average eccentric graph.*

Corollary 4.3. *If $G, \overline{G} \in F_{22}$, then G and $\overline{G}$ are C-average eccentric graphs.*

Corollary 4.4. *Let G be a connecded graph with $r(G) > 1$. If $\overline{G}$ is disconnected with each component complete, then G is a C average eccentric graph.*

Corollary 4.5. *If $G \in F_4$ without isolated vertices, then G is a C-average eccentric graph.*

Corollary 4.6. *If G is disconnected with exactly one isolated vertex, then G is a C-average eccentric graph.*

Lemma 4.7. *If G is disconnected, then $\overline{AE_C(G)}$ is also a disconnected graph with each component complete.*

Proof. By Remark 2.4, the result follows. □

Theorem 4.8. *Let G be a connected graph with $r(G) \geq 1$ and $d(G) \geq 2$. If $\overline{G}$ is disconnected with at least one non complete component, then G is not a C-average eccentric graph.*

Proof. Suppose there exists a graph H such that $AE_C(H) = G$. If H is disconnected, then by Lemma 4.7, $\overline{AE_C(H)}$ is disconnected with each component complete which is a contradiction to the fact that $\overline{G}$ is disconnected with at least one non complete component. Hence H must be connected. If $r(H) = 1$ and $d(H) = 1$, then $AE_C(H) = H = G$, a contradiction. If $r(H) = 1$ and $d(H) = 2$, then by Theorem 2.10, $AE_C(H)$ is a disconnected graph, a contradiction. So $r(H) > 1$. By Proposition 3.1, $AE_C(H) \subseteq \overline{H}$. Hence H is isomorphic to a spanning subgraph of $\overline{G}$. Since $\overline{G}$ is disconnected, H is disconnected, a contradiction to $r(H) > 1$. □

Theorem 4.9. *If $G \in F_{22}$ and $\overline{G} \in F_{23}$, then G is not a C-average eccentric graph.*

Proof. Suppose there exists a graph H such that $AE_C(H) = G$. If H is disconnected, then by Lemma 4.7, $\overline{AE_C(H)}$ is disconnected, a contradiction to $\overline{G}$ is connected. Hence H must be connected. If $H \in F_{11} \cup F_{12}$, then by Theorem 3.2 and Theorem 2.10, $AE_C(H)$ is either a complete graph or a disconnected graph, a contradiction to $G \in F_{22}$. So $r(H) > 1$. By Proposition 3.1, $AE_C(H) \subseteq \overline{H}$. Hence H is isomorphic to a spanning subgraph of $\overline{G}$. Since $r(\overline{G}) = 2$ and $d(\overline{G}) = 3, r(H) \geq 2$ and $d(H) \geq 3$. Let $u \in V(\overline{G})$. Then u is adjacent to all the vertices v in G such that $d_{\overline{G}}(u,v) \geq 2$. $d_H(u,v) \geq 2$ whenever $d_{\overline{G}}(u,v) \geq 2$. So there exists a pair of non adjacent vertices u and v in $\overline{G}$ such that $uv \notin E(AE_C(H))$, a contradiction to $uv \in E(G)$. From these, we conclude that $AE_C(H)$ is not equal to G, a contradiction. □

Using the same proof technique used in Theorem 4.9, the following propositions have been obtained.

Proposition 4.10. *If $G \in F_{22}$ and $\overline{G} \in F_{24}$, then G is not a C-average eccentric graph.*

Proposition 4.11. *If $G \in F_{22}$ and $\overline{G} \in F_3$, then G is not a C-average eccentric graph.*

Proposition 4.12. *If $G \in F_{23}$ and $\overline{G} \in F_{23}$, then G is not a C-average eccentric graph.*

Theorem 4.13. *If $G \in F_{22}$ and $\overline{G} \in F_{22}$, then G is a C-average eccentric graph.*

Proof. By Theorem 4.1, the result follows. □

Lemma 4.14. *If G is a totally disconnected graph on $n \geq 2$ vertices, then G is not a C-average eccentric graph.*

Proof. Suppose there exists a graph H such that $AE_C(H) = G$. If H is disconnected, by definition, $AE_C(H)$ has at least one edge, a contradiction. So H is to be connected. For any connected graph, there is a pair of peripheral vertices, say u and v. Also $d(u,v) = \left\lceil \frac{e(u)+e(v)}{2} \right\rceil$ and hence $uv \in E(AE_C(H))$, a contradiction. □

Theorem 4.15. *Let G be any graph such that $\overline{G}$ is either connected with at most one full degree vertex or disconnected with each component complete. Then G is a C-average eccentric graph if and only if G is a C-average eccentric graph of itself or its complement.*

Proof. If G is a C-average eccentric graph of itself or its complement, then G is a C-average eccentric graph.

Suppose G is a C-average eccentric graph.

Case 1. G is connected and $\overline{G}$ is connected.

Subcase 1.1. Suppose $G \in F_{22}$. Then by Theorem 4.9, Proposition 4.10 and Proposition 4.11, $\overline{G} \notin F_{23}$, $\overline{G} \notin F_{24}$ and $\overline{G} \notin F_3$, respectively. If $\overline{G} \in F_{22}$, then by Theorem 4.1, $AE_C(\overline{G}) = G$.
Subcase 1.2. Suppose $G \in F_{23}$. Then by Proposition 4.12, $\overline{G} \notin F_{23}$. If $\overline{G} \in F_{22}$, then by Theorem 4.1, $AE_C(\overline{G}) = G$.
Subcase 1.3. Suppose $G \in F_{24}$. Then $\overline{G} \in F_{22}$ and by Theorem 4.1, $AE_C(\overline{G}) = G$.
Subcase 1.4. Suppose $G \in F_3$. Then $\overline{G} \in F_{22}$ and by Theorem 4.1, $AE_C(\overline{G}) = G$.
Case 2. G is connected and $\overline{G}$ is disconnected. In this case, $G \in F_{11} \cup F_{12} \cup F_{22}$
Subcase 2.1. Suppose $G \in F_{11}$. Then by Theorem 3.2, $AE_C(G) = G$.
Subcase 2.2. Suppose $G \in F_{12} \cup F_{22}$ Then by assumption, $\overline{G}$ is disconnected in which each component is complete. By Theorem 4.1, $AE_C(\overline{G}) = G$.
Case 3. G is disconnected. Then $\overline{G} \in F_{11} \cup F_{12} \cup F_{22}$. If $\overline{G} \in F_{11}$, then G is totally disconnected and by Lemma 4.14, G is not a C-average eccentric graph, a contradiction. Hence $\overline{G} \in F_{12} \cup F_{22}$. If $\overline{G} \in F_{22}$, by Theorem 4.1, $AE_C(\overline{G}) = G$. If $\overline{G} \in F_{12}$ has only one full degree vertex, then by Theorem 4.1, $AE_C(\overline{G}) = G$. If $\overline{G} \in F_{12}$ has more than one full degree vertices, then by Theorem 4.14, G is not a C-average eccentric graph, a contradiction.

Thus in all the cases, if G is a C-average eccentric graph, then G is a C-average eccentric graph of itself or its complement. □

References

[1] R. Aravamuthan and B. Rajendran, *Graph equations involving antipodal graphs*, Presented at the Seminar on Combinatories and Applications held at ISI, Culcutta during 14 - 17 December, 40 - 43 (1982).

[2] R. Aravamuthan and B. Rajendran, *On antipodal graphs*, Discrete Math., **49**, 193 - 195 (1984).

[3] F. Buckley and F. Harary, *Distance in graphs*, Addition-Wesley, Reading, 1990.

[4] KM. Kathiresan and G. Marimuthu, *A study on radial graphs*, Ars Combin., **96**, 353 - 360 (2010).

[5] D.B. West, *Introduction to graph theory*, Prentice-Hall of India, New Delhi, 2003.

[6] R.R. Singleton, *There is no irregular moore graph*, Amer. Math. Monthly, **75**, 42 - 43 (1968).

Author information

T. Sathiyanandham and S. Arockiaraj, Department of Mathematics, Government Arts and Science College, Sivakasi 626124, Tamilnadu, India.
E-mail: `sathiyan.maths@gmail.com, psarockiaraj@gmail.com`