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Domination in k-zero-divisor hypergraph of some class of commutative rings

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Abstract Let R be a commutative ring with identity and let $Z(R,k)$ be the set of all k-zero-divisors in R and $k>2$ an integer. The *k-zero-divisor hypergraph* of R, denoted by $\mathcal{H}_k(R)$, is a hypergraph with vertex set $Z(R,k)$, and for distinct elements $x_1, x_2, ..., x_k$ in $Z(R,k)$, the set $\{x_1, x_2, ..., x_k\}$ is an edge of $\mathcal{H}_k(R)$ if and only if $\prod_{i=1}^{k} x_i = 0$ and the product of any $(k-1)$ elements of $\{x_1, x_2, ..., x_k\}$ is nonzero. In this paper, we determine the domination number of $\mathcal{H}_k(R)$ for some commutative rings.

1 Introduction

Attaching various graphs to an algebraic structure to understand its properties is a classical and useful technique. In this direction, various graph parameters can be related to algebraic properties of the structure under consideration and lead to a better understanding of its theory. For example, one can attach a graph to a commutative ring R with unity by considering its non-zero zero-divisors as its vertices and connect two of them by an edge if their product is zero [6]. This graph is called the zero-divisor graph of R, denoted by $\Gamma(R)$, and is well-studied in the literature [4]. In view of this, Ch. Eslahchi and A. M. Rahimi [9] have introduced and investigated a graph called the *k-zero-divisor hypergraph* of a commutative ring and later was studied extensively in [11, 12]. For $k=2$, the graph is exactly the same as the zero-divisor graph of a ring.

A hypergraph $\mathcal{H}$ is a pair $(V(\mathcal{H}), E(\mathcal{H}))$ of disjoint sets, where $V(\mathcal{H})$ is a non empty finite set whose elements are called vertices and the elements of $E(\mathcal{H})$ are nonempty subsets of $V(\mathcal{H})$ called edges. The hypergraph $\mathcal{H}$ is called k-uniform whenever every edge e of $\mathcal{H}$ is of size k. The number of edges containing a vertex $v \in V(\mathcal{H})$ is its degree $d_{\mathcal{H}}(v)$. For basic definitions on hypergraphs, one may refer [3]. A set D of vertices in a hypergraph $\mathcal{H}$ is a dominating set if for every $x \in V \setminus D$ there exists $y \in D$ such that x and y are adjacent i.e., there exists $e \in E$ such that $x, y \in e$. The minimum cardinality of a dominating set is called the domination number of $\mathcal{H}$ and is denoted by $\gamma(\mathcal{H})$, one may refer [1].

Throughout this paper, we assume that R is a commutative ring with identity, $Z(R)$, its set of zero-divisors and $R^{\times}$, its group of units and F is a field. For any set X, let X^* denote the nonzero elements of X. For basic definitions on rings, one may refer [7, 10].

2 Domination of $\mathcal{H}_k(R)$

In this section, we determine the domination number of the k-zero-divisor hypergraph of some classes of commutative rings. From the definition, we have the following observations.

Remark 2.1. [11, Remark 2.1] Let $R = F_1 \times F_2 \times \cdots \times F_n$, where each F_i is a field and $3 \le k \le n$.

If $\mathbb{A}_\ell = \{x = (a_1, a_2, \ldots, a_n) \in R :$ exactly ℓ components in the n tuples of x are zero$\}$ for $1 \le \ell \le n-k+1$. Then $Z(R,k) = \bigcup_{\ell=1}^{n-k+1} \mathbb{A}_\ell$.

Remark 2.2. Let $m = p^n$ be a positive integer and $A_d = \{x \in \mathbb{Z}_m : (x, m) = d\}$ where p is prime, d divides m and $n \geq 3$. Then

(i) $Z(\mathbb{Z}_m, 3) = \bigcup_{i=1}^{n-2} A_{p^i}$,

(ii) $Z(R, k) = \bigcup_{i=1}^{n-k+1} A_{p^i}$ and $A_{p^i} \cap A_{p^j} = \emptyset$ for all $i \neq j$.

Proposition 2.3. *Let $(R_i, \mathfrak{m}_i)$ be a local ring with $\mathfrak{m}_i \neq \{0\}$ for $1 \leq i \leq n$ and let $R = R_1 \times \cdots \times R_n$. Let $z_i \in \mathfrak{m}_i^*$ for $1 \leq i \leq n$. Then $z_i \in Z(R_i, 3)$ if and only if $z = (0, \ldots, 0, z_i, 0 \ldots, 0) \in Z(R, 3)$*

Proof. Suppose that $z_i \in Z(R_i, 3)$. Then by definition of $Z(R_i, 3)$, there exists distinct elements x_i, y_i of R_i other than z_i such that $x_i y_i z_i = 0$ and $x_i y_i, x_i z_i, y_i z_i$ are nonzero elements of R_i. Let $x = (0, \ldots, 0, x_i, 0, \ldots, 0)$, $y = (0, \ldots, 0, y_i, 0, \ldots, 0)$, $z = (0, \ldots, 0, z_i, 0, \ldots, 0) \in R$. Then $xyz = 0$ and xy, xz, yz are nonzero elements of R. Hence $z \in Z(R, 3)$.

Suppose that $z = (0, \ldots, 0, z_i, 0, \ldots, 0) \in Z(R, 3)$. Then there exists distinct elements $x = (0, \ldots, 0, a_i, 0, \ldots, 0)$, $y = (0, \ldots, 0, b_i, 0, \ldots, 0) \in Z(R, 3)$ such that $xyz = 0$ and so $a_i b_i z_i = 0$. By definition of $Z(R, 3)$, xy, xz, yz are nonzero elements of R and so $a_i b_i, b_i z_i$ and $a_i z_i$ are nonzero elements of R_i. From this, we get $z_i \in Z(R_i, 3)$. □

Proposition 2.4. *Let $(R, \mathfrak{m})$ be a finite local ring with $\mathfrak{m}^t = 0$. If $t = 2$, then $Z(R, k) = \emptyset$ for any $k \geq 3$.*

Proof. Suppose $Z(R, k) \neq \emptyset$ for any $k \geq 3$. Let $x_1 \in Z(R, k)$. Then there exists $x_2, x_3, \ldots, x_k \in Z(R, k)$ such that $\{x_1, x_2, \ldots, x_k\}$ is an edge. i.e., $x_1 x_2 \ldots x_k = 0$. Since $t = 2$, the product of two vertices from $\{x_1, x_2, \ldots, x_k\}$ is zero, which is a contradiction. Hence $Z(R, k) = \emptyset$. □

Proposition 2.5. *Let $(R, \mathfrak{m})$ be a finite local ring with $\mathfrak{m}^t = 0$, then $Z(R, k) = \emptyset$ for all $t < k$.*

Proof. Suppose $Z(R, k) \neq \emptyset$ for any $t < k$. Let $x_1 \in Z(R, k)$. Then there exists $x_2, x_3, \ldots, x_k \in Z(R, k)$ such that $\{x_1, x_2, \ldots, x_k\}$ is an edge. i.e., $x_1 x_2 \ldots x_k = 0$. Since $t < k$, the product of t vertices from $\{x_1, x_2, \ldots, x_k\}$ is zero, which is a contradiction. Hence $Z(R, k) = \emptyset$. □

The next theorem shows that k-zero-divisor hypergraph of a local ring $\mathbb{Z}_{p^t}$ is connected.

Theorem 2.6. *Let $R = \mathbb{Z}_{p^t}$, where $t \geq 3$. For $k \leq t$, then $\mathcal{H}_k(R)$ is connected, $diam(\mathcal{H}_k(R)) \leq 2$ and $gr(\mathcal{H}_k(R)) = 2$ or ∞.*

Proof. Let x, $y \in Z(R, k)$ such that $x \neq y$. If $x, y \in e$ for some edge e in $\mathcal{H}_k(R)$, then $d(x, y) = 1$. Suppose $x, y \notin e$ for every edge e in $\mathcal{H}_k(R)$. Since $x, y \in Z(R, k)$, $x \in A_{p^i}$ and $y \in A_{p^j}$ for some i, j. For each $w \in Z(R, k)$ and $w \neq z$, $w, z \in e$ for some edge e in $\mathcal{H}_k(R)$, where $z \in A_p$. From this, we get $x \neq z$ and $y \neq z$ and so there exists $x_i \in A_{p^l}$, for some $1 \leq l \leq t - k + 1$ and $1 \leq i \leq t - 2$ such that $\{z, x, x_1, x_2, \ldots, x_{k-2}\}$, $\{z, y, x_1, x_2, \ldots, x_{k-2}\}$ are edges in $\mathcal{H}_k(R)$ and so $d(x, y) = 2$. Hence it is clear that $diam(\mathcal{H}_k(R)) \leq 2$.

Suppose $3 \leq k < t$. Let $e_1 = \{x_1, x_2, \ldots, x_{k-2}, z, z'\}$, $e_2 = \{x_1, x_2, \ldots, x_{k-2}, z, z'\}$ are two edges in $\mathcal{H}_k(R)$, where $z, z' \in A_p$, $x_i \in A_{p^l}$, for some $1 \leq l \leq t - k + 1$ and hence $z' - e_1 - z - e_2 - z'$ form a cycle of length two and so $gr(\mathcal{H}_k(R)) = 2$.

Suppose $k = t$. If $|Z(R, k)| \geq t + 1$, then $e_1 = \{x_1, x_2, \ldots, x_{k-2}, z, z'\}$ and $e_2 = \{x_1, x_2, \ldots, x_{k-2}, z, z'\}$ are two edges in $\mathcal{H}_k(R)$, where $z, z', x_i \in A_p$ and hence $z' - e_1 - z - e_2 - z'$ form a cycle of length two and so $gr(\mathcal{H}_k(R)) = 2$. If $|Z(R, k)| = t$, then $e = \{x_1, x_2, \ldots, x_{k-2}, x_{k-1}, x_k\}$ is the only edge in $\mathcal{H}_k(R)$. Hence $gr(\mathcal{H}_k(R)) = \infty$. □

Note that if $R = \mathbb{Z}_{p^2}$, $\mathbb{Z}_8$ or $F_1 \times F_2$, where p is prime and F_i's are field, then $Z(R, 3) = \emptyset$. We exclude these rings while studying 3-zero-divisor hypergraph.

Theorem 2.7. *Let $R = \mathbb{Z}_{p^n}$, where p is prime and $n \geq 3$. Then $\gamma(\mathcal{H}_3(R)) = 1$*

Proof. For any $x \in Z(R, 3)$, $\{x, y, a\}$ is an edge of $\mathcal{H}_3(R)$ for all $a \in A_p$ and some $y \in Z(R, 3)$. Hence $\{a\}$ is a dominating set of $\mathcal{H}_3(R)$ for some $a \in A_p$ and so $\gamma(\mathcal{H}_3(R)) = 1$. □

Theorem 2.8. *Let $R = \mathbb{Z}_{p^n} \times F$ be a commutative ring, where p is prime, $n \geq 2$, F is a field and $Z(\mathbb{Z}_{p^n}, 3) \neq \emptyset$. Then $\gamma(\mathcal{H}_3(R)) = 1$.*

Proof. Consider the set $D = \{x = (z, u)\} \subset Z(R, 3)$, where $u \in F^*$ and $z \in A_p$. Suppose that $y \in Z(R, 3) \setminus D$. If $y = (0, v) \in Z(R, 3)$, where $v \in F^*$, then there exists $w = (a, u) \in Z(R, 3)$, where $a \in A_{p^{n-1}}$, such that $xyw = 0$ and none of xy, xw and yw are zero. Hence x dominates every element of $(0) \times F^*$.

If $y = (z, 0) \in Z(R, 3)$, where $z \in Z(\mathbb{Z}_{p^n}, 3)$, then there exists $w = (a, u) \in Z(R, 3)$, where $a \in A_{p^i}$, $1 \leq i \leq n - 2$, such that $\{x, y, w\}$ is an edge and so x dominates $Z(\mathbb{Z}_{p^n}, 3) \times (0)$.

Consider the cases when $y = (z, u') \in Z(R, 3)$, where $u' \in F^*$, $z \in Z(\mathbb{Z}_{p^n})^*$, and when $y = (z, u') \in Z(R, 3)$, where $u' \in F^*$, $z \in Z(\mathbb{Z}_{p^n})^*$. In the former case, there exists $w = (v, 0) \in Z(R, 3)$, where $v \in \mathbb{Z}_{p^n}^{\times}$, such that $\{x, y, w\}$ is an edge. In the later case, there exists $w = (a, 0) \in Z(R, 3)$, where $a \in Z(\mathbb{Z}_{p^n}, 3)$, such that the product $xyw = 0$ and xy, xw, yw are non-zero. Thus, we conclude that x dominates every element of $Z(R, 3)$. Hence D is a dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) = 1$. □

Theorem 2.9. *Let $R = \mathbb{Z}_{p_1^{n_1}} \times \mathbb{Z}_{p_2^{n_2}}$, where p_1, p_2 are prime and $n_1, n_2 \geq 2$. Then $\gamma(\mathcal{H}_3(R)) = 2$.*

Proof. Let $a = (a_1, a_2) \in Z(R, 3)$, where $a_i \in A_{p_i}$ for $i = 1, 2$. Then a dominates $Z(R, 3) \setminus B$, where $B = \{(b_1, b_2) : b_i \in A_{p_i^{n_i - 1}}$, for $i = 1, 2\}$. In order to find a vertex which dominates the set B, let us consider the vertex $c = (c_1, c_2) \in Z(R, 3)$, where $c_1 \in Z(\mathbb{Z}_{p_1^{n_1}})^*$ and $c_2 \in \mathbb{Z}_{p_2^{n_2}}^{\times}$. For each $d \in B$, there exist $f = (f_1, f_2) \in Z(R, 3)$, where $f_1 \in \mathbb{Z}_{p_1^{n_1}}^{\times}$ and $f_2 \in Z(\mathbb{Z}_{p_2^{n_2}})^*$ such that the product cdf is zero and none of cd, cf, df are zero and so c dominates the set B. Hence $\{a, c\}$ is a dominating set of $\mathcal{H}_3(R)$ and $\gamma(\mathcal{H}_3(R)) \leq 2$.

Suppose that $\gamma(\mathcal{H}_3(R)) = 1$. Then there exists a subset S of $Z(R, 3)$ such that S is a dominating set of $\mathcal{H}_3(R)$ and $|S| = 1$.

Let $z_1 \in A_{p_1^i}$, $z_2 \in A_{p_2^j}$ for some $i \in \{1, 2, \ldots, n_1 - 1\}$ and $j \in \{1, 2, \ldots, n_2 - 1\}$.

If $(z_1, z_2) \in S$, then there exists a vertex $(a, b) \in Z(R, 3)$, $a \in A_{p_1^{n_1 - i}}$, $b \in A_{p_2^{n_2 - j}}$, such that the product $(z_1, z_2)(a, b) = (0, 0)$ and so the vertices of this nature do not fall under any edge of $\mathcal{H}_3(R)$.

Consider the vertex in S of the form $(z, v) \in Z(\mathbb{Z}_{p_1^{n_1}}) \times \mathbb{Z}_{p_2^{n_2}}^{\times}$. Then the vertices (z, v) and $(0, v')$ are not adjacent in $\mathcal{H}_3(R)$, for all $v' \in \mathbb{Z}_{p_2^{n_2}}^{\times}$.

Finally let us consider the vertex in S of the form $(z, 0) \in Z(R, 3)$, where $z \in Z(\mathbb{Z}_{p_1^{n_1}}, 3)$. Then vertices $(z, 0)$ and $(0, v')$ are not adjacent in $\mathcal{H}_3(R)$, for all $v' \in \mathbb{Z}_{p_2^{n_2}}^{\times}$. Thus there does not exist a dominating set of cardinality one and so $\gamma(\mathcal{H}_3(R)) = 2$. □

Theorem 2.10. *Let $R = F_1 \times F_2 \times F_3$, where each F_i is a field. Then $\gamma(\mathcal{H}_3(R)) = 1$ if and only if $R \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times F_3$, where F_3 is a field.*

Proof. Suppose $\gamma(\mathcal{H}_3(R)) = 1$. Let us assume that at least two of F_1, F_2, F_3 are of cardinality more than 2. Note that $Z(R, 3) = B_1 \cup B_2 \cup B_3$, where $B_1 = F_1^* \times F_2^* \times 0$, $B_2 = 0 \times F_2^* \times F_3^*$, $B_3 = F_1^* \times 0 \times F_3^*$. Clearly $|B_i| \geq 2$ for all $i \in \{1, 2, 3\}$.

Let x be any element of $Z(R, 3)$. Then $x \in B_i$ for some $i \in \{1, 2, 3\}$ and x dominates every element of B_j for $j \neq i$ and x does not dominate the set B_i, a contradiction. Hence $R \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times F_3$.

Conversely, assume that $R = \mathbb{Z}_2 \times \mathbb{Z}_2 \times F_3$, where F_3 is a field. Then $Z(R, 3) = \{(1, 1, 0), (0, 1, w), (1, 0, w) : w \in F_3^*\}$. Consider the set $D = \{x = (1, 1, 0)\}$. If $y = (0, 1, w) \in Z(R, 3)$, where $w \in F_3^*$, then there exists $z = (1, 0, w) \in Z(R, 3)$ such that $\{x, y, z\}$ is an edge. Hence D is a dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) = 1$. □

Theorem 2.11. *Let $R = F_1 \times F_2 \times F_3$ be a commutative ring and $R \ncong \mathbb{Z}_2 \times \mathbb{Z}_2 \times F_3$, where each F_i is a field. Then $\gamma(\mathcal{H}_3(R)) = 2$.*

Proof. By Theorem 2.10, $\gamma(\mathcal{H}_3(R)) \geq 2$. Since $R \ncong \mathbb{Z}_2 \times \mathbb{Z}_2 \times F_3$, at least two F_i's have cardinality more than two. Without loss of generality, we assume that $|F_i| \geq 3$ for $i \in \{2, 3\}$. Note that $Z(R, 3) = B_1 \cup B_2 \cup B_3$, where $B_1 = F_1^* \times F_2^* \times 0$, $B_2 = 0 \times F_2^* \times F_3^*$, $B_3 = F_1^* \times 0 \times F_3^*$.

Consider the set $D = \{a = (u, v, 0), b = (0, v, w)\}$, where $a \in B_1$ and $b \in B_2$. Then a dominates every element of $B_2 \cup B_3$ and b dominates every element of B_1. Hence D is a domination set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) = 2$. □

Theorem 2.12. *Let $R = F_1 \times F_2 \times \mathbb{Z}_{p^n}$ be a commutative ring, where each F_i is a field, p is prime and $n \geq 2$. Then $\gamma(\mathcal{H}_3(R)) = 2$.*

Proof. Consider $D = \{x = (u, v, z), y = (0, v, z)\}$, where $u \in F_1^*$, $v \in F_2^*$, $z \in A_p$. Then the vertex x dominates $Z(R, 3) \smallsetminus B$ where $B = \{(u, v, 0) : u \in F_1^*,\ v \in F_2^*\}$. Also the vertex $y \in D$ dominates B. Hence D is a dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) \leq 2$.

Let $g = (1, 1, z) \in Z(R, 3)$, for some $z \in Z(\mathbb{Z}_{p^n})^*$. Then the vertex g does not dominate any element of B. Clearly the vertices $(0, 1, 1)$ and $(1, 0, 1)$ do not dominate any element of the sets $\{(0, v, w) : v \in F_2^*, w \in \mathbb{Z}_{p^n}^\times\}$ and $\{(u, 0, w) : u \in F_1^*, w \in \mathbb{Z}_{p^n}^\times\}$ respectively.

Let $z \in Z(\mathbb{Z}_{p^n})^*$ with $zd = 0$ for some $d \in Z(\mathbb{Z}_{p^n})^*$. Then vertices $(1, 0, z)$ and $(0, 1, d)$ are not adjacent in $\mathcal{H}_3(R)$. Also the vertex $(0, 0, 1)$ does not dominate any element of the set $\{(0, 0, z') : z' \in Z(\mathbb{Z}_{p^n}, 3)\}$. Thus we conclude that any dominating set of $\mathcal{H}_3(R)$ must contain more than one element and hence $\gamma(\mathcal{H}_3(R)) = 2$. □

Theorem 2.13. *Let $R = F \times \mathbb{Z}_{p_1^{n_1}} \times \mathbb{Z}_{p_2^{n_2}}$ be a commutative ring, where F is a field, p_1, p_2 are prime and $n_1, n_2 \geq 2$. Then $\gamma(\mathcal{H}_3(R)) = 2$.*

Proof. Consider $D = \{x = (u, a, b), y = (u, v, b)\}$, where $u \in F^*$, $v \in \mathbb{Z}_{p_1^{n_1}}^\times$, $a \in A_{p_1}$, $b \in A_{p_2}$. Then the vertex x is adjacent to every element of the set $Z(R, 3) \setminus \{(0, z, w) : z \in A_{p_1^{n_1-1}},\ w \in A_{p_2^{n_2-1}}\}$. Also the vertex y dominates the set $\{(0, p, q) : p \in A_{p_1^{n_1-1}},\ q \in A_{p_2^{n_2-1}}\}$. Hence D is a dominating set of $\mathcal{H}_3(R)$ and by using similar arguments given in Theorem 2.12, we get $\gamma(\mathcal{H}_3(R)) = 2$. □

Theorem 2.14. *Let $R = \mathbb{Z}_{p_1^{n_1}} \times \mathbb{Z}_{p_2^{n_2}} \times \mathbb{Z}_{p_3^{n_3}}$ be a commutative ring, where each p_i's are prime and $n_i \geq 2$ for $i = \{1, 2, 3\}$. Then $\gamma(\mathcal{H}_3(R)) = 2$.*

Proof. Let $D = \{x = (u, b, c), y = (a, v, 0)\} \subseteq Z(R, 3)$, where $a \in A_{p_1}$, $b \in A_{p_2}$, $c \in A_{p_3}$, $u \in \mathbb{Z}_{p_1^{n_1}}^\times$, $v \in \mathbb{Z}_{p_2^{n_2}}^\times$. Then the vertex x dominates the set $Z(R, 3) \setminus (P \cup Q \cup S)$, where $P = \mathbb{Z}_{p_1^{n_1}}^\times \times 0 \times 0$, $Q = Z(\mathbb{Z}_{p_1^{n_1}}, 3) \times 0 \times 0$, and $S = \{(0, b', c') :\ b' \in A_{p_2^{n_2-1}},\ c' \in A_{p_3^{n_3-1}}\}$. Also the vertex y dominates the set$P \cup Q \cup S$ in $\mathcal{H}_3(R)$. From this, we get D is a dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) \leq 2$.

Note that $\mathcal{H}_3(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3)$ is a subhypergraph of $\mathcal{H}_3(R)$. By Theorem 2.11, $\gamma(\mathcal{H}_3(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3)) \geq 2$ and hence $\gamma(\mathcal{H}_3(R)) = 2$. □

Theorem 2.15. *Let R be an Artinian reduced ring and F be a field and $|Max(R)| \geq 3$. If $k = |Max(R)|$, then*

$$\gamma(\mathcal{H}_k(R)) = \begin{cases} 1 & \text{if } R = \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1)\ times} \times F \\ 2 & otherwise. \end{cases}$$

Proof. Since R is reduced, $R \cong F_1 \times \cdots \times F_n$, where each F_i is a field and $|Max(R)| = n$. By Remark 2.2, $Z(R, k) = \bigcup_{\ell=1}^{n-k+1} \mathbb{A}_\ell$.

Case 1: $R = \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1)\ times} \times F$.

Let $x = (1, 1, \ldots, 1, 0) \in Z(R, k)$. For any $y \in Z(R, k)$ and $y \neq x$, there is an edge e of $\mathcal{H}_k(R)$ such that $x, y \in e$. Hence $\{x\}$ is a dominating set of $\mathcal{H}_k(R)$ and so $\gamma(\mathcal{H}_k(R)) = 1$.

Case 2: $R \ncong \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1)\ times} \times F$. Then at least two F_i's have more than two elements. Without loss of generality, we assume that $|F_\ell| \geq 3$ and $|F_t| \geq 3$ for some $\ell \neq t$.

Consider the set $D = \{x = (1, 1, \ldots, 1, 0), y = (0, 1, 1, \ldots, 1, 1)\}$. Then the vertex x dominates the set $Z(R, 3) \setminus B$ where $B = \{(u_1, u_2, \ldots, u_{n-1}, 0) : u_i \in F_i^*\}$. Also the vertex y dominates the set B. Hence D is a dominating set of $\mathcal{H}_k(R)$ and so $\gamma(\mathcal{H}_k(R)) \leq 2$.

Suppose that $\gamma(\mathcal{H}_k(R)) = 1$. Then there exists a subset S of $Z(R, k)$ such that S is a dominating set of $\mathcal{H}_k(R)$ and $|S| = 1$. If $x = (u_1, \ldots, u_{j-1}, 0, u_{j+1}, \ldots, u_n) \in S$, where $u_i \in F_i^*$ for all $i \neq j$, then the vertices x and $(v_1, \ldots, v_\ell, \ldots, v_{j-1}, 0, v_{j+1}, \ldots, v_t, \ldots, v_n) \in Z(R, k)$ are not adjacent in $\mathcal{H}_k(R)$, where $v_i \in F_i^*$, which is a contradiction. Hence any dominating set of $\mathcal{H}_k(R)$ contains at least two elements. Hence $\gamma(\mathcal{H}_k(R)) = 2$. □

Theorem 2.16. *Let $R = F_1 \times F_2 \times \cdots \times F_n$ be a ring, where each F_i is a field and $3 \leq k < n$. Let D be any dominating set in $\mathcal{H}_k(R)$ with minimum cardinality among all dominating sets. Then, for every $1 \leq \ell \leq n - k + 1$, D must contain at least one vertex from $\mathbb{A}_\ell$, where*

$$A_\ell = \{x = (a_1, a_2, \ldots, a_n) \in R : \textit{exactly } \ell \textit{ components in the } n \textit{ tuples of } x \textit{ are zero}\}.$$

Proof. Let $x = (u_1, \ldots, u_\ell, \underbrace{0, 0, \ldots, 0}_{\ell \text{ terms}}, u_{2\ell+1}, \ldots, u_n) \in \mathbb{A}_\ell$, where $u_i \in F_i^*$.

Now $\mathbb{B}_\ell = \{(a_1, \ldots, a_\ell, \underbrace{b_{\ell+1}, \ldots, b_{2\ell}}_{\ell \text{ terms}}, a_{2\ell+1}, \ldots, a_n) \in R : a_s \in F_s, b_j \in F_j\} \setminus \{\{(0, \ldots, 0, \underbrace{b_{\ell+1}, b_{\ell+2}, \ldots, b_{2\ell}}_{\ell \text{ terms}}, 0, \ldots, 0) \in R : b_i \in F_i\} \cup \{(a_1, \ldots, a_\ell, \underbrace{0, \ldots, 0}_{\ell \text{ terms}}, a_{2\ell+1}, \ldots, a_n) \in R : a_i \in F_i\} \cup \{(u_1, \ldots, u_\ell, \underbrace{b_{\ell+1}, \ldots, b_{2\ell}}_{\ell \text{ terms}}, u_{2\ell+1}, \ldots, u_n) \in R : u_i \in F_i^*, b_j \in F_j\}\}$. Since $n > 3$, the vertices in $\mathbb{B}_\ell$ contains at least two vertices that are adjacent only to vertices in $\mathbb{A}_\ell$. Suppose that D does not contain any vertex from $\mathbb{A}_t$ for some t. Since D is a dominating set, D will contain vertices of $\mathbb{B}_t$. Consider $D' = (D - \mathbb{B}_t) \cup \{y\}$ where y is an element of $\mathbb{A}_t$. Then $|D'| < |D|$ and D' is a dominating set of $\mathcal{H}_k(R)$, a contradiction. Hence D will contain at least one element from $\mathbb{A}_\ell$ for every ℓ. □

Theorem 2.17. *Let R be an Artinian reduced ring, F be a field and $n = |Max(R)| \geq 3$. If $3 \leq k < n$, then*

$$\gamma(\mathcal{H}_k(R)) = \begin{cases} n - k + 1 & \textit{if } R \cong \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1) \text{ times}} \times F \\ n - k + 2 & \textit{if otherwise} \end{cases}$$

Proof. Since R is reduced, $R \cong F_1 \times \cdots \times F_n$, where each F_i is a field and $|Max(R)| = n$. By Remark 2.1 , $Z(R, k) = \bigcup_{\ell=1}^{n-k+1} \mathbb{A}_\ell$.

Case 1: $R \cong \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1) \text{ times}} \times F$.

Consider the set $D = \{x_1, x_2, \ldots, x_{n-k+1}\} \subseteq Z(R, k)$, where $x_1 = (1, \ldots, 1, 0), x_2 = (1, \ldots, 1, 0, 0), \ldots, x_{n-k} = (\underbrace{1, \ldots, 1}_{k \text{ terms}}, 0, \ldots, 0)$, $x_{n-k+1} = (\underbrace{1, \ldots, 1}_{k-1 \text{ terms}}, 0, \ldots, 0)$. Then the vertex x_1 dominates the set $\{(y_1, y_2, \ldots, y_{n-1}, u_n) \in Z(R, k) : y_i \in \mathbb{Z}_2, u_n \in F_n^*\}$ and the vertex x_1 does not dominate any elements of the set B_1, where $B_1 = \{(y_1, \ldots, y_{n-1}, 0) \in Z(R, k) : y_i \in \mathbb{Z}_2\}$. Now the vertex x_2 dominates the set $\{(y_1, \ldots, y_{n-2}, 1, 0) \in Z(R, k) : y_i \in \mathbb{Z}_2\}$ and the vertex x_2 does not dominate any vertex in $B_2 = \{(y_1, \ldots, y_{n-2}, 0, 0) \in Z(R, k) : y_i \in \mathbb{Z}_2\}$. Proceeding like this, finally the vertex x_{n-k+1} dominates $\{(y_1, \ldots, y_{k-1}, 1, 0, \ldots, 0) : y_i \in \mathbb{Z}_2, \ y_i \neq 0 \text{ for all } i\}$. Hence D is the dominating set of $\mathcal{H}_k(R)$ and so $\gamma(\mathcal{H}_k(R)) \leq n - k + 1$.

In view of Theorem 2.16, we have $\gamma(\mathcal{H}_k(R)) \geq n - k + 1$ and hence $\gamma(\mathcal{H}_k(R)) = n - k + 1$.

Case 2: $R \ncong \underbrace{\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2}_{(n-1) \text{ times}} \times F$.

Consider the set $D = \{y_1, y_2, \ldots, y_{n-k+2}\}$, where $y_1 = (1, \ldots, 1, 0), y_2 = (1, \ldots, 1, 0, 0), \ldots, y_{n-k} = (\underbrace{1, \ldots, 1}_{k \text{ terms}}, 0, \ldots, 0)$, $y_{n-k+1} = (\underbrace{1, \ldots, 1}_{k-1 \text{ terms}}, 0, \ldots, 0)$, $y_{n-k+2} = (0, 1, \ldots, 1) \in Z(R, k)$.

Then the vertex y_1 dominates the set $\{(w_1, w_2, \dots, w_{n-1}, u_n) \in Z(R,k) : w_i \in F_i, u_n \in F_n^*\}$ and the vertex y_1 does not dominate any elements in the set $\{(w_1, w_2, \dots, w_{n-1}, 0) \in Z(R,k) : w_i \in F_i\}$. Now the vertex y_2 dominates the set $\{(w_1, w_2, \dots, w_{n-2}, u_{n-1}, 0) \in Z(R,k) : w_i \in F_i, u_{n-1} \in F_{n-1}^*\}$ and the vertex y_2 does not dominate any vertex in $\{(w_1, w_2, \dots, w_{n-2}, 0, 0) \in Z(R,k) : w_i \in F_i\} \cup \{(w_1, w_2, \dots, w_{n-1}, 0) \in Z(R,k) : w_i \in F_i\}$. Proceeding like this, y_{n-k+1} dominates the set $\{(w_1, w_2, \dots, w_{k-1}, u_k, 0, \dots, 0) : w_i \in F_i, u_k \in F_k^*, w_i \neq 0 \text{ for all } i\}$ and the vertex y_ℓ does not dominate any elements in the set P, where $P = \{(w_1, \dots, w_{k-1}, 0, \dots, 0) : w_i \in F_i$ and at least two w_i's are non-zero$\} \cup \dots \cup$
$\{(w_1, \dots, w_{n-2}, 0, 0) : w_i \in F_i$ and at least two w_i's are non-zero$\} \cup \{(w_1, \dots, w_{n-1}, 0) : w_i \in F_i$ and at least two w_i's are non-zero$\}$, where $1 \leq \ell \leq n-k+1$. Now the vertex y_{n-k+2} dominates the set P. Hence D is a dominating set of $\mathcal{H}_k(R)$ and so $\gamma(\mathcal{H}_k(R)) \leq n-k+2$.

Suppose that $\gamma(\mathcal{H}_k(R)) \leq n-k+1$. Then by Theorem 2.16, there exists a dominating set $D' = \{x_1, x_2, \dots, x_{n-k+1}\}$, where $x_\ell = (a_1, \dots, a_{\ell-1}, \underbrace{0, 0, \dots, 0}_{\ell \ terms}, a_{\ell+\ell-1}, \dots, a_{n-1}, a_n) \in \mathbb{A}_\ell$ and $1 \leq \ell \leq n-k+1$. Since $|F_i| \geq 3$, vertices in D' do not dominate vertices in $\{(a_1, \dots, a_{\ell-1}, \underbrace{0, 0, \dots, 0}_{\ell \ terms}, a_{\ell+\ell-1}, \dots, a_{n-1}, a_n) : 1 \leq \ell \leq n-k+1\}$, a contradiction. Hence $\gamma(\mathcal{H}_k(R)) = n-k+2$. □

Theorem 2.18. *Let $R = \mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \dots \times \mathbb{Z}_{p_n^{\alpha_n}}$ be a commutative ring with identity, where each p_i is prime and $\alpha_i \geq 2$, $n \geq 3$, $p_1^{\alpha_1} \leq p_2^{\alpha_2} \leq \dots \leq p_n^{\alpha_n}$. Then $\gamma(\mathcal{H}_3(R)) = n-1$.*

Proof. Consider the set $D = \{x_1, \dots, x_{n-1}\}$ where $x_i = (z_1, z_2, \dots, z_{i-1}, u_i, z_{i+1}, \dots, z_n)$, $z_i \in A_{p_i}$, $u_i \in \mathbb{Z}_{p_i^{\alpha_i}}^\times$ for $1 \leq i \leq n-1$. Then the vertex x_1 dominates the set $Z(R,3) \setminus (A' \cup B' \cup C')$, where $B' = \bigcup_{i=2}^{n-1} \{(0, 0, \dots, 0, a_i, a_{i+1}, \dots, a_{n-1}, a_n) : a_i \in A_{p_i^{\alpha_i - 1}}\}$, $C' = \{(a_1, 0, \dots, 0, 0) : a_1 \in Z(\mathbb{Z}_{p_1^{\alpha_1}}, 3)\}$ and $A' = \{(u_1, 0, \dots, 0, 0) : u_1 \in \mathbb{Z}_{p_1^{\alpha_1}}^\times\}$. Also the vertex x_2 dominates the set $C' \cup A' \cup \{(0, a_2, \dots, a_{n-1}, a_n) \in B' : a_i \in A_{p_i^{\alpha_i - 1}}\}$ and $\{x_1, x_2\}$ does not dominate any elements in the set $\bigcup_{i=3}^{n-1} \{(0, 0, \dots, 0, a_i, a_{i+1}, \dots, a_{n-1}, a_n) : a_i \in A_{p_i^{\alpha_i - 1}}\}$. Now the vertex x_3 dominates the set $\{(0, 0, a_3, a_4, \dots, a_{n-1}, a_n) \in B' : a_i \in A_{p_i^{\alpha_i - 1}}\}$ and $\{x_1, x_2, x_3\}$ does not dominate any elements in the set $\bigcup_{i=4}^{n-1} \{(0, \dots, 0, a_i, a_{i+1}, \dots, a_{n-1}, a_n) : a_i \in A_{p_i^{\alpha_i - 1}}\}$. Proceeding like this, the vertex x_{n-2} dominates the set $\{(0, 0, \dots, 0, a_{n-2}, a_{n-1}, a_n) \in B' : a_i \in A_{p_i^{\alpha_i - 1}}\}$ and $\{x_1, x_2, \dots, x_{n-2}\}$ does not dominate any elements of the set H, where $H = \{(0, 0, \dots, 0, a_{n-1}, a_n) \in B' : a_i \in A_{p_i^{\alpha_i - 1}}\}$ but x_{n-1} dominates the set H. Hence D is a dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) \leq n-1$.

Since $\mathcal{H}_3(\mathbb{Z}_3^n)$ is a subhypergraph of $\mathcal{H}_3(R)$ and by Theorem 2.17, $\gamma(\mathcal{H}_3(\mathbb{Z}_3^n)) \geq n-1$. Hence $\gamma(\mathcal{H}_3(R)) = n-1$. □

Theorem 2.19. *Let $R = \mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \dots \times \mathbb{Z}_{p_n^{\alpha_n}} \times F_1 \times F_2 \times \dots \times F_m$ be a finite commutative non-local ring, where each F_j is field, $\alpha_i \geq 2$, $m \geq 1$, $n \geq 2$ and $n + m \geq 3$. Then $\gamma(\mathcal{H}_3(R)) = n+m-1$.*

Proof. Consider the set $D = \{x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_{m-1}\}$, where $x_1 = (z_1, z_2, \dots, z_n, u_1, u_2 \dots, u_m)$, $x_2 = (u_1, u_2, \dots, u_{n-1}, 0, 0, \dots, 0)$, $x_3 = (u_1, u_2, \dots, u_{n-2}, 0, 0, \dots, 0), \dots$, $x_{n-1} = (u_1, u_2, 0, \dots, 0, 0, \dots, 0)$, $x_n = (u_1, 0, \dots, 0, 0, \dots, 0)$, $y_1 = (0, \dots, 0, z_n, u_1, u_2, \dots, u_{m-1}, 0)$, $y_2 = (0, \dots, 0, z_n, u_1, u_2, \dots, u_{m-2}, 0, 0), \dots, y_{m-2} = (0, \dots, 0, z_n, u_1, u_2, 0, \dots, 0)$, $y_{m-1} = (0, \dots, 0, z_n, u_1, 0, \dots, 0, 0)\}$, where $z_i \in A_{p_i}$ and u_i's are units.

Then the vertex x_1 dominates the set $Z(R,3) \setminus (P' \cup Q' \cup S')$, where $P' = \{(0, \dots, 0, b_1, b_2, \dots, b_{m-1}, b_m) : b_i = 0 \text{ or } b_i \in F_i^\times\}$, $S' = \{(a_1, \dots, a_{n-1}, a_n, 0, \dots, 0) : a_i \in A_{p_i^{\alpha_i - 1}}\}$ and $Q' = \{(a_1, \dots, a_{n-1}, a_n, 0, \dots, 0) :$ at least one $a_i = 0$ and at least two $a_i's \in A_{p_i^{\alpha_i - 1}}\}$. Also the vertex x_2 dominates the set $S' \cup \{(a_1, a_2, \dots, a_{n-1}, a_n, 0, \dots, 0) \in Q' : 0 \neq a_n \in A_{p_n^{\alpha_n - 1}}\}$ and vertices

$\{x_1, x_2\}$ do not dominate any elements in the set P' and $\{(a_1, a_2, \ldots, a_{n-1}, 0, 0, \ldots, 0) \in Q' : a_i \in A_{p_i^{\alpha_i - 1}}\}$. Proceeding like this, the vertex x_{n-1} dominates the set $\{(a_1, a_2, a_3, 0, \ldots, 0) \in Q' : 0 \neq a_3 \in A_{p_3^{\alpha_3 - 1}}\}$ and vertices $\{x_1, x_2, \ldots, x_{n-1}\}$ do not dominate any elements in the set P' and H, where $H = \{(a_1, a_2, 0, 0, \ldots, 0) \in Q' : a_i \in A_{p_i^{\alpha_i - 1}}\}$ but x_n dominates the set H. Now the vertex y_1 dominates the set $\{(0, \ldots, 0, b_1, \ldots, b_{m-1}, u_m) \in P' : u_m \in F_m^\times\}$ and the vertices $\{x_1, x_2, \ldots, x_n, y_1\}$ do not dominate any elements in the set $\{(0, \ldots, 0, b_1, b_2, \ldots, b_{m-1}, 0) \in P' : b_i = 0 \text{ or } b_i \in F_i^\times\}$. The vertex y_2 dominates the set $\{(0, \ldots, 0, b_1, b_2, \ldots, u_{m-1}, 0) \in P' : u_{m-1} \in F_{m-1}^\times\}$ and vertices $\{x_1, x_2, \ldots, x_n, y_1, y_2\}$ do not dominate any elements in the set $\{(0, \ldots, 0, b_1, b_2, \ldots, b_{m-2}, 0, 0) \in P' : b_i = 0 \text{ or } b_i \in F_i^\times\}$. Proceeding like this, the vertex y_{m-2} dominates the set $\{(0, \ldots, 0, b_1, b_2, u_3, 0 \ldots, 0) \in P' : u_3 \in F_3^\times\}$ and the vertices $\{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{m-2}\}$ do not dominate any elements in the set H', where $H' = \{(0, \ldots, 0, b_1, b_2, 0, \ldots, 0, 0) \in P' : b_i \in F_i^\times\}$ but y_{m-1} dominates the set H'. Hence D is the dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) \leq n + m - 1$.

Since $\mathcal{H}_3(\mathbb{Z}_3^n \times \mathbb{Z}_2^m)$ is a subhypergraph of $\mathcal{H}_3(R)$. By Theorem 2.17, $\gamma(\mathcal{H}_3(\mathbb{Z}_3^n \times \mathbb{Z}_2^m)) \geq n + m - 1$. Hence $\gamma(\mathcal{H}_3(R)) = n + m - 1$. □

Theorem 2.20. *Let $R = \mathbb{Z}_{p^n} \times F_1 \times F_2 \times \cdots \times F_m$ be a finite commutative non-local ring, where each F_j is field, p is prime, $n \geq 2$ and $m \geq 3$. Then $\gamma(\mathcal{H}_3(R)) = m - 1$.*

Proof. Consider the set $D = \{y_1, y_2, \ldots, y_{m-1}\}$, where $y_1 = (z, u_1, \ldots, u_{m-1}, 0)$,$y_2 = (z, u_1, \ldots, u_{m-2}, 0, u_m), \ldots, y_{m-2} = (z, u_1, u_2, 0, u_4, \ldots, u_m)$, $y_{m-1} = (z, u_1, 0, u_3, \ldots, u_m)$, where $z \in A_p$ and u_i's are units.

Then y_1 dominates the set $Z(R, 3) \setminus \{P \cup P'\}$, where $P = \{(0, b_1, \ldots, b_{m-1}, 0) : b_i = 0 \text{ or } b_i \in F_i^\times\}$ and $P' = \{(v, u_1, \ldots, u_{m-1}, 0) : v \in \mathbb{Z}_{p^n}^\times\ u_i \in F_i^\times\} \cup \{(z', 0 \ldots, 0, u_m) : z' \in A_{p^{n-1}},\ u_m \in F_m^\times\}$. Now the vertex y_2 dominates the set $\{(0, b_1, \ldots, b_{m-2}, u_{m-1}, 0) \in P : u_{m-1} \in F_{m-1}^\times\} \cup P'$ and the vertices $\{y_1, y_2\}$ does not dominate any elements in the set $\{(0, b_1, \ldots, b_{m-2}, 0, 0) \in P : b_i = 0 \text{ or } b_i \in F_i^\times\}$. The vertex y_3 dominates the set $\{(0, b_1, \ldots, b_{m-3}, u_{m-2}, 0, 0) \in P : u_{m-2} \in F_{m-2}^\times\}$ and $\{y_1, y_2, y_3\}$ does not dominate any elements in the set $\{(0, b_1, \ldots, b_{m-3}, 0, 0, 0) \in P : b_i = 0 \text{ or } b_i \in F_i^\times\}$. Proceeding like this, the vertex y_{m-2} dominates the set $\{(0, b_1, b_2, u_3, 0 \ldots, 0) \in P : u_3 \in F_3^\times\}$ and $\{y_1, y_2, \ldots, y_{m-2}\}$ does not dominate any elements in the set H, where $H = \{(0, b_1, b_2, 0, \ldots, 0, 0) \in P : b_i \in F_i^\times\}$ but y_{m-1} dominates the set H. Hence D is the dominating set of $\mathcal{H}_3(R)$ and so $\gamma(\mathcal{H}_3(R)) \leq m - 1$.

Since $\mathcal{H}_3(\mathbb{Z}_3 \times \mathbb{Z}_2^m)$ is a subhypergraph of $\mathcal{H}_3(R)$. By Theorem 2.17, $\gamma(\mathcal{H}_3(\mathbb{Z}_3 \times \mathbb{Z}_2^m)) \geq m - 1$. Hence $\gamma(\mathcal{H}_3(R)) = m - 1$. □

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