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A Matrix Form of Spectral Scaling in Quasi-Newton Algorithm

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Abstract In this work, a matrix form in spectral quasi-Newton algorithm of type BFGS, named $DH-SQN^{EI}$, is suggested. It is a diagonal matrix containing the proposed spectral parameter in BFGS direction along with an approximated Hessian matrix inverse, which is updated by own formula. This is done all to find more effective algorithms for accelerating the solutions in BFGS algorithm. The approximation of Hessian matrix may suffer from the ill-conditioned problem or initialized poorly. Beside the new algorithm, there are some properties which are studied to this proposed one. The analyses were conducted under suitable assumptions. Thoroughly, it provides the sufficient descent, global convergent and superlinear rate of convergence. A list of randomly 50 test functions with various dimensions offered the numerical results. It is concluded that $DH-SQN^{EI}$ algorithm is superior to the standard BFGS (QN^{BFGS}) as well as a type of spectral scaling BFGS algorithm (SNQ^{Ch}); by using Armijo line.

1 Introduction

Real life problems are issues that make scientist almost try to deal with solving them. There are many approaches in mathematics to evaluate these models, in which optimization is one of them. Unconstrained optimization problem is an active part in modeling problems.
Now, in minimization case, consider an unconstrained model with twice differentiable objective function, $f:\mathcal{R}^n\to\mathcal{R}$, and bounded below given as

$$\min f\left(X\right), \quad X\in\mathcal{R}^n \tag{1.1}$$

Numerous numerical methods are invited to solving problem (1.1). Quasi-Newton (QN) methods are among the recommended methods due to the efficiency of the methods, which is convergence globally with super-linear rate. In numerical optimization, there are many steps or iteration to obtain the desired solution. It means for minimizing (1.1),

$$x_{k+1}=x_k+\alpha_k d_k, \quad k=0,1,\dots \tag{1.2}$$

for the line search $\alpha_k\in\mathcal{R}$ and the direction search $d_k\in\mathcal{R}^n$. In quasi-Newton approaches, d_k is defined by

$$d_k=-B_k^{-1}g_k \tag{1.3}$$

where, g_k is gradient and B_k^{-1} is the inverse of Hessian matrix the objective function at x_k. Among most suitable procedures of QN, there is BFGS method, which is described as an effective approach (for more details, see chapter 11 from [1]). BFGS is an abbreviated form to the name of four scientists named: Broyden, Fletcher, Goldfrab and Shanno; [2],[3],[4] and [5]. In this algorithm, the approximation of the Hessian matrix, usually it denotes by B_k, which is used in optimizing problems. This approximated amount starts with any positive defined matrix B_0, usually identity matrix, and then it is updated with iterations maintaining the positive definedness. The method used the following formula

$$B_{k+1}=B_k-\frac{B_ks_ks_k^{'}B_k}{s_k^{'}B_ks_k}+\frac{y_ky_k^{'}}{s_k^{'}y_k} \tag{1.4}$$

where,

$$s_k = x_k - x_{k-1} \quad , \quad y_k = g_k - g_{k-1} \tag{1.5}$$

However, the formula (1.4) is for approximation Hessian matrix and for finding the direction, an inverse is needed. Therefore, the updated inverse of Hessian matrix for BFGS is given by

$$H_{k+1} = H_k + \frac{(s_k + H_k y_k)^{'} y_k s_k s_k^{'}}{(s_k^{'} y_k)^2} - \frac{H_k y_k s_k^{'} + s_k y_k^{'} H_k}{s_k^{'} y_k} \tag{1.6}$$

for s_k and y_k are defined in (1.5) and H_k is the approximation of Hessian matrix inverse [6]. Moreover, BFGS has a good self-correcting property, that steps make the iterations run slowly results from improperly estimated of H_k or poorly chose of initial matrix H_0 (see [7] page 200). For this property, Biggs [8] developed a self-scaling parameter using non-quadratic function.

A new quasi-Newton equation is a problem that many researchers deal with, see [9] and [10]. Adding to this, there is a new class for QN methods with non-monotonic line search proposed by Latif and Hamko [11]. This is the result of the importance of QN and its equation in driving more new effective algorithms.

Now, the spectral scaling is our area of interest. In 2010, Cheng and Li [12] proposed a spectral scaling of BFGS which has the property of self-correcting and more effective than conventional BFGS. Nakayama et al. [13] proposed a memoryless QN symmetry rank-1 formula using a spectral scaling parameter of [12] to be sufficient descent property to tackle the problem of large. Nakayama and Narushima [14] prove the global convergence for that formula, which is based on spectral scaling of [12]. At last, Nakayama [15] had been hybridizing the designed formula of rank-1 memoryless QN by parameter [12] with three-term CG. Also, Lv et al. [16] showed the efficient of memoryless BFGS algorithm with one-parameter scaling formula of Cheng and Li and proved that this parameter is minimizing all eigenvalues of the formula given by them. As more expansion, Bidabadi [17] began to involve the spectral scaling approaches of BFGS on constrained problem starting with nonlinear least square problems. Chen and Cheng [18] extend spectral scaling BFGS ideas to the Broyden with quadratic termination. The reached algorithms were scaled with one parameter; however, Andrei [19] introduced a BFGS spectral scaling using double parameters. He proved that his method is more efficient than other scaled formula and standard BFGS. On the other hand, conjugate gradient (CG) methods take a wide space of this topic, since CG does not need a large storage for matrices in evaluation; this makes researchers to have more findings in this area. The first idea was done by Barzilai and Borwein [20]. Li et al. [10] found spectral CG with using QN direction and equation, it was spectral scaling parameter and three-term modified Polak–Ribière–Polyak (PRP); which showed the superiority of their algorithm from the generated CG. Furthermore, Al-Naemi and Sheekoo[21] utilized the gradient coefficient in designing new spectral scaling for non-linear CG in large-scale problems with line search satisfying strong Wolf's condition. Spectral parameter nested with a CG one was new designed given by Wang et al.[22] and they saw its usefulness for large-scale problem solving. Another step,[23] proposed a new spectral CG described as a fast algorithm; which it is used the new direction combined spectral parameter with the previous direction nested in it. In 2022, [24], there was a new idea in spectral scaling where, authors used a convex combination between two CG coefficients and presented as a spectral parameter. In addition, they prove the global convergence of their method. Again for constrained optimization with boundedness condition, Nakayama et al. [25] presented a QN method with property memoryless derived from spectral scaling of Broydon class that combined with modified Yuan-Lu's memoryless algorithm using Armijo line search. Finally, a real live problem takes spectral algorithms as a tool for analyzing them. For instance, the data on the drug abuse analysis used this type of algorithm; see [26].

With all above workings, especially the spectral parameter of [12] and all recent works about it, we suggest a spectral parameter in a matrix form adding to the approximation of inverse of Hessian matrix to accelerate the solution. The Hessian matrix approximation may suffer from the ill-conditioned problem and make iterations run slowly that is from a large condition number to that matrix. Sometimes, poor selection of the initial matrix to the approximate Hessian matrix may slow down reaching the solution. This makes more work to do with the Hessian approximation. Therefore, in this paper, we think of a parameter to add to the inverse Hessian approximation matrix. It is of matrix form to add it only to the diagonal elements of $\mathrm{H_k}$. It is important to select a matrix to maintain the positive definedness property in minimum type of problem. It is setting in the direction to make it in the QN direction for its effectiveness among other methods.

2 The Proposed Algorithm

In this work, a new idea is used in spectral QN algorithm of type BFGS. In the search direction, there is a new term containing the spectral scaling parameter presented. The parameter is gathered with the main diagonal of H_k in a QN direction of type BFGS. In other words, we use the matrix form used outside the updating formula of Hessian inverse matrix. Thus, if I_n is identity matrix with n representing the dimension of objective function, then the proposed term is given by

$$M_k = \left(\frac{y_k^T s_k}{2 \|y_k\|^2} \right)^{1/2} I_n \tag{2.1}$$

We suggest the function of the minimizer of all eigenvalues of the spectral scaling formula as it is given in [16]. In more details, it is the function that contains $\frac{y_k^T s_k}{\|y_k\|^2}$. In addition, the positive definiteness makes it always positive.
Then, the direction search would be as follows

$$d_k = \begin{cases} -H_k g_k & k = 0 \\ -(H_k + M_k) g_k & k \geq 1 \end{cases} \tag{2.2}$$

Therefore, the algorithm steps are given as follows:

STEP 1: Give initial values as: H_0 identity matrix or any positive definite matrix, $X_0 \in \mathcal{R}^n$ initial point to f; and inaccuracy $\epsilon = 1 \times 10^{-7}$

STEP 2: Begin from $d_0 = -H_0 g_0$, then $k = 1$

STEP 3: Criteria for stopping:$\|g_k\| \leq \epsilon$, $\|x_{k+1} - x_k\| < \epsilon \|x_k\|$, or dimension of f; n times 1000 represented as a maximum number of iterations.

STEP 4: Find α_k inexact step length which satisfies strong Wolfe conditions.

STEP 5: Calculate s_k and y_k from (1.5)

STEP 6: Update H_k by (1.6) and M_k by (2.1)

STEP 7: Find the search direction $\mathrm{d_k}$ by (2.2)

STEP 8: Calculate $\mathrm{x_{k+1}}=\mathrm{x_k}+\alpha_\mathrm{k}\mathrm{d_k}$, where α_k is Armijo line search.

STEP 9: Put $\mathrm{k} = \mathrm{k} + 1,$ and return to **STEP 3**.

3 The Algorithm Convergence Analysis

The convergence analysis of our algorithm is discussed in this part.

3.1 Assumptions

The analysis of convergence needs a list of assumptions. These are imperative for gaining seamless pattern in solving problems.

(i) An objective function f is continuous with differentiability twice.

(ii) Hessian matrix of f $\;(Hess(f\,(x))$) meets the Lipschitz continuous properties, that is

$$\|Hess(f\,(x)) - Hess(f\,(x^*))\| \leq c\,\|x - x^*\|$$

where, $c > 0; \quad c \in \mathcal{R}$ and $\forall x \in N(x^*)$; in which $N(x^*)$ is neighborhood of x^*

(iii) For a set of convex level of twice continuous objective function $V = \{x : f(x) \leq f(x_0)\}$, $\exists c_1, c_2 \in \mathcal{R}$ such that $c_1\|z\|^2 \leq z^{'} Hess(f\,(x))z \;\leq c_2\|z\|^2 \quad \forall\, z \in \mathcal{R}^n$, $x \in V$ and $Hess(f\,(x))$ is Hessian matrix of f.

3.2 Sufficient Descent Condition

In this section, to show that proposed algorithm fulfills the descent property, it is enough to show the positive definedness of the matrix that added to the direction. So, by the definition; any symmetric matrix is positive defined it is satisfying the quadratic form. Therefore, let z be any nonzero vector in $\mathcal{R}^n$, then

$$z\left(H_k+M_k\right)z^{'}=z\left(H_k+\left(\frac{y_k^T s_k}{2\left\|y_k\right\|^2}\right)^{1/2} I_n\right)z^{'}$$

$$=zH_k z^{'}+\left(\frac{y_k^T s_k}{2\left\|y_k\right\|^2}\right)^{1/2} zI_n z^{'}$$

but, H_k is positive defined, then $zH_k z^{'}>0$, also $zI_n\ z^{'}>0$; and $\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2}>0$
Hence, (H_k+M_k) is a positive defined matrix. Now, in sufficient descent condition; for $k=0$, it is obvious that it holds. And other step,

$$g_k^T d_k=-g_k^T\left(H_k+M_k\right)g_k$$

but the matrix (H_k+M_k) is proved to be positive defined, and $g_k$nonzero vector, then

$$g_k^T\left(H_k+M_k\right)g_k>0, k\geq 1$$

Hence, the direction has descent direction.

3.3 The Global Convergence Analysis

The section presents the global convergence of the proposed algorithm. It is proved with the usage of the Armijo line search with descent property is satisfies one of them or both

$$f\left(x_{k+1}\right)-f\left(x_k\right)\leq -c_1\frac{\left(g_k^T d_k\right)^2}{\left\|d_k\right\|^2} \tag{3.1}$$

$$f\left(x_{k+1}\right)-f\left(x_k\right)\leq -c_2\ g_k^T d_k \tag{3.2}$$

where c_1 and c_2 any positive constant, see Lemma 7 in [27].

Theorem 3.1. *Let assumption 1 hold, search direction d_k is given by (2.2) satisfied descent condition, with Armijo line search is Then,*

Proof. The technique of contradiction is used to prove this. Suppose that δ is positive constant with $\|g_k\|>\delta$, $k\geq 1$; then

$$d_k=-\left(H_k+M_k\right)g_k$$

where $\mathrm{H_k}$ is defined by (1.6) and $\mathrm{M_k}$ by (2.1).
So,

$$\left\|d_k\right\|=\left\|-\left(H_k+M_k\right)\ g_k\right\|\leq\left\|H_k+M_k\right\|\left\|g_k\right\|$$

$$\frac{\left\|d_k\right\|}{\left\|g_k\right\|^2}\leq\frac{\left\|H_k+M_k\right\|}{\left\|g_k\right\|}$$

With the boundedness of Hessian matrix inverse approximation H_k, we have $\|H_k\|<z$. Also, $M_k$is bounded, since the quantity $\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2}$ is a function of $\frac{y_k^T s_k}{\|y_k\|^2}$which is one of the two presented step size in [20], then $\|M_k\|<u$.
Therefore;

$$\frac{\left\|d_k\right\|}{\left\|g_k\right\|^2}\leq\frac{z\ u}{\left\|g_k\right\|}<\frac{z\ u}{\delta}$$

Now, all amounts are positive, then by squaring both sides we get the following

$$\frac{\|g_k\|^4}{\|d_k\|^2} \geq \frac{\delta^2}{z^2 u^2}$$

and,

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} \geq \infty$$

Here, with the sufficient descent property, $g_k^T d_k < -\|g_k\|^2$, hence

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} < \sum_{k=0}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2}$$

Then this result is contradicting to (3.1) and the sufficient descent.

Therefore, the algorithm is global convergence which means

$$\lim_{k\to\infty} inf \, \|g_k\| \; = 0$$

3.4 Superlinear Rate of Convergence

The super linear rate of convergence is proved in this section. There is a need to recall Lemma 4.9 and Lemma 4.10 of [28]. In details, suppose that f is continues and differentiable twice. There are two formulae that tend to the boundedness of each approximation Hessian matrix and its inverse $\frac{\|y_k - Hess(f(x^*))s_k\|}{\|s_k\|} \leq \varepsilon_k$ with $\sum_{k=1}^{\infty} \varepsilon_k < \infty$
where, y_k and s_k are given in (1.5) and $\{\varepsilon_k\}$ is a sequence of constants.

$$\lim_{k\to\infty} \frac{\left\| \left(H_k^{-1} - Hess(f(x^*)) \right) s_k \right\|}{\|s_k\|} = 0 \tag{3.3}$$

Theorem 3.2. *Assume that the (i) and (iii) holds in list of assumptions 3.1, and the sequence of solutions obtained in each steps is convergent, that is $\{x_k\} \to x^*$ and also the sequence of Hessian matrix approximation and its inverse are bounded; $\{H_k\}$ and $\left\{H_k^{-1}\right\}$. If*

$$\lim_{k\to\infty} \frac{\left\| \left(H_k^{-1} - Hess(f(x^*)) \right) d_k \right\|}{\|d_k\|} = 0$$

,
for $x_{k+1} = x_k + d_k$, then

$$\lim_{k\to\infty} \frac{\|x_{k+1} - x^*\|}{\|x_{k+1} - x_k\|} = 0$$

Proof.

$$\begin{aligned}
\left[H_k^{-1} - Hess(f(x^*))\right] d_k &= H_k^{-1}\left(-H_k - M_k\right) g_k - Hess(f(x^*))d_k \\
&= \left(-g_k - H_k^{-1} M_k g_k\right) - Hess(f(x^*))d_k \\
&= -\left(I_n + H_k^{-1} M_k\right) g_k - Hess(f(x^*))d_k \\
&= -\left(I_n + H_k^{-1} M_k\right) g_k + \left(I_n + H_k^{-1} M_k\right) Hess(f(x^*))d_k \\
&\quad - \left(I_n + H_k^{-1} M_k\right) Hess(f(x^*))d_k - Hess(f(x^*))d_k \\
&= -\left(I_n + H_k^{-1} M_k\right) \left[g_k - Hess(f(x^*))d_k\right] - \left(I_n + H_k^{-1} M_k - I_n\right) Hess(f(x^*))d_k
\end{aligned}$$

$$= -\left(I_n + H_k^{-1} M_k\right) g_{k+1} - H_k^{-1} M_k Hess(f\left(x^*\right)) d_k + o(\|d_k\|)$$

So that we obtain,

$$\left\| \left(H_k^{-1} - Hess(f\left(x^*\right))\right) d_k \right\| = \left(1 + O\left(1\right)\right) \|g_{k+1}\| + o(\|d_k\|)$$

This means that,

$$\lim_{k \to \infty} \frac{\left\| \left(H_k^{-1} - Hess(f\left(x^*\right))\right) d_k \right\|}{\|d_k\|} = 0$$

so that,

$$\lim_{k \to \infty} \frac{\|g_{k+1}\|}{\|x_{k+1} - x_k\|} = 0$$

However, with $Hess(f\left(x^*\right)) = 0$ and

$$g_{k+1} - Hess(f\left(x^*\right)) - Hess(f\left(x^*\right))\left(x_{k+1} - x^*\right) = o\left(\|x_{k+1} - x^*\|\right)$$

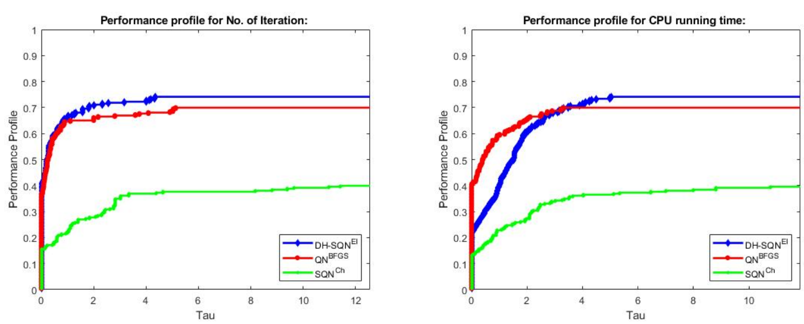
Hence, we get;

$$\lim_{k \to \infty} \frac{\|x_{k+1} - x^*\|}{\|x_{k+1} - x_k\|} = 0$$

.

4 Numerical Discussions and Results

In this part, the result and all finding will present. In the work, our proposed algorithm is in race with two other algorithms to find more efficient methods among them. One of them is the standard QN method of type BFGS and the other is the spectral QN algorithm, that presented by Cheng and Li [12]. The plot of performance profile (PP-Plot) is taken to show the outcomes of the contest. This plot is suggested by [29]. It is a cumulative distribution function, which is a curve over the probability values. The good point it is only present the significant variation among the approaches. Two criterions are involved number of iteration and the time of running the central processor unit (CPU). A set of 50 test functions are engaged along with various dimensions to those are multivariate functions. The Table 1 presents the name of the whole test problems, they are collected from [30] and [31]. There is also a column which shows the initial points of starting functions. And at the last, the total of our run data become 263 data, in which they used to plot the PP-Plot.



(a) Number of Iterations (b) CPU Running Time

Figure 1: Performance Profile Curves

The Figure 1 reveals an act of PP-Plot for three participated algorithms. The proposed algorithm $DH - SQN^{EI}$ takes the first position in terms of iteration numbers and CPU time running. It is the fact that not all algorithms are finding the solutions exactly, but our algorithm is more effective in solving problems according to this list of functions than others.

The functions are randomly selected with various dimensions. Some big dimensions also are taken as 100, 200, 300, and 500 as shown in Table 1. The suggested algorithm again was in the first in comparison to other two algorithms. The QN^{BFGS}algorithm was followed our methods but $SQN^{Ch}$was in the last of the contest. In running programs, the MATLAB 2018a is used and all codes were written with it. In the algorithms, all used inexact step length with Armijo condition. Moreover, the termination of all algorithms was with accuracy $\epsilon = 1 \times 10^{-7}$, $\|x_{k+1} - x_k\| < \epsilon \|x_k\|$ or the dimension of function times 1000. At last, note that; the last column of the Table 1 is intial values for the test functions with using the notation $\otimes$ for the Kronecker product and 1_n for column of ones.

Table 1: List of Test Functions with Used Dimension and Intial Values

ID	**Test Function Name**	**Dimension**	**Initial Point**
1	Zirilli_2	n=2	X_0 =(-1,2)
2	Aluffi Pentini	n=2	X_0 = (-10,10)
3	Brent	n=2	X_0 = (0,0)
4	Ursem_1	n=2	X_0 = (-0.5,2)
5	Jennrich Sampson	n=2	X_0 = (0.15,0.15)
6	Keane	n=2	X_0 = (0,1)
7	Zettl	n=2	X_0 = (0,0)
8	Zakharov	n=2	X_0 = (1,10)
9	ARWHEAD	n=2	X_0 = (1,1)
10	Camel & Six Hump	n=2	X_0 = (1,4)
11	Camel Three Hump	n=2	X_0 = (3.5,4.5)
12	Wolfe Schwefel	n=3	X_0 = (0,1,2)
13	Extended DENSCHNB	n=2,4,10	$X_0 = -0.5 * 1_n$
14	Sphere	n=2,4,6,8,10	$X_0 = 1_n$
15	Extended Freudenstein Roth	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (0.5, -2)$
16	Extended DENSCHNF	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (2, 0)$
17	Cliff	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (0, -1)$
18	CUBE	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (-1.2, 1)$
19	Extended White & Holst	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (-1.2, 1)$
20	Extended Beale	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (1, 0.8)$
21	Generalized Tridiagonal 1	n=1,2,3,4,10,15	$X_0 = 1_n \otimes (0.5, -2)$
22	Extended Tridiagonal 1	n=2,4,6,8,20,30	$X_0 = 1_n$
23	Diagonal 5	n=2,4,6,8,20,30	$X_0 = 1_n$
24	Diagonal 6	n=2,4,6,8,20,30	$X_0 = 1_n$
25	Diagonal 7	n=2,4,6,8,20,30	$X_0 = 1_n$
26	Diagonal 8	n=2,4,6,8,20,30	$X_0 = 1_n$
27	Raydan 1	n=2,4,6,8,20,30	$X_0 = 1_n$
28	Extended DENSCHNB	n=2,4,6,8,20,30	$X_0 = 1_n$
29	COSINE	n=2,4,6,8,20,30	$X_0 = 1_n$
30	Full_Hessian_FH3	n=2,4,6,8,20,30	$X_0 = 1_n$
31	Quartc	n=2,4,6,8,20,30	$X_0 = 2 * 1_n$
32	HIMMELH	n=2,4,6,8,20,30	$X_0 = 1.5 * 1_n$
33	HIMMELBG	n=2,4,6,8,20,30	$X_0 = 1.5 * 1_n$
34	LIARWHD	n=2,4,6,8,20,30	$X_0 = 4 * 1_n$
35	NONSCOMP	n=2,4,6,8,20,30	$X_0 = 3 * 1_n$

36	Extended Penalty	n=2,4,6,8,20,30	$X_0 = (1, \ldots, n)$
37	Ex_BD1_Block_Diagonal	n=2,4,6,8,20,30	$X_0 = 1.5 * 1_n$
38	POWER	n=2,4,6,8,20,30	$X_0 = 1_n$
39	DQDRTIC	n=2,4,6,8,20,30	$X_0 = 3 * 1_n$
40	SINE	n=2,8,20,30,300	$X_0 = 1_n$
41	Schwefel_2_4	n=1,2,3,4,10,15,50,100, 150	$X_0 = 1_n \otimes (0, 10)$
42	Almost Perturbed Quadratic	n=1,2,3,4,10,15,50,100, 150, 250	$X_0 = 1_n \otimes (0, 1)$
43	Raydan 2	n=2,4,6,8,20,30,100,200, 300	$X_0 = 1_n$
44	Generalized Quartic	n=2,4,6,8,20,30,100,200, 300	$X_0 = 1_n$
45	Hager	n=2,4,6,8,20,30,100,200, 300	$X_0 = 1_n$
46	Diagonal 2	n=2,4,10,100,200, 300,500	$X_0 = (1/n) * 1_n$
47	Diagonal 3	n=2,4,6,8,10,15,200, 300,500	$X_0 = 1_n$
48	Dixon Price	n=2,4,6,8,20,30,100,200, 300,500	$X_0 = 1_n$
49	Perturbed Quatratic	n=2,4,6,8,20,30,100,200, 300,500	$X_0 = 0.1 * 1_n$
50	ENGVAL1	n=2,4,6,8,20,30,100,200, 300,500	$X_0 = 2 * 1_n$

5 Conclusions

Spectral scaling is one of interesting area for many researchers. In this work, the new idea was proposed in this field. The suggested algorithm present the spectral parameter as a matrix form adding to the Hessian matrix approximation formula in the last step to accelerate obtaining the solution faster in consuming times by CPU and less number of iterations for the BFGS type of QN with Armijo line search. This was the new algorithm $DH - SQN^{EI}$. The $DH - SQN^{EI}$ algorithm had a descent property with the convergence globally and superlinear rate of it. Practically, there was a competition among three algorithms, $DH - SQN^{EI}$; QN^{BFGS} and the last one was SQN^{CH} for less iteration numbers and faster in reaching the solution. The outcomes show the improvements in BFGS method with $DH - SQN^{EI}$. In other word, all results reveal how $DH - SQN^{EI}$was more plentiful than both standard BFGS and SQN^{CH}in terms of the two mentioned comparison criterions.

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