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ON SOFT NANO SEMI GENERALIZED CLOSED SETS IN SOFT NANO TOPOLOGICAL SPACES AND APPLICATION IN DECISION MAKING PROBLEMS

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Abstract In this paper, the notion of soft nano semi generalized closed sets in soft nano topological spaces is introduced. Also, the related properties are studied. The concept of multi criteria group decision making using soft nano topology is discussed by taking a real world problem and the concept of similarity of soft sets is used to refine the result obtained.

1 Introduction

Soft set theory is initiated by Molodtsov [9] in 1999 to deal with the problems of uncertainties arising while dealing with real world problems. Further, Shabir and Naz [10], Sabir and Bashir [7] and Naim Cagman et al [5] have continued the study of properties of soft sets and soft topological spaces. The notion of Nano topology was introduced by Lellis Thivagar [11]. Based on that, Benchalli et al [3] introduced the notion of soft nano topological spaces using soft set equivalence relation on the universal set. Also, the notion of soft nano continuity and weaker forms of soft nano open sets and weaker forms of soft nano continuous functions in soft nano topological spaces are introduced and studied in [2], [4]. F.Feng [6] has discussed soft set based group decision making. He has studied the application of soft rough approximations in multi criteria group decision making under uncertainty.The concept of similarity measure of soft sets is proposed by P.Majumadar and S.K.Samanta [8]. In the present paper, the notion of soft nano semi generalized closed sets in soft nano topological spaces is introduced. Also, the related properties are studied. Further, based on the work of F.Feng [6] and P.Majumdar and S.K.Samanta [8], the concept of multi criteria group decision making using soft nano topology is discussed by taking a real world problem and the concept of similarity of soft sets is used to refine the result obtained.

2 Preliminaries

Definition 2.1. [9]: Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \to P(U)$.
In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A) . Clearly, a soft set need not be a set.

Definition 2.2. [1]: Let R be a relation on (F, A), then
(i) R is reflexive if $H_1(a, a) \in R, \forall a \in A$
(ii) R is symmetric if $H_1(a, b) \in R \Rightarrow H_1(b, a) \in R, \forall (a, b) \in AXA$
(iii) R is transitive if $H_1(a, b) \in R, H_1(b, c) \in R \Rightarrow H_1(a, c) \in R, \forall a, b, c \in A$

Definition 2.3. [1] : A soft set relation R on a soft set (F, A) is called an equivalence relation if it is reflexive, symmetric and transitive.

Definition 2.4. [3] Let U be a non-empty finite set of objects called the universe and E be a set of parameters. Let R be a soft equivalence relation on U. The triplet (U, R, E) is said to be the soft approximation space. Let $X \subseteq U$.
(i) The soft lower approximation of X with respect to R and the set of parameters E is $(L_R(X), E) = \cup\{R(x) : R(x) \subseteq X\}$
(ii) The soft upper approximation of X with respect to R and the set of parameters E is $(U_R(X), E) = \cup\{R(x) : R(x) \cap X \neq \phi\}$
(iii) The soft boundary region of X with respect to R and the set of parameters E is $(B_R(X), E) = (U_R(X), E) \setminus (L_R(X), E)$

Definition 2.5. [3] Let U be a non-empty universal set and E be a set of parameters. Let R be a soft equivalence relation on U. Let $X \subseteq U$.
Let $(\tau_R(X), U, E) = \{U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)\}$. Then,
$(\tau_R(X), U, E)$ is a soft topology on (U, E), called as the Soft nano topology with respect to X.

Definition 2.6. [8] If $E_1 = E_2$ then similarity between (F_1, E_1) and (F_2, E_2) is defined by $S(F_1, F_2) = \frac{\sum_i \overrightarrow{F_1}(e_i).\overrightarrow{F_2}(e_i)}{\sum_i [\overrightarrow{F_1}(e_i)^2 \bigcup \overrightarrow{F_2}(e_i)^2]}$

3 Soft nano semi generalized closed sets

Here we extend the concept of soft nano generalized closed sets to soft nano semi generalized closed sets that are independent of soft nano generalized closed sets.

Definition 3.1. Let $(\tau_R(X), U, E)$ be a soft nano topological space and (A, E) be a soft subset. Then, (A, E) is called a soft nano semi generalized closed set (SNsg-closed) if $SNscl(A, E) \subseteq (G, E)$, whenever $(G, E) \in SNSO(X, E)$ and $(A, E) \subseteq (G, E)$.
The complement of SNsg-closed set is called soft nano semi generalized open (SNsg-open) set.

Definition 3.2. The smallest SNsg-closed set containing a soft set (A, E) is called the SNsg-closure and is denoted by $SNsgcl(A, E)$.
The largest SNsg-open set contained in (A, E) is called the SNsg-interior of (A, E) and is denoted by $SNsgint(A, E)$.

Example 3.3. Let $U = \{a, b, c, d\}, E = \{m_1, m_2, m_3\}$ and $X = \{a, b\} \subseteq U$ with $U/R = \{F(m_1), F(m_2), F(m_3)\} = \{\{a\}, \{c\}, \{b, d\}\}$. Then,
$(L_R(X), E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$
$(U_R(X), E) = \{(m_1, \{a, b, d\}), (m_2, \{a, b, d\}), (m_3, \{a, b, d\})\}$
$(B_R(X), E) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}$
Now, $\{\tau_R(X), U, E\} = \{U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)\}$ is a soft nano topology on U.
Here soft nano open sets are $U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)$
Soft nano closed sets are $U, \phi, (L_R(X), E)', (U_R(X), E)', (B_R(X), E)'$
Soft nano generalized-closed sets are $U, \phi, (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E), (B_6, E), (B_7, E)$
where $(B_1, E) = \{(m_1, \{c\}), (m_2, \{c\}), (m_3, \{c\})\}$
$(B_2, E) = \{(m_1, \{a, c\}), (m_2, \{a, c\}), (m_3, \{a, c\})\}$
$(B_3, E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$
$(B_4, E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$
$(B_5, E) = \{(m_1, \{b, c, d\}), (m_2, \{b, c, d\}), (m_3, \{b, c, d\})\}$
$(B_6, E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$
Soft nano semi generalized-closed sets are $U, \phi, (C_1, E), (C_2, E), (C_3, E), (C_4, E), (C_5, E), (C_6, E), (C_7, E), (C_8, E), (C_9, E), (C_{10}, E), (C_{11}, E)$
where $(C_1, E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$
$(C_2, E) = \{(m_1, \{b\}), (m_2, \{b\}), (m_3, \{b\})\}$
$(C_3, E) = \{(m_1, \{c\}), (m_2, \{c\}), (m_3, \{c\})\}$
$(C_4, E) = \{(m_1, \{d\}), (m_2, \{d\}), (m_3, \{d\})\}$
$(C_5, E) = \{(m_1, \{a, c\}), (m_2, \{a, c\}), (m_3, \{a, c\})\}$
$(C_6, E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$

$(C_7, E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$
$(C_8, E) = \{(m_1, \{b, c, d\}), (m_2, \{b, c, d\}), (m_3, \{b, c, d\})\}$
Now let us consider (i) $(A, E) = \{(m_1, \{a, c\}), (m_2, \{a, c\}), (m_3, \{a, c\})\}$. It can be verified that the soft set (A, E) is SNsg-closed set. Also, it is SNg-closed set.
(ii)$(A, E) = \{(m_1, \{d\}), (m_2, \{d\}), (m_3, \{d\})\}$. It can be verified that the soft set (A, E) is SNsg-closed set but not SNg-closed set.
(iii) $(A, E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$. It can be verified that the soft set (A, E) is SNg-closed set but not SNsg-closed set.

Remark 3.4. Example 3.3 shows that the families of soft nano semi generalized closed sets and soft nano generalized closed sets are independent.

Theorem 3.5. *Every soft nano semi closed set is soft nano semi generalized closed set.*

Proof : Let (A, E) be a soft nano semi closed set in $(\tau_R(X), U, E)$ and (G, E) be a soft nano semi open set over U such that $(A, E) \subseteq (G, E)$ and hence $SNscl(A, E) = (A, E) \subseteq (G, E)$. By definition, (A, E) is soft nano semi generalized closed set.

Remark 3.6. The converse of the above theorem is not true.
Let $U = \{a, b, c, d\}, E = \{m_1, m_2, m_3\}$ and $X = \{a, b\} \subseteq U$ with $U/R = \{F(m_1), F(m_2), F(m_3)\} = \{\{a\}, \{d\}, \{b, c\}\}$.
Then, $(L_R(X), E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$
$(U_R(X), E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$
$(B_R(X), E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$
Now, $(\tau_R(X), U, E) = \{U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)\}$ is a soft nano topology on U.
Here soft nano open sets are $U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)$
Soft nano closed sets are $U, \phi, (L_R(X), E)', (U_R(X), E)', (B_R(X), E)'$
Soft nano semi open sets are $U, \phi, (B_1, E), (B_2, E), (B_3, E), (B_4, E)$
where $(B_1, E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$
$(B_2, E) = \{(m_1, \{a, d\}), (m_2, \{a, d\}), (m_3, \{a, d\})\}$
$(B_3, E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$
$(B_4, E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$
Soft nano semi generalized-closed sets are
$U, \phi, (C_1, E), (C_2, E), (C_3, E), (C_4, E), (C_5, E), (C_6, E), (C_7, E), (C_8, E)$
where $(C_1, E) = \{(m_1, \{b\}), (m_2, \{b\}), (m_3, \{b\})\}$
$(C_2, E) = \{(m_1, \{d\}), (m_2, \{d\}), (m_3, \{d\})\}$
$(C_3, E) = \{(m_1, \{a, d\}), (m_2, \{a, d\}), (m_3, \{a, d\})\}$
$(C_4, E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$
$(C_5, E) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}$
$(C_6, E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$
$(C_7, E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$
$(C_8, E) = \{(m_1, \{b, c, d\}), (m_2, \{b, c, d\}), (m_3, \{b, c, d\})\}$
Now, consider $(C_1, E) = \{(m_1, \{b\}), (m_2, \{b\}), (m_3, \{b\})\}$, which is a soft nano semi generalized closed set but not soft nano semi closed set.

Theorem 3.7. *A soft set (A, E) is SNsg-closed over U if and only if $SNscl(A, E) - (A, E)$ has no non empty soft nano semi closed set.*

Proof : Let (A, E) be a SNsg-closed set and (F, E) be a soft nano semi closed subset of $SNscl(A, E) - (A, E)$. Then, $(SNscl(A, E) - (A, E))' \subseteq (F, E)'$ and $(F, E)'$ is soft nano semi-open set. That is $(SNscl(A, E) \cap (A, E)')' \subseteq (F, E)'$. Therefore, $(A, E) \cup (SNsint(A, E)') \subseteq (F, E)'$. Thus, $(F, E)'$ is soft nano semi-open and $(A, E) \subseteq (F, E)'$. Since (A, E) is SNsg-closed, $SNscl(A, E) \subseteq (F, E)'$. Thus, $(F, E) \subseteq (SNscl(A, E))'$. Hence, $(F, E) \subseteq (SNscl(A, E)) \cap (SNscl(A, E))' = \phi$. Therefore, $(F, E) = \phi$. That is,$SNscl(A, E) - (A, E)$ has no non empty soft nano semi closed set.
Conversely, let $SNscl(A, E) - (A, E)$ has no non empty soft nano semi closed set. Let (G, E) be a soft nano semi open set over U such that $(A, E) \subset (G, E)$. If $SNscl(A, E) \nsubseteq (G, E)$, then $SNscl(A, E) \cap (G, E)' \neq \phi$. Also, $SNscl(A, E) \cap (G, E)' \subseteq SNscl(A, E) - (A, E)$, since

$(A,E) \subseteq (G,E)$. Thus, $SNscl(A,E) \cap (G,E)'$ is a non empty soft nano semi closed subset of $SNscl(A,E) - (A,E)$, which is a contradiction. Thus, $SNscl(A,E) \subseteq (G,E)$, whenever (G,E) is a soft nano semi open set and $(A,E) \subseteq (G,E)$. Hence, (A,E) is a soft nano semi generalized closed (SNsg-closed) set over U.

In the following theorem, the condition under which a soft nano semi generalized closed set is soft nano semi closed is discussed.

Theorem 3.8. *Let (A,E) be a SNsg-closed set. Then, (A,E) is soft nano semi closed set if and only if $SNscl(A,E) - (A,E)$ is soft nano semi closed set.*

Proof : Proof is straight forward.

The various cases of soft approximations for soft nano semi generalized closed sets are discussed below.

Theorem 3.9. *If $L_R(X,E) = U_R(X,E)$ in a soft nano topological space $(\tau_R(X), U, E)$, then any soft sets $(A,E) \subseteq [L_R(X,E)]'$ and $[L_R(X,E)]' \cup (B,E)$ where $(B,E) \subseteq L_R(X,E)$ are the only soft nano semi generalized closed sets over U.*

Proof : If $L_R(X,E) = U_R(X,E)$,then $(\tau_R(X), U, E) = \{U, \phi, L_R(X,E)\}$. Then, ϕ and any soft set $(A,E) \supseteq L_R(X,E)$ are the only soft nano semi open set over U. If $(A,E) \subseteq L_R(X,E)$, then $SNscl(A,E) = U$ and the soft nano semi open sets containing (A,E) are those soft sets (B,E) for which $L_R(X,E) \subseteq (B,E)$. Then, $SNscl(A,E) \subseteq (G,E)$, for every soft nano semi open set (G,E) such that $(A,E) \subseteq (G,E)$. Hence, (A,E) is not SNsg-closed set. If $(A,E) \subseteq [L_R(X,E)]'$, then $SNscl(A,E) = (A,E)$, since any soft subset of $[L_R(X,E)]'$ is soft nano semi closed over U. Thus, $SNscl(A,E) = (A,E) \subseteq (G,E)$, whenever (G,E) is soft nano semi open set and $(A,E) \subseteq (G,E)$. Therefore, $(A,E)$is SNsg-closed set. If $(A,E) \supseteq L_R(X,E)$ and $(A,E) \neq U$, $SNcl(A,E) = U$ and the soft nano semi open sets containing (A,E) are (A,E) and U. Therefore, $SNscl(A,E) \subsetneq (A,E)$. Thus, any soft set $(A,E) \supseteq L_R(X,E)$ and $(A,E) \neq U$ is not SNsg-closed set.If $(A,E) \supseteq [L_R(X,E)]'$, then $SNscl(A,E) \subseteq (G,E)$ whenever (G,E) is soft nano semi open and $(G,E) \subseteq (A,E)$, since U is the only soft nano semi open set containing (A,E). Thus, if $(A,E) \supseteq [L_R(X,E)]'$, then (A,E) is SNsg-closed. When (A,E) has at least one element of $L_R(X,E)$ and exactly one element,say w of $[L_R(X,E)]'$, where $L_R(X,E)$ is not a singleton set, and $SNscl(A,E) = U$. But, $L_R(X,E) \cup (w,E)$ is a soft nano semi open set containing (A,E) and $SNscl(A,E) = U \subsetneq L_R(X,E) \cup (w,E)$. Therefore, (A,E) is not a SNsg-closed set. Thus, the only SNsg-closed set over U are soft subsets of $[L_R(X,E)]'$ and any soft set $(A,E) \supset [L_R(X,E)]'$.

Theorem 3.10. *If $L_R(X,E) = \phi$ and $U_R(X,E) \neq U$ then the only SNsg-closed sets over U are soft subsets of $U_R(X,E)$ and any soft set $(A,E) \supset [U_R(X,E)]'$.*

Proof : Suppose that $L_R(X,E) = \phi$ and $U_R(X,E) \neq U$, then $(\tau_R(X), U, E) = \{U, \phi, U_R(X,E)\}$. Here, ϕ and the soft sets (A,E) for which $(A,E) \supseteq U_R(X,E)$ are the only soft nano semi open sets over U. Thus, the soft sets (A,E) for which $(A,E) \subseteq [U_R(X,E)]'$ are the only soft nano semi closed sets over U. Suppose that $(A,E) \subseteq U_R(X,E)$, then $SNscl(A,E) = U$. But, $U_R(X,E)$ is a soft nano semi open set containing (A,E), for which $SNscl(A,E) \subsetneq (G,E)$. Then, (A,E) is not SNsg-closed set. If $(A,E) \supset U_R(X,E)$ and $(A,E) \neq U$, then $SNscl(A,E) = U$. But, for $(G,E) = (A,E)$, which is a soft nano semi open set containing itself, $SNscl(A,E) \subsetneq (G,E)$. Thus, (A,E) is not SNsg-closed set. Now, suppose that $(A,E) \subseteq [U_R(X,E)]'$, then $SNscl(A,E) = (A,E)$ and hence for every soft nano semi open set (G,E) such that $(A,E) \subseteq (G,E)$, $SNscl(A,E) \subseteq (G,E)$. Hence, (A,E) is SNsg-closed set. Suppose that $(A,E) \supset [U_R(X,E)]'$, then U is the only soft nano semi open set containing (A,E) and hence $SNscl(A,E) \subseteq (G,E)$ whenever (G,E) is soft nano semi open and $(A,E) \subseteq (G,E)$. Then, (A,E) is SNsg-closed set. Suppose that (A,E) has one element, say x of $U_R(X,E)$ and at least one element of $[U_R(X,E)]'$, then $SNscl(A,E) = U$. Since any soft set containing $U_R(X,E)$ is soft nano semi open over U, we have $(A,E) \cup U_R(X,E)$ and any soft set containing $(A,E) \cup U_R(X,E)$ are soft nano semi open sets containing (A,E). But,

$SNscl(A, E) = U \subsetneq (A, E) \cup U_R(X, E)$. Therefore, (A, E) is not SNsg-closed set over U. Thus, the only soft subsets of $[U_R(X, E)]'$ and any soft set $(A, E) \supset [U_R(X, E)]$ are SNsg-closed over U, when $L_R(X, E) = \phi$ and $U_R(X, E) \neq U$.

Theorem 3.11. *If $U_R(X, E) = U$ and $L_R(X, E) \neq \phi$ in a soft nano topological space $(\tau_R(X), U, E)$, then each soft subset of soft nano topological space is SNsg-closed set.*

Proof : If $U_R(X, E) = U$ and $L_R(X, E) \neq \phi$ in a soft nano topological space $(\tau_R(X), U, E)$, then $U, \phi, L_R(X, E), B_R(X, E)$ are the only soft sets over U which are soft nano open, soft nano semi open and soft nano semi closed sets over U. Suppose that $(A, E) \subseteq L_R(X, E)$, then $L_R(X, E)$ and U are the only soft nano semi open sets containing (A, E) and $SNscl(A, E) = L_R(X, E)$. Thus, $SNscl(A, E) \subseteq (G, E)$ whenever (G, E) is soft nano semi open set and $(A, E) \subseteq (G, E)$. Thus, (A, E) is SNsg-closed. Suppose that $(A, E) \subseteq B_R(X, E)$, then $B_R(X, E)$ and U are the only soft nano semi open sets containing (A, E) and $SNscl(A, E) = B_R(X, E)$. Thus, (A, E) is SNsg-closed set. Suppose that $(A, E) \supset L_R(X, E)$ or $(A, E) \supset B_R(X, E)$, then U is the only soft nano semi open set containing (A, E) and hence (A, E) is SNsg-closed set. Suppose (A, E) contains at least one element of $L_R(X, E)$ and one element of $B_R(X, E)$, then U is the only soft nano semi open set containing (A, E). Therefore, (A, E) is SNsg-closed set. Thus, each soft subset over U is SNsg-closed, if $U_R(X, E) = U$ and $L_R(X, E) \neq \phi$.

4 Application in decision making

Example 4.1. Suppose that a company wants to fill 2 positions. There are 6 candidates who applied for those positions. Let the set of candidates be $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and the set of parameters be $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
Where $e_1, e_2, e_3, e_4, e_5, e_6$ stands for experience, computer knowledge, training, higher education , good health and young age respectively. The following is the information about 6 candidates.

Table 1. Data in terms of soft set

E/U	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	0	1	0	1	1
e_2	0	1	1	1	0	0
e_3	1	1	1	1	1	1
e_4	0	1	0	1	0	1
e_5	1	1	1	1	1	0
e_6	0	1	0	0	0	1

Let $S = (F, E)$ be a soft set given as in table 1.
We have the set of equivalence classes is given by
$U/R = \{\{u_1, u_5\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_6\}\}$. Using the method discussed in [3], one can verify that the $CORE(SR) = \{e_1, e_3, e_4, e_5\}$ i.e. experience, training, higher education and good health are the key attributes to select a candidate. Based on the CORE set, let $X = \{u_2, u_4, u_6\}$ be the set of candidates selected in first round of the interview. Further, we refine the result obtained as the company needs only two candidates. For that the company considers two decision makers or experts: one of them is from the HR department and the other one from the Board of members. Let $G = \{T_1, T_2\}$ be the expert group. The primary evaluation of the expert group can be represented by the evaluation soft set (ν, G) over U given by $X_1 = \nu(T_1) = \{u_1, u_2, u_6\}, X_2 = \nu(T_2) = \{u_3, u_4, u_5\}$.
The tabular representation is given by

Let us consider $P = (U, G)$ as the soft approximation space.
Now, $\underline{v(T_1)} = \underline{apr(X_1)} = \{u_2, u_6\}$, $\underline{v(T_2)} = \underline{apr(X_2)} = \phi$ and

Table 2. Evaluation soft set

G/U	u_1	u_2	u_3	u_4	u_5	u_6
T_1	1	1	0	0	0	1
T_2	0	0	1	1	1	0

$\overline{v(T_1)} = \overline{apr(X_1)} = U$, $\overline{v(T_2)} = \overline{apr(X_2)} = U$

Tabular representations of soft sets $\underline{G_1}$ and $\overline{G_1}$ are given by Table 3 and Table 4 as follows:

Table 3. Tabular representation of soft set $\underline{G_1}$

G/U	u_1	u_2	u_3	u_4	u_5	u_6
T_1	0	1	0	0	0	1
T_2	0	0	0	0	0	0

Table 4. Tabular representation of soft set $\overline{G_1}$

G/U	u_1	u_2	u_3	u_4	u_5	u_6
T_1	1	1	1	1	1	1
T_2	1	1	1	1	1	1

Now we can define fuzzy sets $\mu_{\underline{G_1}}, \mu_{G_1}$ and $\mu_{\overline{G_1}}$ as follows :
$\mu_{\underline{G_1}}(u_k) = (1/2)\sum_{j=1}^{2} C_{\underline{v}(T_j)}(u_k)$, $\mu_{G_1}(u_k) = (1/2)\sum_{j=1}^{2} C_{v(T_j)}(u_k)$
$\mu_{\overline{G_1}}(u_k) = (1/2)\sum_{j=1}^{2} C_{\overline{v}(T_j)}(u_k)$, where $k = 1, 2, 3, ..., 6$. By calculating using the above formulae, we have

Now, $\mu_{\overline{G_1}} = \{(u_1, 1), (u_2, 1), (u_3, 1), (u_4, 1), (u_5, 1), (u_6, 1)\}$
$\mu_{G_1} = \{(u_1, 1/2), (u_2, 1/2), (u_3, 1/2), (u_4, 1/2), (u_5, 1/2), (u_6, 1/2)\}$
$\mu_{\underline{G_1}} = \{(u_1, 0), (u_2, 1/2), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 1/2)\}$
Consider a set of parameters as $D = \{H, M, L\}$, where H, M, L corresponds to "high confidence", "middle confidence" and "low confidence", respectively. Let weighting vector be $W = (0.25, 0.5, 0.25)$ and we define
$v(u_k) = 0.25\alpha(H)(u_k) + 0.5\alpha(M)(u_k) + 0.25\alpha(L)(u_k)$
From the table, the ranking of all the candidates with respect to their weighted evaluation values: $u_2 = u_6 > u_1 > u_3 = u_4 = u_5$
Therefore, finally the candidates $\{u_2, u_6\}$ are selected for the two positions.
Further, using the concept of similarity of soft sets, we can refine our conclusion. For this, we first construct a soft set model for selection of candidates based on the CORE set and the soft sets for the final selected candidates $\{u_2, u_6\}$.
Next, we find the similarity measure of these soft sets of final selected candidates with the soft set model of the selection process. If they are significantly similar, then we conclude that the person is possibly get selected in the interview.
Let the universal set contains only two elements y and n, which indicates 'yes' and 'no' respectively. Let $U = \{y, n\}$ and E be the set of parameters, $E = \{e_1, e_3, e_4, e_5\}$.
Let the model soft set for selection process be (G, E) and is constructed based on CORE set as follows,

Now, we construct the soft set model for the selected candidate u_2 with respect to parameters

Table 5. Fuzzy soft set (α, c) with weighted evaluation values

	u_1	u_2	u_3	u_4	u_5	u_6
L	1	1	1	1	1	1
M	1/2	1/2	0	0	0	1/2
H	0	1/2	0	0	0	1/2
$v(.)$	0.5	0.625	0.25	0.25	0.25	0.625

Table 6. Model soft set for selection process

(G,E)	e_1	e_3	e_4	e_5
y	1	1	1	1
n	0	0	0	0

from CORE set as follows:

Table 7. Soft set model for the candidate u_2 and u_6

(u_2, E)	e_1	e_3	e_4	e_5
y	0	1	1	1
n	1	0	0	0

(u_6, E)	e_1	e_3	e_4	e_5
y	1	1	1	0
n	0	0	0	1

Now, the concept of similarity of soft sets is used to refine the result obtained. Thus, we shall find the similarity measure of the soft sets (u_2, E), (u_6, E) with (G, E).
Thus, $S(G, u_2) = 3/4 = 0.75 > 1/2$, $S(G, u_6) = 3/4 = 0.75 > 1/2$
Therefore, they are significantly similar. Thus, we can conclude that both the candidates u_2, u_6 are possibly get selected for the two positions in the interview. Here one can note that, the selected candidates has common attributes $\{e_3, e_4, e_6\}$ that is training, higher education and young age, where as in first round the preference was given to attributes $\{e_1, e_3, e_4, e_5\}$ i.e. experience, training, higher education and good health by based on the CORE set.

5 Conclusion

In this paper, the notion of soft nano semi generalized closed sets in soft nano topological spaces is introduced. Also, the related properties are studied.In the example , firstly we calculate the CORE for the given data. Further, the concept of multi criteria group decision making is applied and the candidates are arranged with respect to their weighted evaluation values and hence the conclusion can be drawn. To refine the result obtained, we further use the concept of similarity of soft sets and hence the conclusion can be written in better and stronger way.

References

[1] K.V.Babita, J.J.Sunil, Soft set relations and functions, *Computers and Mathematics with Applications*, **60**, 1840-1849,(2010).

[2] Benchalli.S.S, P.G.Patil, Nivedita.S.Kabbur and J.Pradeepkumar, Weaker forms of soft nano open sets, *Journal of Computer and Mathematical Sciences*, **8(11)**, 589-599,(2017).

[3] Benchalli.S.S., P.G.Patil, Nivedita Kabbur and Pradeepkumar.J, On soft nano continuity in soft nano topological spaces and its applications, *Annals of Fuzzy Mathematics and Informatics*, **15(3)**,265âĂŞ283,(2018).

[4] Benchalli.S.S., P.G.Patil, Nivedita Kabbur and Pradeepkumar.J, On δ-Operation in Soft Nano Topological Spaces , *Journal of Computer and Mathematical Sciences*, **9(8)**,1001-1016,(2018).

[5] N.Cagman, S.Karatas and S.Enginoglu, Soft topology, *Computers and Mathematics with Applications*, **62**,351-358,(2011).

[6] F.Feng, Soft rough sets applied to multicriteria group decision making, *Annals of Fuzzy Mathematics and Informatics*, **2(1)**, 69-80, (2011).

[7] S.Hussain and B.Ahmad, Some properties of soft topological spaces, *Computers and Mathematics with Applications*, **62**, 4058-4067,(2011).

[8] P.Majumdar, S.K.Samanta, Similarity measure of soft sets, *New Mathematics and Natural Computation*, **4(1)**,1-12,(2008).

[9] D.Molodtsov, Soft set Theory-First results, *Computers and Mathematics with Applications*, **37**,19-31,(1999).

[10] M.Shabir and M.Naz, On soft topological spaces, *Computers and Mathematics with Applications*, **61**, 1786-1799,(2011).

[11] L.Thivagar, C.Richard, On Nano Continuity, *Mathematical Theory and Modeling*, **3(7)**,32-37,(2013).

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