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More on Neutrosophic Soft Nano Topological Space

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Abstract This paper is to combine the idea of neutrosophic soft set and nano topology dependent on neutrosophic right neighbourhood to present another numerical model named as redefined neutrosophic soft nano topology.Further, we will study the idea of neutrosophic soft approximation and their properties are derived and demonstrated with examples. Finally, we delineate that the traditional nano topological model can be seen as an exceptional instance of the proposed model in this paper.

1 Introduction

Smarandache [14] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The Notion of neutrosophic crisp set and topological spaces were the contribution of Salama and Alblowi [11][12]. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, databases, image processing and decision making problems. In 1999, Molodtsov [3] posited the concept of soft set as a new mathematical tool for dealing with uncertainties, that was free from the difficulties that have troubled the usual theoretical approaches. Maji et al. [4] first applied soft set to solve the decision making problems with the help of rough set approach. Shabir and Naz [6] introduced the notion of soft topological spaces,which are defined over an initial universe with a fixed set of parameters. They showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces based on nano topology were also done in [8] [9]. Neutrosophic soft set is defined by Maji [5] . The theory of nano topology is introduced by Thivagar et al.[7]. The main objective of this study is to introduce a new theory called neutrosophic soft nano topology.Further we will study the neutrosophic right neighbourhood is defined. In addition,we define the notion of neutrosophic soft approximation space. Moreover, their properties of neutrosophic soft($\mathscr{N}_S$ shortly) lower, neutrosophic soft ($\mathscr{N}_S$ shortly) upper approximations and neutrosophic soft($\mathscr{N}_S$ shortly) boundary region are included along with supported proofs and given examples.

The remainder of this paper is sorted out as follows. Some fundamental ideas required to build our work are quickly reviewed in section 2. In section 3, we investigate the idea of neutrosophic soft nano topology. The Section 4 finishes up the paper with certain properties on the redefined neutrosophic soft nano interior and the redefined neutrosophic soft nano closure.

2 Preliminaries

Here we mention some definitions which will be used in the rest of the paper.

Definition 2.1. [7]: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup\limits_{x\in\mathcal{U}} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in\mathcal{U}} \{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [7]: Let $\mathcal{U}$ be an universe, R an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Definition 2.3. [3] A soft set (F, A) denoted by F_A on the universe $\mathcal{U}$ and $P(\mathcal{U})$ is the power set of $\mathcal{U}$ is defined by the set of ordered pairs $F_A = \{(e, F(e)) : e \in E, F(e) \in P(\mathcal{U})\}$, where $F : E \to P(\mathcal{U})$ such that $F(e) = \emptyset$ if $e \notin A$. Here, F is called an approximate function of the soft set F_A.The set F(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter $e \in E$.

Definition 2.4. [14]: Let X be a non empty set. A neutrosophic set A having the form $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) respectively of each $x \in X$ to the set A. Also $^-0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^+$ for all $x \in X$.

Definition 2.5. [5]: Let $\mathcal{U}$ be an initial universe set and E be a set of parameters. Let $NS(\mathcal{U})$ denote the set of all neutrosophic soft set(NSs) of $\mathcal{U}$. Then,a neutrosophic soft set N over $\mathcal{U}$ is a set defined by a set valued function f_N representing a mapping $f_N : E \longrightarrow NS(\mathcal{U})$ where f_N is called approximate function of the neutrosophic soft set N. In otherwords,the neutrosophic soft set is a parameterized family of some elements of the set $NS(\mathcal{U})$ and therefore it can be written as a set of ordered pairs, $N = \{(e, \{\mu_{f_{N(e)}}(x), \sigma_{f_{N(e)}}(x), \gamma_{f_{N(e)}}(x)) : x \in \mathcal{U} : e \in E\}$, where $\mu_{f_{N(e)}}(x), \sigma_{f_{N(e)}}(x), \gamma_{f_{N(e)}}(x)) \in [0, 1]$ respectively called the degree of membership function , the degree of indeterminacy and the degree of non membership of $f_{N(e)}$. Since supremum of each μ, σ, γ is 1 so the inequality $0 \leq \mu_{f_{N(e)}}(x) + \sigma_{f_{N(e)}}(x) + \gamma_{f_{N(e)}}(x) \leq 3$ is obvious.

Definition 2.6. [5]: Let F_E, G_E be neutrosophic soft set over the universe $\mathcal{U}$. if $F_E = \{(x, \mu_{F_e}(x), \sigma_{F_e}(x), \gamma_{F_e}(x)) : x \in X\}$ and $G_E = \{(x, \mu_{G_e}(x), \sigma_{G_e}(x), \gamma_{G_e}(x)) : x \in X\}$ are two neutrosophic soft sets in $X \subseteq \mathcal{U}$ then

(i) $F_E \subseteq G_E$ if and only if $\mu_{F_e}(x) \leq \mu_{G_e}(x)$, $\sigma_{F_e}(x) \leq \sigma_{G_e}(x)$ and $\gamma_{F_e}(x) \geq \gamma_{F_e}(x)$ $\forall e \in E$ $\forall x \in X$.

(ii) $F_E = G_E$ if and only if $F_E \subseteq G_E$ and $G_E \subseteq F_E$.

(iii) $F_E^c = \{(e, < x, \gamma_{F_e}(x), 1 - \sigma_{F_e}(x), \mu_{F_e}(x) >) : x \in X : e \in E\}$ [Complement of F_E].

(iv) $F_E \cap G_E = \{(e, < x, \min\{\mu_{F_e}(x), \mu_{G_e}(x)\}, \min\{\sigma_{F_e}(x), \sigma_{G_e}(x)\}, \max\{\gamma_{F_e}(x), \gamma_{G_e}(x)\} >) : x \in X : e \in E\}$.

(v) $F_E \cup G_E = \{(e, < x, \max\{\mu_{F_e}(x), \mu_{G_e}(x)\}, \max\{\sigma_{F_e}(x), \sigma_{G_e}(x)\}, \min\{\gamma_{F_e}(x), \gamma_{G_e}(x)\} >) : x \in X : e \in E\}$.

Remark 2.7. [5]: Let X be a non empty neutrosophic set. We consider the neutrosophic empty set $0_{\mathcal{N}}$ as $0\mathcal{N} = \{(x, 0, 0, 1) : x \in \mathcal{U}\}$ and the neutrosophic whole set $1\mathcal{N}$ as $1\mathcal{N} = \{(x, 1, 1, 0) : x \in \mathcal{U}\}$.

Definition 2.8. :[2] Let H_A be a neutrosophic soft set on a universe $\mathcal{U}$. For any element $h \in \mathcal{U}$, a neutrosophic right neighborhood, with respect to $e \in A$ is defined as follows:
$h_e = \{h_i \in \mathcal{U} : \mu_e(h_i) \geq \mu_e(h), \sigma_e(h_i) \geq \sigma_e(h), \nu_e(h_i) \geq \nu_e(h)$.

Definition 2.9. :[2] Let H_A be a neutrosophic soft set on a universe $\mathcal{U}$. For any element $h \in \mathcal{U}$, a neutrosophic right neighborhood, with respect to all parameters A is defined as follows:
$h]_A = \cap\{h(e_i) : e_i \in A\}$.

3 Redefined Neutrosophic Soft Nano Topological Space

In this section, we introduce the notion of neutrosophic soft nano topology by means of neutrosophic soft approximations namely neutrosophic soft ($\mathscr{N}_S$ shortly) lower, $\mathscr{N}_S$ and $\mathscr{N}_S$ boundary region. Further, we also define their properties and examples are discussed.

Definition 3.1. Let H_A be a neutrosophic soft set on universe $\tilde{\mathcal{U}}$. Let $(\tilde{\mathcal{U}}, h]_A)$ be a neutrosophic soft approximation space,where $h]_A$ is a neutrosophic right neighbourhood relation on H_A.Then,$\mathscr{N}_S$ lower,$\mathscr{N}_S$ approximations and $\mathscr{N}_S$ boundary region of $H \subseteq \tilde{\mathcal{U}}$ are defined as follows:

(i) $\mathcal{L}_{\underline{N}}(H) = \bigcup\{h]_A : h \in \mathcal{U}, h]_A \subseteq H\}$.

(ii) $U_{\overline{N}}(H) = \bigcup\{h]_A : h \in \mathcal{U}, h]_A \cap H \neq \emptyset\}$.

(iii) $\mathcal{B}_N(H) = U_{\overline{N}}(H) - \mathcal{L}_{\underline{N}}(H)$.

Definition 3.2. Let $\tilde{\mathcal{U}}$ be an universe, $h]_A$ be a neutrosophic right neighbourhood relation on $\tilde{\mathcal{U}}$ and H_A be a neutrosophic soft set in $\tilde{\mathcal{U}}$ and if the collection $\tilde{\tau}_N(H) = \{\tilde{\phi}, \tilde{\mathcal{U}}, L_{\underline{N}}(H), U_{\overline{N}}(H), B_N(H)\}$ forms a topology then it is said to be a nano topology induced by neutrosophic soft set. We call $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ as the redefined neutrosophic soft nano topological space. The elements of $\tilde{\tau}_N(H)$ are called redefined neutrosophic soft nano open sets.

Example 3.3. [2] Let U be a universal set of bikes under consideration and E is the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence involving generalized neutrosophic words. Consider E = $\{$beautiful, cheap, expensive, wide, modern, ingoodrepair, costly, comfortable$\}$. In this case, to define a neutrosophic soft set means to point out beautiful bike, cheap bike and so on. Suppose that, there are five bikes in the universe U , given by U = $\{b_1, b_2, b_3, b_4, b_5\}$ and the set of parameters A = $\{e_1, e_2, e_3, e_4\}$, where each e_i is a specific criterion for bikes: e_1 stands for (beautiful),e_2 stands for (cheap), e_3 stands for (modern), e_4 stands for (comfortable) . Suppose that $F(beautiful) = \{< b_1, 0.6, 0.6, 0.2 >, < b_2, 0.4, 0.6, 0.6 >, < b_3, 0.6, 0.4, 0.24 >, < b_4, 0.6, 0.3, 0.3 >, < b_5, 50.8, 0.2, 0.3 >\}$ $F(cheap) = \{< b_1, 0.8, 0.4, 0.3 >, < b_2, 0.6, 0.2, 0.4 >, < b_3, 0.8, 0.1, 0.3 >, < b_4, 0.8, 0.2, 0.2 >, < b_5, 0.8, 0.3, 0.2 >\}$. $F(modern) = \{< b_1, 0.7, 0.4, 0.3 >, < b_2, 0.6, 0.4, 0.3 >, < b_3, 0.7, 0.2, 0.5 >, < b_4, 0.5, 0.2, 0.6 >, < b_5, 0.7, 0.3, 0.4 >\}$. $F(comfortable) = \{< b_1, 0.8, 0.6, 0.4 >, < b_2, 0.7, 0.6, 0.6 >, < b_3, 0.7, 0.6, 0.4 >, < b_4, 0.7, 0.5, 0.6 >, < b_5, 0.9, 0.5, 0.7 >\}$. In order to store a neutrosophic soft set in a following table, In this table, the entries are c_{ij} corresponding to the bike b_i and the parameter e_j , where c_{ij} = (true membership value of b_i , indeterminacy-membership value of b_i , falsity membership value of b_i) in F_{e_j}. Table 1, represents the neutrosophic soft set F_A as follows.

Table 1.Tabular representation of neutrosophic soft set F_A

$\mathcal{U}$	e_1	e_2	e_3	e_4
b_1	$(0.6, 0.6, 0.2)$	$(0.8, 0.4, 0.3)$	$(0.7, 0.4, 0.3)$	$(0.8, 0.6, 0.4)$
b_2	$(0.4, 0.6, 0.6)$	$(0.6, 0.2, 0.4)$	$(0.6, 0.4, 0.3)$	$(0.7, 0.6, 0.6)$
b_3	$(0.6, 0.4, 0.2)$	$(0.8, 0.1, 0.3)$	$(0.7, 0.2, 0.5)$	$(0.7, 0.6, 0.4)$
b_4	$(0.6, 0.3, 0.3)$	$(0.8, 0.2, 0.2)$	$(0.5, 0.2, 0.6)$	$(0.7, 0.5, 0.6)$
b_5	$(0.8, 0.2, 0.3)$	$(0.8, 0.3, 0.2)$	$(0.7, 0.3, 0.4)$	$(0.9, 0.5, 0.7)$

We can find the neutrosophic right neighbourhood as follows:

Neutrosophic right neighbourhoods

$e_i \in A$	e_1	e_2	e_3	e_4
$b_1(e_i)$	$\{b_1\}$	$\{b_1\}$	$\{b_1\}$	$\{b_1\}$
$b_2(e_i)$	$\{b_1, b_2\}$	$\{b_1, b_2, b_4, b_5\}$	$\{b_1, b_2\}$	$\{b_1, b_2, b_3\}$
$b_3(e_i)$	$\{b_1, b_3\}$	$\{b_1, b_3, b_4, b_5\}$	$\{b_1, b_3, b_5\}$	$\{b_1, b_3\}$
$b_4(e_i)$	$\{b_1, b_3, b_4\}$	$\{b_4, b_5\}$	$\{b_1, b_2, b_3, b_4, b_5\}$	$\{b_1, b_2, b_3, b_4\}$
$b_5(e_i)$	$\{b_5\}$	$\{b_5\}$	$\{b_1, b_5\}$	$\{b_5\}$

It follows that,$b_1](A) = \{b_1\}, b_2](A) = \{b_1, b_2\}, b_3](A) = \{b_1, b_3\}, b_4](A) = \{b_4\}$ and $b_5](A) = \{b_5\}$.

Let us consider $\tilde{\mathcal{U}} = \{b_1, b_2, b_3, b_4, b_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ and $A = \{e_1, e_2, e_3, e_4\}$then $b_1](A) = \{b_1\}, b_2](A) = \{b_1, b_2\}, b_3](A) = \{b_1, b_3\}$, $b_4](A) = \{b_4\}, b_5](A) = \{b_5\}$.Let $H = \{b_2, b_4\}$.Then $\mathcal{L}_{\underline{N}}(H) = \{b_4\}$,$U_{\overline{N}}(H) = \{b_1, b_2, b_4\}$ and $\mathcal{B}_N(H) = \{b_1, b_2\}$.Hence $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, \{b_4\}, \{b_1, b_2, b_4\}, \{b_1, b_2\}\}$

Proposition 3.4. *[2]*Let $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ be a redefined neutrosophic soft nano topological space with respect to $H_A$and let $P, Q \subseteq \tilde{\mathcal{U}}$.Then the following properties hold:

Proof. (a) $\mathcal{L}_{\underline{N}}(P) \subseteq P \subseteq U_{\overline{N}}(P)$.

(b) $\mathcal{L}_{\underline{N}}(\emptyset) = \emptyset = U_{\overline{N}}(\tilde{\phi})$.

(c) $\mathcal{L}_{\underline{N}}(\mathcal{U}) = \mathcal{U}$=$U_{\overline{N}}(P)$.

(d) $P \subseteq Q$ which implies that $\mathcal{L}_{\underline{N}}(P) \subseteq \mathcal{L}_{\underline{N}}(Q)$

(e) $P \subseteq Q$ which implies that $U_{\overline{N}}(P) \subseteq U_{\overline{N}}(Q)$.

(f) $\mathcal{L}_{\underline{N}}(P \cap Q) = \mathcal{L}_{\underline{N}}(P) \cap \mathcal{L}_{\underline{N}}(Q)$

(g) $\mathcal{L}_{\underline{N}}(P \cup Q) \supseteq \mathcal{L}_{\underline{N}}(P) \cup \mathcal{L}_{\underline{N}}(Q)$

(h) $U_{\overline{N}}(P \cap Q) \subseteq U_{\overline{N}}(P) \cap U_{\overline{N}}(Q)$.

(i) $U_{\overline{N}}(P \cap Q) = U_{\overline{N}}(P) \cap U_{\overline{N}}(Q)$.

□

Example 3.5. By Example 3.3 ,if $P = \{b_1\}$,then $\mathcal{L}_{\underline{N}}(P) = \{b_1\}$ and $U_{\overline{N}}(P) = \{b_1, b_2, b_3\}$.
Hence,$U_{\overline{N}}(P) \neq P and P \neq U_{\overline{N}}(P)$.

Example 3.6. By Example 3.3 ,if $P = \{b_2\}$ and $Q = \{b_2, b_4\}$,then $\mathcal{L}_{\underline{N}}(P) = \{\emptyset\}, \mathcal{L}_{\underline{N}}(P) = \{b_4\}$ and $U_{\overline{N}}(P) = \{b_1, b_2\}, U_{\overline{N}}(Q) = \{b_1, b_2, b_4\}$.
Hence,$\mathcal{L}_{\underline{N}}(P) \neq \mathcal{L}_{\underline{N}}(Q)$ and $U_{\overline{N}}(P) \neq U_{\overline{N}}(Q)$.

Example 3.7. By Example 3.3 ,if $P = \{b_1\}$ and $Q = \{b_2\}$,then $\mathcal{L}_{\underline{N}}(P) = \{\emptyset\}, \mathcal{L}_{\underline{N}}(P) = \{b_1\}$ and $U_{\overline{N}}(P) = \{b_1, b_2\}, U_{\overline{N}}(P) = \{b_1, b_2, b_4\}$.
Hence,$\mathcal{L}_{\underline{N}}(P) \neq \mathcal{L}_{\underline{N}}(Q)$ and $U_{\overline{N}}(P) \neq U_{\overline{N}}(Q)$.

Example 3.8. By Example 3.3 ,if $P = \{b_1, b_4\}$ and $Q = \{b_2, b_4\}$,then $U_{\overline{N}}(P) = \{b_1, b_2, b_3, b_4, \}$, $U_{\overline{N}}(Q) = \{b_1, b_2, b_4\} and U_{\overline{N}}(P \cap Q) = \{b_4\}$.
Hence,$U_{\overline{N}}(P \cap Q) = U_{\overline{N}}(P) \cap U_{\overline{N}}(Q)$.

Proposition 3.9. *[2]*Let H_A be a neutrosophic soft set on a universe $\mathcal{U}$ and $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ be a neutrosophic soft nano topological space with respect to H_A. Also let $P \subseteq \tilde{\mathcal{U}}$.Then the following properties hold:

(a) $\mathcal{L}_{\underline{N}}(\mathcal{L}_{\underline{N}}(P)) = \mathcal{L}_{\underline{N}}(P)$.

(b) $\mathcal{L}_{\underline{N}}(U_{\overline{N}}(P)) = U_{\overline{N}}(P)$.

Proof. (a) Let $Y = \mathcal{L}_{\underline{N}}(P)$ and $y \in Y = \bigcup\{h]_A : h \in \mathcal{U}, h]_A \subseteq X\}$.Then,there exists some $h]_A$ containing y,such that $h]_A \subseteq Y$. So $y \in \mathcal{L}_{\underline{N}}(Y)$.Hence,$Y \subseteq \mathcal{L}_{\underline{N}}(Y)$.Thus ,$\mathcal{L}_{\underline{N}}(P) \subseteq \mathcal{L}_{\underline{N}}(\mathcal{L}_{\underline{N}}(P))$.Also,from property(a) of Proposition 3.9 we have $\mathcal{L}_{\underline{N}}(P) \subseteq P$ by using property (d),of proposition 3.2, we get $\mathcal{L}_{\underline{N}}(\mathcal{L}_{\underline{N}}(P)) \subseteq \mathcal{L}_{\underline{N}}(P)$.Therefore $\mathcal{L}_{\underline{N}}(\mathcal{L}_{\underline{N}}(P)) = \mathcal{L}_{\underline{N}}(P))$.

(b) Let $Y = U_{\overline{N}}(X)$ by using property (a), of Proposition 3.9, we have $\mathcal{L}_{\underline{N}}(Y) \subseteq Y$.Conversely, let $y \in Y = \bigcup\{h]_A : h \in \mathcal{U}, h]_A \cap X \neq \emptyset\}$,hence there exist $, h]_A$containg y such that $y \in, h]_A$. It follows that $y \in \mathcal{L}_{\underline{N}}(P)$. Then $Y = \mathcal{L}_{\underline{N}}(P)$.But $Y = U_{\overline{N}}(X)$. Hence $\mathcal{L}_{\underline{N}}(U_{\overline{N}}(P)) = U_{\overline{N}}(P)$

□

Proposition 3.10. *[2]*Let H_A be a neutrosophic soft set on a universe $\tilde{\mathcal{U}}$ and $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ be a redefined neutrosophic soft nano topological space with respect to H_A.Moreover, let $P, Q \subseteq \tilde{\mathcal{U}}$.Then the following properties are

(a) $U_{\overline{N}}(U_{\overline{N}}(P)) = U_{\overline{N}}(P)$.

(b) $U_{\overline{N}}(\mathcal{L}_{\underline{N}})(P) = \mathcal{L}_{\underline{N}}(P)$.

(c) $\mathcal{L}_{\underline{N}}(P)^c = [U_{\overline{N}}(P)]^c$.

(d) $U_{\overline{N}}(P)^c = [\mathcal{L}_{\underline{N}}(P)]^c$.

(e) $\mathcal{L}_{\underline{N}}(P - Q) = \mathcal{L}_{\underline{N}}(P) - \mathcal{L}_{\underline{N}}(Q)$.

(f) $U_{\overline{N}}(P - Q) = U_{\overline{N}}(P) - U_{\overline{N}}(Q)$

Remark 3.11. The following counter example proves (a) of Proposition 3.10. By example 3.3,if $P = \{b_2\}$,then $U_{\overline{N}}(P) = \{b_1, b_2\}$ and $U_{\overline{N}}(U_{\overline{N}}(P)) = \{b_1, b_2, b_3\}$. Hence $U_{\overline{N}}(U_{\overline{N}}(P)) \neq U_{\overline{N}}(P)$.

Remark 3.12. The following counter example proves (b) of Proposition 3.10 From example 3.3, if $P = \{b_1\}$,then $(\mathcal{L}_{\underline{N}})(P) = \{b_1\}$ and $U_{\overline{N}}(\mathcal{L}_{\underline{N}})(P) = \{b_1, b_2, b_3\}$. Hence $U_{\overline{N}}(\mathcal{L}_{\underline{N}})(P) \neq U_{\overline{N}}(P)$.

Remark 3.13. The following counter example proves (c) of Proposition 3.10 . By example 3.3, if $P = \{b_2\}$, then $(\mathcal{L}_{\underline{N}})(P)^c = \{b_1, b_3, b_4, b_5\}$ and $[U_{\overline{N}}(P)]^c = \{b_3, b_4, b_5\}$. Therefore $\mathcal{L}_{\underline{N}}(P)^c \neq [U_{\overline{N}}(P)]^c$.

Remark 3.14. The following counter example proves (d) of Proposition 3.10. By example 3.3, if $P = \{b_1, b_3, b_4, b_5\}$,then $(\mathcal{L}_{\underline{N}})(P)^c = \{b_1, b_2\}$ and $[U_{\overline{N}}(P)]^c = \{b_2\}$. Therefore $U_{\overline{N}}(P)^c = [\mathcal{L}_{\underline{N}}(P)]^c$.

Remark 3.15. The following counter example proves (e),(f) of Proposition 3.10. By example 3.3, if $P = \{b_1, b_2\}$ and $Q = \{b_1, b_3\}$, then $(\mathcal{L}_{\underline{N}})(P) = \{b_1, b_2\}$ and $(\mathcal{L}_{\underline{N}})(Q) = \{b_1, b_3\}$,$(\mathcal{L}_{\underline{N}})(P - Q) = \emptyset$, $U_{\overline{N}}(P) = \{b_1, b_2, b_3\}$,$U_{\overline{N}}(Q) = \{b_1, b_2, b_3\}$,$U_{\overline{N}}(P - Q) = \{b_1, b_2\}$. Therefore $\mathcal{L}_{\underline{N}}(P - Q) \neq \mathcal{L}_{\underline{N}}(P) - \mathcal{L}_{\underline{N}}(Q)$. $U_{\overline{N}}(P - Q) \neq U_{\overline{N}}(P) - U_{\overline{N}}(Q)$

Remark 3.16. In redefined neutrosophic soft nano topological space, the $\mathscr{N}_S$ boundary cannot be empty. Since the difference between $\mathscr{N}_S$ upper and $\mathscr{N}_S$ approximations is defined here as the maximum and minimum of the values in the neutrosophic soft sets.

Definition 3.17. Let $\tilde{\mathcal{U}}$ be a non-empty finite universe, R an equivalence relation on $\tilde{\mathcal{U}}$ and H a neutrosophic subset on $\tilde{\mathcal{U}}$. Then we define the redefined neutrosophic soft nano topologies as follows:

(i) The collection $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N\}$, is the indiscrete redefined neutrosophic soft nano topology on $\mathcal{U}$.

(ii) If $\mathcal{L}_{\underline{N}}(H) = U_{\overline{N}}(H) = H$, then the redfined neutrosophic soft nano topology, $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, \mathcal{L}_{\underline{N}}(H), B_N(H)\}$.

(iii) If $\mathcal{L}_{\underline{N}}(H) = B_N(H)$, then $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, \mathcal{L}_{\underline{N}}(H), U_{\overline{N}}(H)\}$ is a redefined neutrosophic soft nano topology

(iv) If $U_{\overline{N}}(H) = B_N(H)$ then $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, U_{\overline{N}}(H), B_N(H)\}$.

(v) The collection $\tilde{\tau}_N(H) = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, \mathcal{L}_{\underline{N}}(H), U_{\overline{N}}(H), B_N(H)\}$ is the redefined discrete neutrosophic soft nano topology on $\tilde{\mathcal{U}}$.

4 Redefined Neutrosophic soft nano closure and interior

In this section we have investigated redefined neutrosophic soft($\mathscr{N}_S$ shortly nano closure and redefined neutrosophic soft($\mathscr{N}_S$ shortly) nano interior on redefined neutrosophic soft nano topological space. Based on this we also prove some properties.

Definition 4.1. If $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ is a redefined neutrosophic soft nano topological space with respect to neutrosophic soft subset of $\tilde{\mathcal{U}}$ and if G is any neutrosophic soft subset of $\tilde{\mathcal{U}}$, then the redefined neutrosophic soft nano interior of G is defined as the union of all redefined neutrosophic soft nano open subsets of G and it is denoted by $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(G)$. That is, $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(G)$ is the largest redefined neutrosophic soft nano open subset of G. The redefined neutrosophic soft nano closure of G is defined as the intersection of all redefined neutrosophic soft nano closed sets containing G and it is denoted by $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(G)$. That is, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(G)$ is the smallest redefined neutrosophic soft nano closed set containing G.

Example 4.2. Let us consider Example 3.3 and let $G = \{b_3, b_4\} \subseteq \mathcal{U}$. Then the redefined neutrosophic soft nano interior of G i.e $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(G)$=$\{b_4\}$. Also,the redefined neutrosophic soft nano closed sets are $[\tilde{\tau}_N(H)]^c = \{\tilde{\phi}_N, \tilde{\mathcal{U}}_N, \{b_1, b_2, b_3, b_5\}, \{b_3, b_5\}, \{b_3, b_4, b_5\}\}$.Then the neutrosophic soft nano closure of G ie.$N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(G) = \{b_3, b_4, b_5\}$.

Remark 4.3. Let $(\tilde{\mathcal{U}}, \tilde{\tau}_N(H))$ be a redefined neutrosophic soft nano topological space with respect to H, where H is a neutrosophic subset of $\tilde{\mathcal{U}}$. The redefined neutrosophic soft nano closed sets in $\mathcal{U}$ are $\tilde{\phi}_N$,$\tilde{\mathcal{U}}_N$, $(\mathcal{L}_{\underline{N}}(H))^C$, $(U_{\overline{N}}(H))^C$ and $(B_N(H))^C$.

Theorem 4.4. Let $(\mathcal{U}, \tau_N(H))$ be a redefined neutrosophic soft nano topological space with respect to H, where H is a neutrosophic subset of $\tilde{\mathcal{U}}$. Let I and J be neutrosophic subsets of $\tilde{\mathcal{U}}$. Then the following statements hold:

(i) $I \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$.

(ii) I is redefined neutrosophic soft nano closed if and only if $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) = I$.

(iii) $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(\tilde{\phi}_N) = \tilde{\phi}_N$ and $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(\tilde{\mathcal{U}}_N) = \tilde{\mathcal{U}}_N$.

(iv) $I \subseteq J \Rightarrow N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(v) $N_{\mathcal{S}_\mathcal{N}}cl(I \cup J) = N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(vi) $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)(I \cap J) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cap N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(vii) $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)) = N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$.

Proof. :

(i) By definition of redefined neutrosophic soft nano closure, $I \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$.

(ii) If I is redefined neutrosophic soft nano closed, then I is the smallest redefined neutrosophic soft nano closed set containing itself and hence $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$ = I. Conversely, if $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$ = I, then I is the smallest redefined neutrosophic soft nano closed set containing itself and hence I is redefined neutrosophic soft nano closed.

(iii) Since $\tilde{\phi}_N$ and $\tilde{\mathcal{U}}_N$ are redefined neutrosophic soft nano closed in $(\mathcal{U}, \tilde{\tau}_N(H))$, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(\tilde{\phi}_N) = \tilde{\phi}_N$ and $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)(\tilde{\mathcal{U}}_N) = \tilde{\mathcal{U}}_N$.

(iv) If $I \subseteq J$, since $J \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$, then $I \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$. That is, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$ is a redefined neutrosophic soft nano closed set containing A. But $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$ is the smallest redefined neutrosophic soft nano closed set containing A. Therefore, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(v) Since $I \subseteq I \cup J$ and $J \subseteq I \cup J$, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J)$ and $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J)$. Therefore, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J)$. By the fact that $I \cup J \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$, and since $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J)$ is the smallest redefined nano closed set containing $I \cup J$, so $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$. Thus, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cup J) = N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(vi) Since $I \cap J \subseteq I$ and $I \cap J \subseteq J$,$N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I \cap J) \subseteq N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I) \cap N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(J)$.

(vii) Since $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$ is redefined neutrosophic soft nano closed, $N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)) = N_{\mathcal{S}_\mathcal{N}}\widetilde{cl}(I)$.

□

Theorem 4.5. Let $(\mathcal{U}, \tau_N(H))$ be a redefined neutrosophic soft nano topological space with respect to H, where H is a neutrosophic subset of $\widetilde{\mathcal{U}}$. Let K and L be neutrosophic subsets of $\widetilde{\mathcal{U}}$. Then the following statements hold:

(i) $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) \subseteq K$ and $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K)$ is the largest open set.

(ii) $K \subset L \Rightarrow N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) \subset N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(L)$.

(iii) $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K)$ is an open redefined neutrosophic soft set ie.,$N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) \in \tau_N(H)$.

(iv) K is an redefined Neutrosophic soft nano open set iff $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) = K$

(v) $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K)) = N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K)$.

(vi) $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K \cap L) = N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) \cap N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(L)$.

(vii) $N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K) \cup N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(L) \subset N_{\mathcal{S}_\mathcal{N}}\widetilde{int}(K \cap L)$

Conclusion The neutrosophic soft set, which sums up the idea of exemplary sets, for example, fuzzy set,neutrosophic set and soft set.This paper can be presented by redefined neutrosophic soft nano topological space and contemplated their properties and models are given.Finally we explored the redefined neutrosophic soft nano interier and redefined neutrosophic soft nano closure are additionally examined.

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